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Research Paper

On N Job M Machine Flow Shop Schedule Minimizing Total Waiting Time Involving Transportation Time For Job Blocks

Dr. Neeru Chaudhary
Assistant professor Dewan V S Group of institutions, Meerut

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ABSTRACT:- little work has been done in minimizing total waiting time for obtaining on optimal schedule of jobs. The waiting time is to be important for scheduling job on the machines. The idea of minimizing the waiting time or cost may be an economical expression from industry/factory manager's point of view however this minimization of total waiting time is more important to economize .The operation of jobs on machine in order to reduce the final costs of product in the open market completion.One of the earliest results in flow shop scheduling theory is an algorithm by S.M. Jonson (1954) for scheduling jobs in two machine flow shop to minimizing the time at which all jobs are completed. The concept of equivalent job for a job block has been recently inducted into scheduling theory by Maggu and Das (1977).Maggu and Das (1980) first time introduced the concept of transportation time in sequencing n-jobs,2-machine problem and to obtain sequence considering the criteria to minimize total elapsed time .Maggu and Das (1980) originally established a theorem to provide us a decomposition algorithm for determining an optimal schedule for the 2- machine n-job flow shop scheduling problem involving transportation time of jobs .n-jobs two machine flow shop problem which have separately found techniques to minimize total impresses inventory time for all jobs was faintly introduced by Ikram(1977) and two machine flow shop problem in which concept of transportation time and job block have been separately introduced by Thommn Singh (1989) Mahabir Singh(1979).M-Machine flow shop scheduling model has been studied by Maggu,Das and Singhal (1981).Ikram&Thahir Husain(2006) consider a special type of n – jobs 2 machine sequence with criteria of obtaining optimal sequences.

Keywords:- Flowshop scheduling, Transportation Time, Job Blocks, Inventry, Sequence.

I. MATHEMATICAL ANALYSIS

Here a heuristic approach has been devised to give optimal solution for the problem.
1 – <u>Statement of the problem</u> n -Job proceed through two machines A and B in order AB with processing time A_i and B_i.These machines are to be set up at distant places so that a job after completion of machine A with $t_i>0$ for job I before it starts processing on the machine B.
There are two jobs α_k and α_{k+1} which are operated as job block equivalent to a signal job $\beta = (\alpha_k, \alpha_{k+1})$.The problem is to find an optimal schedule rule minimizing the total waiting time for all jobs. This problem must satisfy the following condition according to Ikram (1977) Max $(A_i + t_i) \le$ Min $(B_i + t_i)$
The problem is also investigating a numerical procedure to obtain a sequence of jobs. The optimal algorithm has been described as consequences of equivalent job for job block theorem due to Muggu and Das (1977) which is proved as follows:
1(A) – <u>Theorem</u>
<u>Equivalent job for a job block theorem due to Maggu and Dass(1977) in two machine flow shop problem</u>
In processing a schedule S = $(\alpha_1, \alpha_2, \alpha_3$ ---------α_{k-1}, α_k -----α_n) of n-job on two machine A & B in order AB with no passing allowed. The job block $(\alpha_1\alpha_{k-1})$ having processing times $(A\alpha_k, B\alpha_k, A\alpha_{k+1}, B\alpha_{k+1})$ is equivalent to the single job β where β is equivalent job.Now the processing times of job β on the machines A and B denoted respectively by A_i and B_i are given by
$A_\beta = A\alpha_k + A\alpha_{k+1}$ = min $(B\alpha_k, A\alpha_{k+1})$
$B_\beta = B\alpha_k + B\alpha_{k+1}$ = min $(B\alpha_k, A\alpha_{k+1})$
Proof: Let T_{pq} denote the completion time of job p on machine q for the given sequence s, we consider the following relations

$T\alpha_k B$ =Max $(T\alpha_k A, T\alpha_{k-1}B) + B\alpha_k$
=Max $(T\alpha_k A+B\alpha_k, T\alpha_{k-1}B + B\alpha_k)$
$T\alpha_{k+1}B$ =Max $(T\alpha_{k+1}A, T\alpha_k B) + B\alpha_{k+1}$
= Max $(T\alpha_{k+1}A, T\alpha_k A+B\alpha_k, T\alpha_{k+1}B+B\alpha_k) + B\alpha_{k+1}$
= Max $(T\alpha_{k+1}A+B\alpha_{k+1}, T\alpha_k A+B\alpha_k+B\alpha_{k+1}, T\alpha_{k+1}B+B\alpha_k + B\alpha_{k+1})$
Now $T\alpha_{k+1} = T\alpha_k A + A\alpha_{k+1}$
We have
$T\alpha_{k+1}B$ = Max$(T\alpha_k A+A\alpha_{k+1}+ B\alpha_{k+1}, T\alpha_k A+ B\alpha_k+B\alpha_{k+1}, T\alpha_{k+1}B+ B\alpha_k+ B\alpha_{k+1})$
$T\alpha_{k+2}B$= Max $(T\alpha_{k+2}A, T\alpha_{k+1}B) + B\alpha_{k+2}$
= Max $(T\alpha_{k+2}A, T\alpha_k A + A\alpha_{k+1}+ B\alpha_{k+1}, T\alpha_k A + B\alpha_k+B\alpha_{k+1}, T\alpha_{k+1}B+ B\alpha_k+ B\alpha_{k+1}) + B\alpha_{k+2}$
Now it is obvious that
$T\alpha_{k+2}A = T\alpha_k A + A\alpha_{k+1}+ A\alpha_{k+2}$
Hence
$T\alpha_{k+2}B$ =Max $(T\alpha_k A+ A\alpha_{k+1}+ A\alpha_{k+1}$)
$(T\alpha_k A+ A\alpha_{k+1}+ B\alpha_{k+1}, T\alpha_k A, B\alpha_k+ B\alpha_{k+1}$, $T\alpha_{k+1}B+ B\alpha_k + B\alpha_{\alpha k+1}) + B\alpha_{\alpha k+2}$
Since max
= $(T\alpha_k A+ A\alpha_{k+1}, B\alpha_{k+1}, T\alpha_k+ B\alpha_k+ B\alpha_{k+1})$
$T\alpha_k A$ + max $(A\alpha_{\alpha k+1}, B\alpha_k) + B\alpha_k$
Therefore, we have
$T\alpha_{k+2}B$=max $[(T\alpha_k A+ A\alpha_{k+1} + A\alpha_{k+2}, T\alpha_k A$ + max $(A\alpha_{k+1}, B\alpha_k) + B\alpha_{k+1}, T\alpha_{k+1}B+ B\alpha_k+ B\alpha_{k+1}] + B\alpha_{k+2}$----------------(1)

$T\alpha_{k+2}A = T\alpha_{k-1}A + A\alpha_k + A\alpha_{k+1}+ A\alpha_{k+2}$
$T\alpha_k A+ A\alpha_{k+1} + A\alpha_{k+2}$------------------(2)
Now a sequence S' as
S'= $(\alpha_1 ,\alpha_2,\alpha_3$ ---------$\alpha_{k+1} ,\alpha_{k+2}$ -----$\alpha_n)$
$A_\beta = A\alpha_k+A\alpha_{k+1}$-C -------------------(3)
$B_\beta = B\alpha_k+B\alpha_{k+1}$ –C -------------------(4)
Let $T'_\beta B$ denote the completion time of job p on machine q in the sequence S' so that
$T'_\beta B$ =max $(T'_\beta A, T'_{\alpha_{k-1}}B) + B_\beta$
=max $(T'_\beta A+B_\beta, T'_{\alpha_{k-1}}B + B_\beta)$
$T'_{\alpha_{k+2}}B$ =max $(T'_{\alpha_{k+1}}A' \, T'_\beta B) + B\alpha_{k+2}$
= max $(T'_{\alpha_{k+2}}A , T'_\beta A + B_\beta, T'_{\alpha_{k+1}}B+ B_\beta) \, B\alpha_{k+2}$
-------------------(5)Now it is obvious that
$T'_{\alpha_{k+2}}A = T'_{\alpha_{k+1}}A+A_\beta + A\alpha_{k+2}$
$T'_{\alpha_{k-1}}+ A\alpha_k+ A\alpha_{k+1}$-C+ $A\alpha_{k+2}$
(As $T'_{\alpha_{k-1}}A=T_{\alpha_{k-1}}A$) -------------------(6)

$T'_\beta A = T'_{\alpha_{k-1}}A + A_\beta$
$=T'_{\alpha_{k-1}}A + A\alpha_k+ A\alpha_{k+1}$-C -------------------(7)
$T'_\beta A = T_{\alpha_k}A + A\alpha_{k+1}$-C
Using (3), (4), (5), (6), (7) we have
$T'_{\alpha_{k+2}}B$ = Max $(T_{\alpha_k}A+ A\alpha_{k+1}$-C + $A\alpha_{k+2},$
$T_{\alpha_k}A + A\alpha_{k+1}$-C+ $B\alpha_k + B\alpha_{k+1}$-C,
$T'_{\alpha_{k-1}}+ B\alpha_k+ B\alpha_{k+1}$-C) + $B\alpha_{k+2}$---------------------(8)
Let C=Min $(A\alpha_{k+1}, B\alpha_k)$ ---------------------(9)
Then
$A\alpha_{k+1}$-C+ $B\alpha_k= A\alpha_{k+1}$-Min $(A\alpha_{k+1}, B\alpha_k)$
=Max $(A\alpha_{k+1}, B\alpha_k)$ --------------------(10)

$T'_{\alpha_{k+2}}B = T_{\alpha_{k-1}}B$-------------------(11)
Hence from (8), (9), (10) & (11) we have
$T'_{\alpha_{k+2}}B$ = Max $(T_{\alpha_k}A+ A\alpha_{k+1} + A\alpha_{k+2}$-C)
$=T_{\alpha_k}A$+ Max $(A\alpha_{k+1}, B\alpha_k) + B\alpha_{k+1}$-C
$=T_{\alpha_{k-1}}B + B\alpha_k+ B\alpha_{k+1}$-C+ $B\alpha_{k+2}$

$=\text{Max}\,(T_{\alpha_k}A + A\alpha_{k+1} + A\alpha_{k+2}, T_{\alpha_k}A$

$\text{Max}\,(A\alpha_{k+1}, B\alpha_k) + B\alpha_{k+1}, T_{\alpha_{k-1}}B + B\alpha_k + B\alpha_{k+1}) + B\alpha_{k+2} - C$

--------------------(12)

Hence from (1) & (12) we have

$T'_{\alpha_{k+2}}B = T_{\alpha_{k+1}}B - C$ ----------------(13)

From (2) & (6) it is obvious that

$T_{\alpha_{k-2}}A = T_{\alpha_{k+2}}A - C$

From equations (13) & (14) it is clear that replacement of job block (α_k, α_{k+1}) in S by job β decreases the completion time on both the machine of the later job α_{k+2} by a constant C in S' as compared for the job α_{k+2} in S. Let T & T' both completion times of sequences S and S' respectively. Then from the above discussion it is that T'=T-C, hence where β replaces jobs (α_k, α_{k+1}) in any sequence S to produce.

A new sequence S' then completion times on all the machines are changed by a value which is independent of the particular sequence hence the substitution does not change the relative merit of different sequence hence job β is equivalent job for job block (α_k, α_{k+1}).

1(B) – <u>Theorem</u>

Let n jobs 1,2,3, --------n be processed through two machines A,B in over AB with no passing allowed that satisfying processing times structural relationship.

$$\text{Max}\ t_{iA} \leq \text{Min}\ t_{iB}$$

Where t_{ix} is the processing time of job I on machine X(X=A,B);(I, j=1,2,3,----n) then for any n job sequences S:$\alpha_1, \alpha_2, \alpha_3$ --------α_n the total waiting T_w is given by

$$T_w = nt\alpha_1 A + \sum_{r-1}^{n-1}(n-r)X_{\alpha r} - \sum_{i-1}^{n} t_{iA}$$

$X_{\alpha r} = t_{\alpha r}B - t_{\alpha r}A, \quad \alpha_r \in (1,2,3,---n)$

Lemma (i) with the notation of theorem for the n jobs sequence

$$S' = (\alpha_1, \alpha_2, \alpha_3 \text{ --------} \alpha_k \text{---------} \alpha_n)$$

$$T_{\alpha n}B = t\alpha_1 A + t\alpha_1 B + t\alpha_2 B \text{----------} t\alpha_n B$$

Where T_{pq} is the completion time of job p on machine q.

Lemma (ii) with the same notation as that of Lemma (i) we now prove that for n job sequence

$\alpha_1, \alpha_2, \alpha_3$ --------α_k---------α_n

$$Y\alpha_1 = 0$$

$Y_{\alpha k} = t\alpha_1 A + \sum_{r-1}^{k-1} X_{\alpha r} - X\alpha_k A \quad$ (K=2, 3, 4 -------n)

Where $Y_{\alpha k}$ is the waiting time of job αk for sequence

$\alpha_1, \alpha_2, \alpha_3$ --------α_k---------α_n

Now we have

$Y\alpha_1 = U\alpha_1 B - T\alpha_1 A = T\alpha_1 A - T\alpha_1 A = 0$

$Y_{\alpha k} = U\alpha_k B - T\alpha_k A$

$Y_{\alpha k} = \max\,(T\alpha_{k-1}B, T\alpha_k A) - T\alpha_k A$

$= \max\,(t\alpha_1 A + t\alpha_1 B + t\alpha_2 B + \text{----------} t\alpha_{k-1}B, t\alpha_1 A + t\alpha_2 A + t\alpha_3 A + \text{----------} t\alpha_k A$

$-(t\alpha_1 A + t\alpha_2 A + t\alpha_3 A + \text{----------} t\alpha_k A)$

$= t\alpha_1 A + t\alpha_1 B + t\alpha_1 B + \text{----} t\alpha_{k-1}B - t\alpha_1 A - t\alpha_2 A - t\alpha_3 A \text{-----------} t\alpha_k A$

$= t\alpha_1 A + (t\alpha_1 B - t\alpha_1 A) + (t\alpha_2 B - t\alpha_2 B) \text{-----------} + (t\alpha_{k-1}B - t\alpha_{k-1}A) - t\alpha_k A$

$= t\alpha_1 A + \sum_{r-1}^{k-1}(t\alpha_r B - t\alpha_r A) - t\alpha_k A$

$= t\alpha_1 A + \sum_{r-1}^{k-1} X_{\alpha r} - t\alpha_k A$

Now we able to proof of the main theorem as follow

From lemma (ii)

$$Y\alpha_1 = 0$$

When k=2, k-1=1

$Y\alpha_2 = t\alpha_1 A + \sum_{r-1}^{k-1} X_{\alpha r} - t\alpha_2 A$

$= t\alpha_1 A + X_{\alpha 1} - t\alpha_2 A$

When k=n, k-1=n-1

$Y\alpha_n = t\alpha_1 A + \sum_{r-1}^{k-1} X_{\alpha r} - t\alpha_2 A$

$Y\alpha_n = t\alpha_1 A + X_{\alpha 1} \text{-------------} X_{\alpha n} - t\alpha_n A$

Hence total waiting time

$T_w = Y\alpha_1 + Y\alpha_2 + \text{--------------} Y\alpha_n$

T_w =0 + ($t\alpha_1 A + X_{\alpha 1}$- $t\alpha_2 A$) + ($t\alpha_1 A + X_{\alpha 1} + X_{\alpha 2} + X_{\alpha 3} - t_1\alpha_3 A$) + ($t\alpha_1 A$ + $X_{\alpha 1} + X_{\alpha 2} + X_{\alpha 3} - X_{\alpha n-1} - t\alpha_n A$)

T_w = ($t\alpha_1 A + t\alpha_1 A + t\alpha_1 A$ ------(n-1) times) + ($X_{\alpha 1} + X_{\alpha 1}$ +-------------(n-1)times) +($X_{\alpha 2} + X_{\alpha 2}$____________$(n-2)+(n-2)$times)+ - + $X_{\alpha n-1}$ −($t\alpha_2 A + t\alpha_3 A - - - - - t\alpha_n A$)

T_w = (n-1) $t\alpha_1 A + (n-1)X_{\alpha 1} + (n-2)X_{\alpha 2}$ + ------+$X_{n-1} - \sum(t_{iA} - t_{\alpha 1}A)$

T_w =$nt\alpha_1 A + \sum_{r-1}^{n-1}(n-r)X_{\alpha r} - \sum t_{iA}$

II. NOTATIONS

n = number of jobs
m = machines A, B
T_{ij} = processing time of i_{th} job on j_{th} the machine
A_i = processing time of i_{th} job on machine A
B_i = processing time of i_{th} job on machine B
B = Equivalent job for the given job- block (α_k , α_{k+1})
T_w = Total waiting time

III. HEURISTIC APPROACH

To solving the problem of minimizing the total waiting time for all jobs we can use following steps.

Step 1:- By the definition of Maggu and Dass (1977) or operator oiw to find processing time for the job β where β is the equivalent job for the given job blocks (α_k , α_{k+1}))

Step 2:- Define t_β= ($t_{\alpha k}, t_{\alpha k+1}$)

Step 3:- Define a new problem from the step 1 by replacing the two jobs (α_k , α_{k+1}) by the single job β with processing time as in step 1 and t_β defined as per step 2.

Step 4:- Define a new problem from step 3,with processing time A_i',B_i given by

$$A_i' = A_i + t_i$$

$$B_i' = B_i + t_i$$

Step 5:- Use Ikram method (1977) to solve the new reduced problem in step 4 to find an optimal or near optimal schedule is minimizing the total waiting time for all jobs.

Step 6:-The optimal schedule in step 5 is optimal or near optimal for the original problem when β is replaced back by the jobs (α_k , α_{k+1}) if there are more than one optimal or near optimal sequence then choose that sequence as optimal or near optimal sequence which corresponds to the most minimum total waiting time T_w for all jobs. Now we can find total waiting time for all jobs by using usual method.

3.1 – Numerical illustration

Example: - Consider the following job scheduling problem with processing time the matrix as follows

Job	Machine	Transportation time	Machine
i	A_i	t_i	B_i
1	3	4	12
2	6	3	18
3	9	2	21
4	12	1	1
5	10	2	16

Table 1.1

With equivalent job β defined by β =(2,5)

By Step 1 β =(2,5)= (α_k , α_{k+1})

Where $\alpha_\beta = 2, \alpha_{k+1} = 5$

Using Maggu and Dass (1977) techniques to find processing time for job blocks as follows

$$A_\beta = A_2 + A_5 - \text{Min}(B_2, A_5)$$

= 6 + 15 – Min (18, 10)

= 21-10=11

$$A_\beta = 11$$

$B_\beta = B_2 + B_5 - \text{Min}(B_2, A_5)$

= 18 + 16 – Min (18, 10)

= 34 – 10 = 14 $B_\beta = 14$

By Step 2
The new reduced problem is

$$T_\beta = \mathrm{Max}\,(t_2, t_3)$$

$= \mathrm{Max}\,(3, 2)$
$= 3$

By Step 3: The new problem is

Job	Machine	Transportation time	Machine
i	A_i	t_i	B_i
1	3	4	12
□	11	3	18
3	9	2	21
4	12	1	15
5	10	2	16

Table 1.2

By step 4: The problem with new processing time A_i', B_i' can be defined from step 3 as follows

Job	Machine	Transportation time	Machine
i	$A_i' = A_i + t_i$	t_i	$B_i' = B_i + t_i$
1	7	4	16
□	14	3	21
3	11	2	23
4	13	1	16
5	12	2	16

Table 1.3

By Step 5: Using Ikram method to find the optimal sequence for the problem

Job	Machine	Machine	
i	$A_i' = A_i + t_i$	B_i'	$X_i = B_i' - A_i'$
□	14	21	7
1	7	16	9
3	11	23	12
4	13	16	3

Table 1.4

The optimal schedule can be obtained 1, 3, 4, β

Min (A_i') = 7

1, 3, 4, β is required optimal sequence

Job	Machine	Machine	
i	A_i	B_i	$X_i = B_i - A_i$
1	7	16	9
3	11	23	12
4	13	16	3
2	14	21	7
5	12	18	6

By Step 6:
Total waiting time for sequence S: $\alpha_1, \alpha_2, \alpha_3$ --------α_k---------α_n is now given by modified Ikram formula

$$T_w = nA\alpha_1 + Z\alpha_{11} + Z\alpha_{22} + Z\alpha_{33} - - - Z\alpha_{n-1,n-1}$$

$$\sum A_i = 40$$

Therefore optimal schedule 1, 3, 4, 2, 5

Here $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5 = 1,3,4,2,5$

$$T_w = 35 + 36 + 6 + 7 - 40$$

= 70

IV. CONCLUSION

The model presented in the section is near to real time of left communication Our study provides a guideline to be system based on optimal continue policy.this problem may be generalized by taking 2-machine,5 jobs.

*Corresponding Author: Dr. Neeru Chaudhary

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