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Research Paper

A Comparative Study of One-Sample t-Test Under Fuzzy Environments

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ABSTRACT:- *This paper proposes a method for testing hypotheses over one sample t-test under fuzzy environments using trapezoidal fuzzy numbers (tfns.). In fact, trapezoidal fuzzy numbers have many advantages over triangular fuzzy numbers as they have more generalized form. Here, we have approached a new method where trapezoidal fuzzy numbers are defined in terms of alpha level of trapezoidal interval data and based on this approach, the test of hypothesis is performed. More over the proposed test is analysed under various types of trapezoidal fuzzy models such as Alpha Cut Interval, Membership Function, Ranking Function, Total Integral Value and Graded Mean Integration Representation. And two numerical examples have been illustrated. Finally a comparative view of all conclusions obtained from various test is given for a concrete comparative study.*

Keywords:- *Trapezoidal Fuzzy Numbers (tfns./TFNS.), Alpha Cut, Test of Hypothesis, Confidence Limits, One-sample t-Test, Ranking Function, Total Integral Value (TIV), Graded Mean Integration Representation (GMIR).*
AMS Mathematics Subject Classification (2010): 62A86, 62F03, 97K80

I. INTRODUCTION

Test of hypotheses concerning some population parameters are an important part of statistical analysis. In traditional mean testing, the observation of sample is generally assumed to be a crisp value and to satisfy some relevant assumptions. However, in the real life, the data sometimes cannot be recorded or collected precisely. The statistical hypotheses testing under fuzzy environments has been studied by many authors using the fuzzy set theory concepts introduced by Zadeh [38]. Viertl [33] investigated some methods to construct confidence intervals and statistical tests for fuzzy data. Wu [37] proposed some approaches to construct fuzzy confidence intervals for the unknown fuzzy parameter. A new approach to the problem of testing statistical hypotheses is introduced by Chachi et al. [15]. Asady [9] introduced a method to obtain the nearest trapezoidal approximation of fuzzy numbers. Gajivaradhan and Parthiban analysed one sample t-test using alpha cut interval method using trapezoidal fuzzy numbers [17]. Abhinav Bansal [5] explored some arithmetic properties of arbitrary trapezoidal fuzzy numbers of the form (a, b, c, d). Moreover, Liou and Wang ranked fuzzy numbers with total integral value [23]. Wang et al. presented the method for centroid formulae for a generalized fuzzy number [35]. Iuliana Carmen BĂRBĂCIORU dealt with the statistical hypotheses testing using membership function of fuzzy numbers [21]. Salim Rezvani analysed the ranking functions with trapezoidal fuzzy numbers [28]. Wang arrived some different approach for ranking trapezoidal fuzzy numbers [35]. Thorani et al. approached the ranking function of a trapezoidal fuzzy number with some modifications [29]. Salim Rezvani and Mohammad Molani presented the shape function and Graded Mean Integration Representation for trapezoidal fuzzy numbers [27]. Liou and Wang proposed the Total Integral Value of the trapezoidal fuzzy number with the index of optimism and pessimism [23].

In this paper, we propose a new statistical fuzzy hypothesis testing of one-sample t-test in which the designated samples are in terms of fuzzy (trapezoidal fuzzy numbers) data. Another idea in this paper is, when we have some vague data about an experiment, what can be the result when the centroid point/ranking grades of those imprecise data are employed in hypothesis testing? For this reason, we have used the centroid/ranking grades of trapezoidal fuzzy numbers (tfns.) in hypothesis testing. In fact, we would like to counter an argument

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that the α-cut interval method can be general enough to deal with one-sample t-test under fuzzy environments. And for better understanding, the proposed fuzzy hypothesis testing technique is illustrated with two numerical examples at each models. Finally a tabular form of all conclusions obtained from various test is given for a concrete comparative study. And the same concept can also be used when we have samples in terms of triangular fuzzy numbers [10].

II. PRELIMINARIES

Definition 2.1. Generalized fuzzy number

A generalized fuzzy number $\widetilde{A}=(a, b, c, d; w)$ is described as any fuzzy subset of the real line $\mathbb{R}$, whose membership function $\mu_{\widetilde{A}}(x)$ satisfies the following conditions:

i. $\mu_{\widetilde{A}}(x)$ is a continuous mapping from $\mathbb{R}$ to the closed interval $[0, \omega],\ 0\le\omega\le 1$,
ii. $\mu_{\widetilde{A}}(x) = 0$, for all $x \in (-\infty, a]$,
iii. $\mu_L(x) = L_{\widetilde{A}}(x)$ is strictly increasing on $[a, b]$,
iv. $\mu_{\widetilde{A}}(x) = \omega$, for all $[b, c]$, as ω is a constant and $0<\omega\le 1$,
v. $\mu_R(x) = R_{\widetilde{A}}(x)$ is strictly decreasing on $[c, d]$,
vi. $\mu_{\widetilde{A}}(x) = 0$, for all $x \in [d, \infty)$ where a, b, c, d are real numbers such that $a < b \le c < d$.

Definition 2.2. A fuzzy set $\widetilde{A}$ is called ***normal*** fuzzy set if there exists an element (member) 'x' such that $\mu_{\widetilde{A}}(x)=1$. A fuzzy set $\widetilde{A}$ is called ***convex*** fuzzy set if $\mu_{\widetilde{A}}(\alpha x_1 + (1-\alpha)x_2) \ge \min\{\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)\}$ where $x_1, x_2 \in X$ and $\alpha \in [0, 1]$. The set $\widetilde{A}_\alpha = \{x \in X / \mu_{\widetilde{A}}(x) \ge \alpha\}$ is said to be the α-cut of a fuzzy set $\widetilde{A}$.

Definition 2.3. A fuzzy subset $\widetilde{A}$ of the real line $\mathbb{R}$ with ***membership function*** $\mu_{\widetilde{A}}(x)$ such that $\mu_{\widetilde{A}}(x) : \mathbb{R} \to [0, 1]$, is called a fuzzy number if $\widetilde{A}$ is normal, $\widetilde{A}$ is fuzzy convex, $\mu_{\widetilde{A}}(x)$ is upper semi-continuous and $\mathrm{Supp}(\widetilde{A})$ is bounded, where $\mathrm{Supp}(\widetilde{A}) = \mathrm{cl}\{x \in \mathbb{R} : \mu_{\widetilde{A}}(x) > 0\}$ and 'cl' is the closure operator.

It is known that for a ***normalized tfn.*** $\widetilde{A} = (a, b, c, d; 1)$, there exists four numbers $a, b, c, d \in \mathbb{R}$ and two functions $L_{\widetilde{A}}(x), R_{\widetilde{A}}(x) : \mathbb{R} \to [0, 1]$, where $L_{\widetilde{A}}(x)$ and $R_{\widetilde{A}}(x)$ are non-decreasing and non-increasing functions respectively. And its membership function is defined as follows:

$\mu_{\widetilde{A}}(x) = \{L_{\widetilde{A}}(x)=(x-a)/(b-a) \text{ for } a \le x \le b;\ 1 \text{ for } b \le x \le c;\ R_{\widetilde{A}}(x)=(x-d)/(c-d) \text{ for } c \le x \le d$ and 0 otherwise}. The functions $L_{\widetilde{A}}(x)$ and $R_{\widetilde{A}}(x)$ are also called the ***left*** and ***right side*** of the fuzzy number $\widetilde{A}$ respectively [16]. In this paper, we assume that $\int_{-\infty}^{\infty} \widetilde{A}(x)\,dx < +\infty$ and it is known that the α-cut of a fuzzy number is $\widetilde{A}_\alpha = \{x \in \mathbb{R} / \mu_{\widetilde{A}}(x) \ge \alpha\}$, for $\alpha \in (0, 1]$ and $\widetilde{A}_0 = \mathrm{cl}\left(\bigcup_{\alpha\in(0,1]} \widetilde{A}_\alpha\right)$, according to the definition of a fuzzy number, it is seen at once that every α-cut of a fuzzy number is a closed interval. Hence, for a fuzzy number $\widetilde{A}$, we have $\widetilde{A}(\alpha) = [\widetilde{A}_L(\alpha), \widetilde{A}_U(\alpha)]$ where $A_L(\alpha) = \inf\{x \in \mathbb{R} : \mu_{\widetilde{A}}(x) \ge \alpha\}$ and $A_U(\alpha) = \sup\{x \in \mathbb{R} : \mu_{\widetilde{A}}(x) \ge \alpha\}$. The left and right sides of the fuzzy number $\widetilde{A}$ are strictly monotone, obviously, $\widetilde{A}_L$ and $\widetilde{A}_U$ are inverse functions of $L_{\widetilde{A}}(x)$ and $R_{\widetilde{A}}(x)$ respectively. Another important type of fuzzy number was introduced in [11] as follows:

Let $a, b, c, d \in \mathbb{R}$ such that $a < b \leq c < d$. A fuzzy number $\widetilde{A}$ defined as $\mu_{\widetilde{A}}(x): \mathbb{R} \to [0, 1]$,

$$\mu_{\widetilde{A}}(x) = \left(\frac{x - a}{b - a}\right)^n \text{ for } a \leq x \leq b;\ 1 \text{ for } b \leq x \leq c;\ \left(\frac{d - x}{d - c}\right)^n \text{ for } c \leq x \leq d;\ 0 \text{ otherwise}$$

where $n > 0,$ is denoted by $\widetilde{A} = (a, b, c, d)_n$. And $L(x) = \left(\frac{x - a}{b - a}\right)^n$, $R(x) = \left(\frac{d - x}{d - c}\right)^n$ can also be termed as left and right spread of the tfn. [Dubois and Prade in 1981].

If $\widetilde{A} = (a, b, c, d)_n$, then [1-4],

$$\widetilde{A}_\alpha = \left[\widetilde{A}_L(\alpha),\ \widetilde{A}_U(\alpha)\right] = \left[a + (b - a)\sqrt[n]{\alpha},\ d - (d - c)\sqrt[n]{\alpha}\right];\ \alpha \in [0, 1].$$

When $n = 1$ and $b = c$, we get a triangular fuzzy number. The conditions $r = 1, a = b$ and $c = d$ imply the closed interval and in the case $r = 1, a = b = c = d = t$ (some constant), we can get a crisp number 't'. Since a trapezoidal fuzzy number is completely characterized by $n = 1$ and four real numbers $a \leq b \leq c \leq d$, it is often denoted as $\widetilde{A} = (a, b, c, d)$. And the family of trapezoidal fuzzy numbers will be denoted by $F^T(\mathbb{R})$. Now, for $n = 1$ we have a normal trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d)$ and the corresponding α - cut is defined by

$$\widetilde{A}_\alpha = \left[a + \alpha(b - a),\ d - \alpha(d - c)\right];\ \alpha \in [0, 1] --- (2.4).$$

And we need the following results which can be found in [20, 22].

Result 2.1. Let $D = \{[a, b],\ a \leq b \text{ and } a, b \in \mathbb{R}\}$, the set of all closed, bounded intervals on the real line $\mathbb{R}$.

Result 2.2. Let $A = [a, b]$ and $B = [c, d]$ be in D. Then $A = B$ if $a = c$ and $b = d$.

III. ONE – SAMPLE t-TEST FOR SINGLE MEAN

In case if we want to test (i) if a random (small) sample $x_i\ (i = 1, 2, ..., n)$ of size n < 30 has been drawn from a normal population with a specified mean, say μ_0 or (ii) if the sample mean differs significantly from the hypothetical value μ_0 of the population mean, then under the null hypothesis H_0:

(a) The sample has been drawn from the population with mean μ_0 or

(b) There is no significant difference between the sample mean $\bar{x}$ and the population mean μ_0, in this case, **the test statistic** is given by

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \text{ where } \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \text{ and } s^2 = \frac{1}{n - 1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$

follows Student's t-distribution with $(n - 1)$ degrees of freedom.

We now compare the calculated value of 't' with the tabulated value at certain level of significance. Let t_α be the tabulated value at α level of significance and 't' be the calculated value and we set the null hypothesis as $H_0: \mu = \mu_0$ and alternative hypothesis as below:

Alternative Hypothesis	**Rejection Region**
$H_A: \mu > \mu_0$	$t \geq t_{\alpha,\, n-1}$ [Upper tailed test]
$H_A: \mu < \mu_0$	$t \leq -t_{\alpha,\, n-1}$ [Lower tailed test]
$H_A: \mu \neq \mu_0$	$\lvert t \rvert \geq t_{\alpha/2,\, n-1}$ [Two tailed test]

That is, if $|t| > t_\alpha \Rightarrow$ The null hypothesis H_0 is rejected (one tailed test) and if $|t| < t_\alpha \Rightarrow$ the null hypothesis H_0 may be accepted (one tailed test) at the level of significance adopted. If $|t| < t_{\alpha/2} \Rightarrow$ the null hypothesis H_0 is accepted (two tailed test).

Now, the $100(1-\alpha)\%$ confidence limits for the population mean μ corresponding to the given sample are given by, $\bar{x} - t_{\alpha/2,\, n-1}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{x} + t_{\alpha/2,\, n-1}\left(\frac{s}{\sqrt{n}}\right)$.

IV. TEST OF HYPOTHESIS FOR INTERVAL DATA

Let $\{[x_i, y_i];\ i = 1, 2, ..., n\}$ be a random sample of size $n(< 30)$ such that $\{x_i;\ i = 1, 2, ..., n\}$ and $\{y_i;\ i = 1, 2, ..., n\}$ are the random samples from two distinct normal population with the population means η and μ and the sample means η_0 and μ_0 of the sample respectively. Then the null hypothesis $H_0: [\eta,\ \mu] = [\eta_0,\ \mu_0]$, that is, $\eta = \eta_0$ and $\mu = \mu_0$ and the alternative hypotheses $H_A: [\eta,\ \mu] < [\eta_0,\ \mu_0]$, that is $\eta < \eta_0$ and $\mu < \mu_0$; $H_A: [\eta,\ \mu] > [\eta_0,\ \mu_0]$, that is $\eta > \eta_0$ and $\mu > \mu_0$ $H_A: [\eta,\ \mu] \neq [\eta_0,\ \mu_0]$, that is $\eta \neq \eta_0$ or $\mu \neq \mu_0$. We now consider the random sample of lower value and upper value of the given interval data be $S_L = \{x_i;\ i = 1, 2, ..., n\}$ and $S_U = \{y_i;\ i = 1, 2, ..., n\}$ respectively. The sample mean of S_L and S_U are s_x and s_y respectively.

The proposed test statistic is given by $t^L = \frac{(\bar{x} - \eta)}{s_x / \sqrt{n}}$ and $t^U = \frac{(\bar{y} - \mu)}{s_y / \sqrt{n}}$

Alternative Hypothesis H_A	**Rejection Region for** α **Level Test**
$H_A: [\eta,\ \mu] > [\eta_0,\ \mu_0]$	$t^L \geq t_{\alpha,\, n-1}$ and $t^U \geq t_{\alpha,\, n-1}$ (Upper tailed test)
$H_A: [\eta,\ \mu] < [\eta_0,\ \mu_0]$	$t^L \leq -t_{\alpha,\, n-1}$ and $t^U \leq -t_{\alpha,\, n-1}$ (Lower tailed test)
$H_A: [\eta,\ \mu] \neq [\eta_0,\ \mu_0]$	$\lvert t^L \rvert \geq t_{\alpha/2,\, n-1}$ or $\lvert t^U \rvert \geq t_{\alpha/2,\, n-1}$ (Two tailed test)

The rejection region of the alternative hypothesis for α level of significance is given by

If $|t^L| < t_{\alpha,\, n-1}$ (One tailed test) and $|t^U| < t_{\alpha,\, n-1}$ (One tailed test)

$\Rightarrow$ The null hypothesis H_0 is accepted.

$\Rightarrow$ The difference between $[\eta,\ \mu]$ and $[\eta_0,\ \mu_0]$ is not significant at α level. Otherwise the alternative hypothesis H_A is accepted

If $|t^L| < t_{\alpha/2,\, n-1}$ (Two tailed test) and $|t^U| < t_{\alpha/2,\, n-1}$ (Two tailed test)

$\Rightarrow$ The null hypothesis H_0 is accepted

$\Rightarrow$ The difference between $[\eta,\ \mu]$ and $[\eta_0,\ \mu_0]$ is not significant at α level. Otherwise the alternative hypothesis H_A is accepted.

Also, the $100(1-\alpha)\%$ confidence limits for the population mean $[\eta,\ \mu]$ corresponding to the given sample are given below:

$$\left[\bar{x} - t_{\alpha/2,n-1}\left(\frac{s_x}{\sqrt{n}}\right),\ \bar{y} - t_{\alpha/2,n-1}\left(\frac{s_y}{\sqrt{n}}\right)\right] < [\eta,\ \mu] < \left[\bar{x} + t_{\alpha/2,n-1}\left(\frac{s_x}{\sqrt{n}}\right),\ \bar{y} + t_{\alpha/2,n-1}\left(\frac{s_y}{\sqrt{n}}\right)\right]$$

Decision table:

Acceptance of null hypotheses $\widetilde{H}_0$		
Lower Level Model	**Upper Level Model**	**Conclusion**
If H_0^L is accepted for all α, $\alpha \in [0,1]$	and H_0^U is accepted for all α, $\alpha \in [0,1]$	then $\widetilde{H}_0$ is accepted for all α, $\alpha \in [0,1]$
If H_0^L is accepted for all α, $\alpha \in [0,1]$	and H_0^U is rejected for all α, $\alpha \in [0,1]$	then $\widetilde{H}_0$ is rejected for all α, $\alpha \in [0,1]$
If H_0^L is rejected for all α, $\alpha \in [0,1]$	and H_0^U is accepted for all α, $\alpha \in [0,1]$	then $\widetilde{H}_0$ is rejected for all α, $\alpha \in [0,1]$
If H_0^L is rejected for all α, $\alpha \in [0,1]$	or H_0^U is rejected for all α, $\alpha \in [0,1]$	then $\widetilde{H}_0$ is rejected for all α, $\alpha \in [0,1]$

Partial acceptance of null hypothesis H_0 at the intersection of certain level of α at both upper level and lower level models can be taken into account for the acceptance of the null hypothesis $\widetilde{H}_0$ **.**

Example-1

The department of career and placement of an inspection committee of a University claims that the placement percentage of the students from its affiliated institution is between 50 and 65. Only 12 colleges in that zone are selected at random. The colleges are observed and the placement percentages of each of the colleges are recorded. Due to some limited resources, the minimum and maximum of the placement percentage of each of the colleges can only be observed. Therefore, the placement percentages of the colleges are taken to be 'intervals' as follows [19]:

[44, 53], [40, 38], [61, 69], [52, 57], [32, 46], [44, 39], [70, 73], [41, 48], [67, 73], [72, 74] [53, 60], [72, 78].

Solution

We now consider the test hypotheses be $H_0 : \mu = \mu_0$ and $H_A : \mu \neq \mu_0$ (Two tailed test).

Here, the size of the sample n = 12 and the population mean is [50, 65] with unknown sample S.D. We use 5% level of significance. Now, the sample mean value of the lower and upper interval values are

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \Rightarrow \bar{x}^L = 54 \text{ and } \bar{x}^U = 59$$

And the sample S.D. of the lower and upper interval values are $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$

$\Rightarrow S^L = 14.06$ and $S^U = 14.30$.

Now, the tabulated value of t at $(n-1) = 11$ degrees of freedom at 5% level of significance is $T_\alpha = 2.20$.

And the lower value of μ_0 is $\mu_0^L = 50$ and the upper value of μ_0 is $\mu_0^U = 65$.

The test statistic:

$$t^L = \frac{\left(\overline{x}^L - \mu_0^L\right)}{s^L/\sqrt{n}} = 0.9855 \text{ and } t^U = \frac{\left(\overline{x}^U - \mu_0^U\right)}{s^U/\sqrt{n}} = -1.4535$$

Since $\left|t^L\right| < T_\alpha$ and $\left|t^U\right| < T_\alpha$, the null hypothesis " the population mean $\mu = \mu_0$" is accepted and the 95% confidence limits for the population mean μ is

$$\left[\overline{x}^L - t_{\alpha/2, n-1}\left(\frac{s^L}{\sqrt{n}}\right),\ \overline{x}^U - t_{\alpha/2, n-1}\left(\frac{s^U}{\sqrt{n}}\right)\right] < [\eta,\ \mu] < \left[\overline{x}^L + t_{\alpha/2, n-1}\left(\frac{s^L}{\sqrt{n}}\right),\ \overline{x}^U + t_{\alpha/2, n-1}\left(\frac{s^U}{\sqrt{n}}\right)\right]$$

$$\Rightarrow [45.0707,\ 49.9183] < [\eta,\ \mu] < [62.9293,\ 68.0817]$$

V. TESTING HYPOTHESIS FOR FUZZY DATA USING TFNS.

Trapezoidal Fuzzy Number to Interval

Let a trapezoidal fuzzy number be $\widetilde{A} = (a, b, c, d)$, then the fuzzy interval [31] in terms of α - cut interval is defined as follows:

$$\widetilde{A}_\alpha = \left[a + (b - a)\alpha,\ d - (d - c)\alpha\right]; \quad 0 \le \alpha \le 1 \text{ --- [5.1]}$$

Suppose that the given sample is a fuzzy data that are trapezoidal fuzzy numbers and if we want to test the hypothesis about the population mean, then we can transfer the given fuzzy data into interval data by using the relation (5.1).

Example-2

The marketing department for a tire and rubber company wants to claim that the average life of a tire, the company recently developed exceeds the well-known average tire life of a competitive brand, which is known to be 32000 miles. Only 24 new tires were tested because the tests are tedious and take considerable time to complete. Six cars of a particular model and brand were used to test the tires. Car model and brand were fixed so that other car-related aspects did not come into play. The situation was that the life of the tires were not known exactly. The obtained life of the tire was around a number. Therefore the tire life numbers were taken to be trapezoidal fuzzy numbers as follows [37, 15]:

$\tilde{x}_1 = (33266,\ 33671,\ 34177,\ 34889)$, $\tilde{x}_2 = (32093,\ 32613,\ 33149,\ 33255)$,

$\tilde{x}_3 = (32585,\ 32885,\ 33215,\ 33787)$, $\tilde{x}_4 = (31720,\ 32143,\ 32819,\ 33497)$,

$\tilde{x}_5 = (32806,\ 33026,\ 33346,\ 33908)$, $\tilde{x}_6 = (31977,\ 32237,\ 32817,\ 33034)$,

$\tilde{x}_7 = (33065,\ 33345,\ 33993,\ 34131)$, $\tilde{x}_8 = (31943,\ 32303,\ 33053,\ 33212)$,

$\tilde{x}_9 = (30743,\ 31325,\ 31994,\ 32460)$, $\tilde{x}_{10} = (32169,\ 32955,\ 33148,\ 33968)$,

$\tilde{x}_{11} = (32415,\ 33101,\ 33955,\ 34072)$, $\tilde{x}_{12} = (32900,\ 33452,\ 33873,\ 34335)$,

$\tilde{x}_{13} = (32687,\ 33195,\ 33431,\ 33908)$, $\tilde{x}_{14} = (30327,\ 30975,\ 31295,\ 31445)$,

$\tilde{x}_{15} = (32185,\ 32723,\ 32960,\ 33186)$, $\tilde{x}_{16} = (31187,\ 31436,\ 31999,\ 32237)$,

$\tilde{x}_{17} = (33423,\ 34017,\ 34258,\ 34771)$, $\tilde{x}_{18} = (33208,\ 33860,\ 34280,\ 34876)$,

$\tilde{x}_{19} = (31639,\ 32481,\ 33026,\ 33542)$, $\tilde{x}_{20} = (30945,\ 31634,\ 32325,\ 32739)$,

$\tilde{x}_{21} = (31511,\ 32038,\ 32727,\ 33064)$, $\tilde{x}_{22} = (30826,\ 31610,\ 32406,\ 32913)$,

$\tilde{x}_{23} = (33063,\ 33762,\ 34181,\ 34449)$, $\tilde{x}_{24} = (33464,\ 34062,\ 34388,\ 34974)$.

Let us consider the testing hypotheses

$\widetilde{H}_0: \tilde{\mu} \cong \widetilde{32000}$ and $\widetilde{H}_A: \tilde{\mu} > \widetilde{32000}$ where $\widetilde{32000}$ means "***around 32000***", which regarded as a "***linguistic data***". Therefore,

$\widetilde{H}_0:$ The average life of the tire is around 32000.

$\widetilde{H}_A:$ The average life of the tire is greater than 32000.

We may assume the membership function of $\widetilde{32000}$ as $\widetilde{32000} = (30000, 32000, 33000, 34000)$. The size of the sample $n = 24$ and the population mean is $\tilde{\mu}_0 = (30000, 32000, 33000, 34000)$ with unknown sample standard deviation.

Using relation (5.1), we convert the fuzzy data into interval data and are listed below:

$\tilde{x}_1 = [33266 + 405\alpha, 34889 - 712\alpha]$, $\tilde{x}_2 = [32093 + 520\alpha, 33255 - 106\alpha]$,

$\tilde{x}_3 = [32585 + 300\alpha, 33787 - 572\alpha]$, $\tilde{x}_4 = [31720 + 423\alpha, 33497 - 678\alpha]$,

$\tilde{x}_5 = [32806 + 220\alpha, 33908 - 562\alpha]$, $\tilde{x}_6 = [31977 + 260\alpha, 33034 - 217\alpha]$,

$\tilde{x}_7 = [33065 + 280\alpha, 34131 - 138\alpha]$, $\tilde{x}_8 = [31943 + 360\alpha, 33212 - 159\alpha]$,

$\tilde{x}_9 = [30743 + 582\alpha, 32460 - 466\alpha]$, $\tilde{x}_{10} = [32169 + 786\alpha, 33968 - 820\alpha]$,

$\tilde{x}_{11} = [32415 + 686\alpha, 34072 - 117\alpha]$, $\tilde{x}_{12} = [32900 + 552\alpha, 34335 - 462\alpha]$,

$\tilde{x}_{13} = [32687 + 508\alpha, 33908 - 477\alpha]$, $\tilde{x}_{14} = [30327 + 648\alpha, 31445 - 150\alpha]$,

$\tilde{x}_{15} = [32185 + 538\alpha, 33186 - 226\alpha]$, $\tilde{x}_{16} = [31187 + 249\alpha, 32237 - 238\alpha]$,

$\tilde{x}_{17} = [33423 + 594\alpha, 34771 - 513\alpha]$, $\tilde{x}_{18} = [33208 + 652\alpha, 34876 - 596\alpha]$,

$\tilde{x}_{19} = [31639 + 842\alpha, 33542 - 516\alpha]$, $\tilde{x}_{20} = [30945 + 689\alpha, 32739 - 414\alpha]$,

$\tilde{x}_{21} = [31511 + 527\alpha, 33064 - 307\alpha]$, $\tilde{x}_{22} = [30826 + 784\alpha, 32913 - 507\alpha]$,

$\tilde{x}_{23} = [33063 + 699\alpha, 34449 - 268\alpha]$, $\tilde{x}_{24} = [33464 + 598\alpha, 34974 - 586\alpha]$.

Now, we obtain the mean value of the lower and upper interval values which are given below:

$$\bar{x}^L = 32168.63 + 529.25\alpha \text{ and } \bar{x}^U = 33610.50 + 408.63\alpha$$

And the sample S. D. of the lower and upper interval values are:

$$S^L = \sqrt{\frac{1}{n-1}\left[\sum_{i=1}^{n}\left(x_i^L - \bar{x}^L\right)^2\right]} \Rightarrow S^L = \sqrt{33316.02\alpha^2 - 80004.54\alpha + 838201.99}$$

and $S^U = \sqrt{740506.58\alpha^2 - 169304.62\alpha + 815219.02}$

Now, we set the null hypothesis: $H_0: \tilde{\mu} = \tilde{\mu}_0$ and the alternative hypothesis: $H_A: \tilde{\mu} > \tilde{\mu}_0$ (Upper tailed test)

Choosing 5% level of significance and the table value of 't' for 23 degrees of freedom is $T_\alpha = 1.714$

And the interval representation of $\tilde{\mu}_0$ is given by

$$\tilde{\mu}_0 = \left[\mu_0^L, \mu_0^U\right] \text{ where } \mu_0^L = 30000 + 2000\alpha \text{ and } \mu_0^U = 34000 - 1000\alpha \,;\, 0 \le \alpha \le 1.$$

And therefore,

$$t^L = \frac{\bar{x}^L - \mu_0^L}{s^L/\sqrt{n}} = \begin{cases} 11.6042 \,; & \alpha = 0 \\ 3.8429 \,; & \alpha = 1 \end{cases}$$

$$\Rightarrow t^L > T_\alpha = 1.714 \text{ for all } \alpha, 0 \le \alpha \le 1.$$

And,

$$t^U = \frac{\overline{x}^U - \mu_0^U}{s^U / \sqrt{n}} = \begin{cases} -2.1134 \; ; & \alpha = 0 \\ 1.8105 \; ; & \alpha = 0.53 \\ 1.8757 \; ; & \alpha = 0.54 \\ 4.2402 \; ; & \alpha = 1 \end{cases}$$

$\Rightarrow t^U > T_\alpha$ for $0.53 \le \alpha \le 1$

5.1. Conclusion

Since for $0.53 \le \alpha \le 1$, $t^L > T_\alpha$ and $t^U > T_\alpha$, the alternative hypothesis is accepted. Therefore, $\widetilde{H}_A$: The average tire life is approximately greater than 32000 is accepted based on the given fuzzy data with condition $0.53 \le \alpha \le 1$.

5.2. Remark

The results obtained from example-2 differ by 0.01 level of lower limit of α $(0.53 \le \alpha \le 1)$ when compared with the results in Wu [37], Chachi et al. [15] which is $(0.54 \le \alpha \le 1)$.

VI. WANG'S CENTROID POINT AND RANKING METHOD

Wang et al. [35] found that the centroid formulae proposed by Cheng are incorrect and have led to some misapplications such as by Chu and Tsao. They presented the correct method for centroid formulae for a generalized fuzzy number $\widetilde{A} = (a, b, c, d; w)$ as

$$\left(\overline{x}_0, \overline{y}_0\right) = \left[\frac{1}{3}\left((a+b+c+d) - \left(\frac{dc - ab}{(d+c)-(a+b)}\right)\right), \left(\frac{w}{3}\right)\left(1 + \left(\frac{c-b}{(d+c)-(a+b)}\right)\right)\right] --- (6.1)$$

And the ranking function associated with $\widetilde{A}$ is $R(\widetilde{A}) = \sqrt{\overline{x}_0^2 + \overline{y}_0^2} --- (6.2)$

For a normalized tfn., we put w = 1 in equations (6.1) so we have,

$$\left(\overline{x}_0, \overline{y}_0\right) = \left[\frac{1}{3}\left((a+b+c+d) - \left(\frac{dc - ab}{(d+c)-(a+b)}\right)\right), \left(\frac{1}{3}\right)\left(1 + \left(\frac{c-b}{(d+c)-(a+b)}\right)\right)\right] --- (6.3)$$

And the ranking function associated with $\widetilde{A}$ is $R(\widetilde{A}) = \sqrt{\overline{x}_0^2 + \overline{y}_0^2} --- (6.4)$.

Let $\widetilde{A}_i$ and $\widetilde{A}_j$ be two fuzzy numbers, (i) $R(\widetilde{A}_i) > R(\widetilde{A}_j)$ then $\widetilde{A}_i > \widetilde{A}_j$ (ii) $R(\widetilde{A}_i) > R(\widetilde{A}_j)$ then $\widetilde{A}_i > \widetilde{A}_j$ and (iii) $R(\widetilde{A}_i) = R(\widetilde{A}_j)$ then $\widetilde{A}_i = \widetilde{A}_j$.

One-sample t-test using Wang's centroid point and ranking function

Example 6.1. Let we consider example 2, using the above relations (6.3) and (6.4), we obtain the ranks of tfns. which are tabulated below:

$R(\widetilde{x}_i)$; i = 1, 2, ..., 24		
34014.17	31626.25	34112.55
32764.78	33062.28	34053.21
33130.90	33370.59	32656.93
32554.29	33635.90	31900.78
33287.17	33303.50	32328.90
32515.21	30987.47	31928.41
33630.61	32747.45	33844.51
32623.44	31714.47	34221.36

Using the above relations (6.3) and (6.4), we obtain the calculated population mean $\mu_0 = 32200$.

And the calculated value of 't' is t = 3.998. The tabulated value of 't' at 5% level of significance with 23 degree of freedom is $T_\alpha = 1.714$. Here, $t > T_\alpha$. So, the null hypothesis $\widetilde{H}_0$ is rejected. Therefore, we conclude that the average life of the tire is greater than 32000.

VII. REZVANI'S RANKING FUNCTION OF TFNS.

The centroid of a trapezoid is considered as the balancing point of the trapezoid. Divide the trapezoid into three plane figures. These three plane figures are a triangle (APB), a rectangle (BPQC) and a triangle (CQD) respectively. Let the centroids of the three plane figures be G_1, G_2 and G_3 respectively. The incenter of these centroids G_1, G_2 and G_3 is taken as the point of reference to define the ranking of generalized trapezoidal fuzzy numbers. The reason for selecting this point as a point of reference is that each centroid point are ***balancing points*** of each individual plane figure and the incenter of these centroid points is much more balancing point for a generalized trapezoidal fuzzy number. Therefore, this point would be a better reference point than the centroid point of the trapezoid.

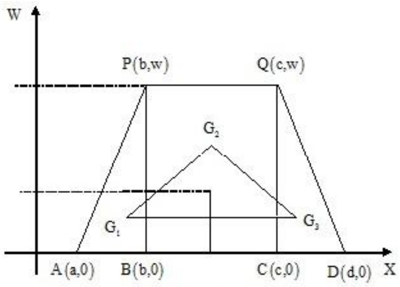

Fig.1 Centroid of centroids

Consider a generalized trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d; w)$. The centroids of the three plane figures are:

$$G_1 = \left(\frac{a+2b}{3}, \frac{w}{3}\right),\ G_2 = \left(\frac{b+c}{2}, \frac{w}{2}\right) \text{ and } G_3 = \left(\frac{2c+d}{3}, \frac{w}{3}\right) --- (7.1)$$

Equation of the line G_1G_3 is $y = \frac{w}{3}$ and G_2 does not lie on the line G_1G_3. Therefore, G_1, G_2 and G_3 are non-collinear and they form a triangle. We define the incenter $I(\bar{x}_0, \bar{y}_0)$ of the triangle with vertices G_1, G_2 and G_3 of the generalized fuzzy number $\widetilde{A} = (a, b, c, d; w)$ as [28]

$$I_{\widetilde{A}}(\bar{x}_0, \bar{y}_0) = \left[\frac{\alpha\left(\frac{a+2b}{3}\right) + \beta\left(\frac{b+c}{2}\right) + \gamma\left(\frac{2c+d}{3}\right)}{\alpha+\beta+\gamma}, \frac{\alpha\left(\frac{w}{3}\right) + \beta\left(\frac{w}{2}\right) + \gamma\left(\frac{w}{3}\right)}{\alpha+\beta+\gamma}\right] --- (7.2)$$

$$\text{where } \alpha = \frac{\sqrt{(c - 3b + 2d)^2 + w^2}}{6}, \beta = \frac{\sqrt{(2c + d - a - 2b)^2}}{3}, \gamma = \frac{\sqrt{(3c - 2a - b)^2 + w^2}}{6}$$

And ranking function of the trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d; w)$ which maps the set of all fuzzy numbers to a set of all real numbers $\left[\text{i.e. } R: \left[\widetilde{A}\right] \to \mathbb{R}\right]$ is defined as $R\left(\widetilde{A}\right) = \sqrt{\bar{x}_0^2 + \bar{y}_0^2} --- (7.3)$ which is the Euclidean distance from the incenter of the centroids. For a normalized tfn, we put w = 1 in equations (7.1), (7.2) and (7.3) so we have,

$$G_1 = \left(\frac{a+2b}{3}, \frac{1}{3}\right),\ G_2 = \left(\frac{b+c}{2}, \frac{1}{2}\right) \text{ and } G_3 = \left(\frac{2c+d}{3}, \frac{1}{3}\right) --- (7.4)$$

$$I_{\widetilde{A}}\left(\bar{x}_0, \bar{y}_0\right)=\left[\frac{\alpha\left(\frac{a+2b}{3}\right)+\beta\left(\frac{b+c}{2}\right)+\gamma\left(\frac{2c+d}{3}\right)}{\alpha+\beta+\gamma}, \frac{\alpha\left(\frac{1}{3}\right)+\beta\left(\frac{1}{2}\right)+\gamma\left(\frac{1}{3}\right)}{\alpha+\beta+\gamma}\right] ---(7.5)$$

where $\alpha=\frac{\sqrt{(c - 3b + 2d)^2+1}}{6}, \beta=\frac{\sqrt{(2c + d - a - 2b)^2}}{3}$ and $\gamma=\frac{\sqrt{(3c - 2a - b)^2+1}}{6}$

And ranking function of the trapezoidal fuzzy number $\widetilde{A}=(a, b, c, d; 1)$ is defined as $R\left(\widetilde{A}\right)=\sqrt{\bar{x}_0^2 + \bar{y}_0^2} ---(7.6)$.

One-sample t-test using Rezvani's ranking function

We now analyse the one-sample t-test by assigning rank for each normalized trapezoidal fuzzy numbers and based on the ranking grades the decisions are observed.

Example 7.1. Let we consider example 2, using the above relations (7.4), (7.5) and (7.6), we obtain the ranks of tfns. which are tabulated below:

$R\left(\widetilde{x}_i\right);\ i = 1, 2, ..., 24$		
33924	31659.50	34137.50
32881	33051.50	34070
33050	33528	32753.50
32481	33662.50	31980
33186	33313	32382.50
32527	31135	32008
33669	32841.5	33971.50
32678	31717.50	34225

Using the above relations (7.4), (7.5) and (7.6), we obtain the calculated population mean $\mu_0=32500$. And the calculated value of 't' is t = 2.572. The tabulated value of 't' at 5% level of significance with 23 degree of freedom is $T_\alpha=1.714$. Here, $t > T_\alpha$. So, the null hypothesis $\widetilde{H}_0$ is rejected. Therefore, we conclude that the average life of the tire is greater than 32000.

VIII. GRADED MEAN INTEGRATION REPRESENTATION (GMIR)

Let $\widetilde{A}=(a, b, c, d; w)$ be a generalized trapezoidal fuzzy number, then the **GMIR** [27] of $\widetilde{A}$ is defined by $P\left(\widetilde{A}\right)=\int_0^w h\left[\frac{L^{-1}(h)+R^{-1}(h)}{2}\right]dh \,/\, \int_0^w hdh$.

Theorem 8.1. Let $\widetilde{A}=(a, b, c, d; 1)$ be a tfn. with normal shape function, where a, b, c, d are real numbers such that $a < b \le c < d$. Then the graded mean integration representation (GMIR) of $\widetilde{A}$ is $P\left(\widetilde{A}\right)=\frac{(a + d)}{2}+\frac{n}{2n + 1}(b - a - d + c)$.

Proof : For a trapezoidal fuzzy number $\widetilde{A}=(a, b, c, d; 1)_n$, we have $L(x)=\left(\frac{x - a}{b - a}\right)^n$ and $R(x)=\left(\frac{d - x}{d - c}\right)^n$ Then, $h = \left(\frac{x - a}{b - a}\right)^n \Rightarrow L^{-1}(h)=a+(b - a)h^{1/n}$;

$$h = \left(\frac{d - x}{d - c}\right)^n \Rightarrow R^{-1}(h)=d-(d - c)h^{1/n}$$

$$\therefore P(\widetilde{A})=\left(\frac{1}{2}\int_0^1 h\left[\left(a+(b - a)h^{1/n}\right)+\left(d-(d - c)h^{1/n}\right)\right]dh\right)/\int_0^1 hdh$$

$$=\left(\frac{1}{2}\left[\frac{(a + d)}{2}+\frac{n}{2n + 1}(b - a - d + c)\right]\right)/\left(1/2\right)$$

Thus, $P(\widetilde{A})=\frac{(a + d)}{2}+\frac{n}{2n + 1}(b - a - d + c)$ Hence the proof.

Result 8.1. If n =1 in the above theorem, we have $P(\widetilde{A})=\frac{a + 2b + 2c + d}{6}$

One-sample t-test using GMIR of tfns.

Example 8.1. Let us consider example 2, using the result-8.1 from above theorem-8.1, we get the GMIR of each tfns. $\widetilde{x}_i$ which are tabulated below:

$P(\widetilde{x}_i)$; i = 1, 2, ..., 24		
33975.16	31640.16	34124
32812	33057.16	34060.67
33095.33	33433.16	32699.17
32523.50	33647.50	31934
33243	33307.83	32350.83
32519.83	31052	31961.83
33645.33	32789.50	33899.67
32644.50	31715.67	34223

Using the above result-8.1, we obtain population mean $\mu_0 = 32333.33$. And the calculated value of 't' is t = 3.368. The tabulated value of 't' at 5% level of significance with 23 degree of freedom is T_α=1.714. Here, $t > T_\alpha$. So, the null hypothesis $\widetilde{H}_0$ is rejected. Therefore, we conclude that the average life of the tire is greater than 32000.

IX. ONE-SAMPLE t-TEST USING TOTAL INTEGRAL VALUE (TIV) OF TFNS.

The membership grades for a normalized tfn. $\widetilde{A}=(a, b, c, d; 1)$ is calculated by the relation [21]

$$\int_{Supp(\widetilde{A})} \mu_{\widetilde{A}}(x)dx = \int_a^b\left(\frac{x - a}{b - a}\right)dx + \int_b^c dx + \int_c^d\left(\frac{x - d}{c - d}\right)dx --- (9.1)$$

Example 9.1. Let us consider example 2, the total integral value of first entry $\widetilde{x}_1=(33266, 33671, 34177, 34889)$ will be

$$\int_{Supp(\widetilde{x}_i)} \mu_{\widetilde{x}_i}(x)dx = \int_{33266}^{33671}\left(\frac{x - 33266}{405}\right)dx + \int_{33671}^{34177} dx + \int_{34177}^{34889}\left(\frac{x - 34899}{-712}\right)dx = 1064.5 = I$$

The total integral value of remaining entries can be calculated in similar way, which have been tabulated below:

$\int_{Supp(\tilde{x}_i)} \mu_{\tilde{x}_i}(x)dx = I;\ i = 1, 2, ..., 24$		
1064.50	1193	794.50
849	996	1044
766	1255.50	1224
1226.50	928	1242.50
711	728.50	1121
818.50	719	1441.50
857	619	902.50
1009.50	806.50	918

Using the above relation (9.1), we obtain the calculated population mean $\mu_0 = 2500$. And the calculated value of 't' is t = -34.83. The tabulated value of 't' at 5% level of significance with 23 degree of freedom is T_α=1.714. Here, $|t| > T_\alpha$. So, the null hypothesis $\widetilde{H}_0$ is rejected. Therefore, we conclude that the average life of the tire is greater than 32000.

X. LIOU AND WANG'S CENTROID POINT METHOD

Liou and Wang [23] ranked fuzzy numbers with total integral value. For a fuzzy number defined by definition (2.3), the total integral value is defined as

$I_T^\alpha(\tilde{x}_i) = \alpha I_R(\tilde{x}_i) + (1 - \alpha) I_L(\tilde{x}_i)---(10.1)$ where

$I_R(\tilde{x}_i) = \int_{Supp(\tilde{x}_i)} R_{\tilde{x}_i}(x)dx = \int_c^d \left(\frac{x\text{-}d}{c\text{-}d}\right)dx---(10.2)$ and

$I_L(\tilde{x}_i) = \int_{Supp(\tilde{x}_i)} L_{\tilde{x}_i}(x)dx = \int_a^b \left(\frac{x\text{-}a}{b\text{-}a}\right)dx---(10.3)$ are the ***right*** and ***left integral values*** of $\tilde{x}_i;\ i = 1, 2, ..., 24$ respectively and $0 \le \alpha \le 1$.

(i) $\alpha \in [0,1]$ is the **index of optimism** which represents the **degree of optimism** of a decision maker. (ii) If $\alpha = 0$, then the total value of integral represents a **pessimistic decision maker's view point** which is equal to left integral value. (iii) If $\alpha = 1$, then the total integral value represents an **optimistic decision maker's view point** and is equal to the right integral value.(iv) If $\alpha = 0.5$ then the total integral value represents a **moderate decision maker's view point** and is equal to the mean of right and left integral values. For a decision maker, the larger the value of α is, the higher is the degree of optimism.

One-sample t-test using LIOU and WANG'S centroid point method:

Example 10.1. Let us consider example 2, using the above equations (10.1), (10.2) and (10.3), we get the centroid point of first member as follows:

$I_L(\tilde{x}_1) = \int_{33266}^{33671} \left(\frac{x\text{-}33266}{405}\right)dx = 405/2$; $I_R(\tilde{x}_1) = \int_{34177}^{34889} \left(\frac{x - 34889}{-712}\right)dx = 356$

Therefore $I_T^\alpha(\tilde{x}_1) = (307\alpha + 405)/2$.

Similarly we can find $I_T^\alpha(\tilde{x}_i);$ for i = 2, ..., 24. and the calculated values are tabulated below:

$I_T^{\alpha}\left(\tilde{x}_i\right)$; for i = 2, ..., 24.		
$(307\alpha+405)/2$	$(291-58\alpha)$	$(594-81\alpha)/2$
$(260-207\alpha)$	$(393+17\alpha)$	$(326-28\alpha)$
$(150+136\alpha)$	$(686-596\alpha)/2$	$(421-163\alpha)$
$(255\alpha+423)/2$	$(276-45\alpha)$	$(689-275\alpha)/2$
$(110+171\alpha)$	$(508-31\alpha)/2$	$(527-190\alpha)/2$
$(260-43\alpha)/2$	$(324-249\alpha)$	$(784-277\alpha)/2$
$(140-71\alpha)$	$(269-155\alpha)$	$(699-431\alpha)/2$
$(360-201\alpha)/2$	$(249-11\alpha)/2$	$(299-6\alpha)$

Using the above relations (10.1), (10.2) and (10.3), we obtain the calculated population mean $\mu_0 = 1000 - 500\alpha$. And the calculated value of 't' is

$$t = \frac{(0.40)(10555\alpha - 17649)}{\sqrt{170390217\alpha^2 - 1486563634\alpha + 3259550979}}$$

The tabulated value of 't' at 5% level of significance with 23 degree of freedom is $T_\alpha = 1.714$. Here, $t < T_\alpha,\ \forall\alpha,\ 0 \le \alpha \le 1$. So, the null hypothesis $\tilde{H}_0$ is accepted in this case. Therefore, we conclude that the average life of the tire is around 32000.

XI. THORANI'S RANKING METHOD

As per the description in Salim Rezvani's ranking method, Thorani et al. [29] presented a different kind of centroid point and ranking function of tfns. The incenter $I_{\tilde{A}}\left(\bar{x}_0,\ \bar{y}_0\right)$ of the triangle [Fig. 1] with vertices G_1, G_2 and G_3 of the generalized tfn. $\tilde{A} = (a, b, c, d; w)$ is given by,

$$I_{\tilde{A}}\left(\bar{x}_0,\ \bar{y}_0\right) = \left[\frac{\alpha\left(\frac{a+2b}{3}\right)+\beta\left(\frac{b+c}{2}\right)+\gamma\left(\frac{2c+d}{3}\right)}{\alpha+\beta+\gamma},\ \frac{\alpha\left(\frac{w}{3}\right)+\beta\left(\frac{w}{2}\right)+\gamma\left(\frac{w}{3}\right)}{\alpha+\beta+\gamma}\right] --- (11.1)$$

$$\text{where } \alpha = \frac{\sqrt{(c - 3b + 2d)^2 + w^2}}{6}, \beta = \frac{\sqrt{(2c + d - a - 2b)^2}}{3}, \gamma = \frac{\sqrt{(3c - 2a - b)^2 + w^2}}{6}$$

And the ranking function of the generalized tfn. $\tilde{A} = (a, b, c, d; w)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as $R\left(\tilde{A}\right) = x_0 \times y_0 --- (11.2)$. For a normalized tfn., we put w = 1 in equations (11. 1) and (11. 2) so we have,

$$I_{\tilde{A}}\left(\bar{x}_0,\ \bar{y}_0\right) = \left[\frac{\alpha\left(\frac{a+2b}{3}\right)+\beta\left(\frac{b+c}{2}\right)+\gamma\left(\frac{2c+d}{3}\right)}{\alpha+\beta+\gamma},\ \frac{\alpha\left(\frac{1}{3}\right)+\beta\left(\frac{1}{2}\right)+\gamma\left(\frac{1}{3}\right)}{\alpha+\beta+\gamma}\right] --- (11.3)$$

$$\text{where } \alpha = \frac{\sqrt{(c - 3b + 2d)^2 + 1}}{6}, \beta = \frac{\sqrt{(2c + d - a - 2b)^2}}{3} \text{ and } \gamma = \frac{\sqrt{(3c - 2a - b)^2 + 1}}{6}$$

And for $\tilde{A} = (a, b, c, d; 1)$, $R\left(\tilde{A}\right) = x_0 \times y_0 --- (11.4)$

One-sample t-test using Thorani's ranking method of tfns.

Example 11.1. Let us consider example 2, using the above relations (11.3) and (11.4), we get the ranks of each tfns. $\tilde{x}_i,\ i = 1, 2, ..., 24$ which are tabulated below:

$R\left(\tilde{x}_i\right)$, i = 1, 2, ..., 24		
14135	13191.50	14223.96
13700.42	13771.50	14195.83
13770.83	13970	13647.29
13533.75	14026	13324.99
13827.50	13880.42	13492.71
13552.91	12972.92	13336.67
14028.75	13684	14154.79
13615.83	13215.62	14260.42

Using the above relations (11.3) and (11.4), we obtain the calculated population mean $\mu_0 = 13541.67$. And the calculated value of 't' is t = 2.57. The tabulated value of 't' at 5% level of significance with 23 degree of freedom is $T_\alpha = 1.714$. Here, $t > T_\alpha$. So, the null hypothesis $\widetilde{H}_0$ is rejected. Therefore, we conclude that the average life of the tire is greater than 32000.

XII. GENERAL CONCLUSION

The decisions obtained from various methods are tabulated below for the acceptance of null hypothesis.

Acceptance of null hypothesis $\widetilde{H}_0$ at 5% level of significance						
α-cut	**Wang**	**Rezvani**	**GMIR**	**TIV**	**L&W**	**Thorani**
×	×	×	×	×	✓	×

Observing the decisions obtained from alpha cut interval method, for example-2, the null hypothesis is rejected for the level of $0.53 \leq \alpha \leq 1$ at both l.l.m and u.l.m. And Liou & Wang's method (L&W) do not provide reliable result as it accepts the null hypotheses for all α. Also for example-2, the decisions obtained from Wang's ranking method, Rezvani's ranking method, GMIR, TIV and Thorani's ranking method provide parallel discussion.

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