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Research Paper

Dynamics of Diffusive Ratio-Dependent Predator-Prey System Along With Leslie-Gower Model and Harvesting

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ABSTRACT: This *paper explores a dynamical analysis of diffusive ratio-dependent predator-prey model with prey harvesting. We consider the predator switches to alternative food when its favorite food density is low (i.e. which obeys Leslie-Gower model). Firstly, we perform the permanence analysis of proposed system, which ensures that the species will always coexist at any time. Then, we study the local stability, global stability and Hopf bifurcation analysis around interior equilibrium point. Finally, we investigate the existence and nonexistence of non-constant positive steady state solutions.*
KEYWORDS: *Predator-prey model, Ratio-dependent function, Stability analysis, Hopf bifurcation*

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I. INTRODUCTION

Inspired by the pioneering work of Lotka and Volterra in [1], the dynamics of interactions between two species(predator-prey) models has been receiving much more attention in the field of both ecology and mathematical ecology. The functional response term plays a vital role in deriving the exact mathematical model and in this regard, many authors have proposed and investigated different types of functional responses, including Holling I-IV, ratio-dependent, Beddington-DeAngelis and Crowley-Martin functional, for more details one can see [2–4]. Note that, when the predators have look for food (and as a result they have to share/compete for food), prey dependent functional response cannot provide better performance. Hence, the ratio-dependent functional response came into core [5], in which per capita growth rate of predator should be a function of the ratio of prey to predator abundance. A substantial works on ratio-dependent predator-prey models has drawn in [6-10] and references therein.

Suppose that the predators have preferred for alternative food when of its favorite food density is low and to exploit this fact, Leslie and Gower in [11] have modified as:

$$\frac{dn}{dt} = nf(n) - pg(n,p), \qquad \frac{dp}{dt} = cp\left(1 - \frac{dp}{n+l}\right), \tag{1}$$

where $n(t)$ and $p(t)$ are the respective population densities of prey and predator at time $t > 0$, $f(n)$ stands the per capita growth rate of n, $g(n.p)$ is the general functional response term, c is the intrinsic growth rate of predator, d and l are positive constants. The studies on dynamics of model given in (1) has been investigated by many authors in the literature (see [12-15] and references cited therein). The diffusion terms into predator-prey models is quite common due to species moves from higher to lower concentration areas as a result of good living environment, food, etc. Therefore, a modified Leslie-Gower formulation of diffusive predator-prey model with ratio-dependent function takes the form of

$$\frac{dn}{dt} = nf(n) - \frac{pn}{n+ap} + d_1\nabla^2 n, \qquad \frac{dp}{dt} = cp\left(1 - \frac{dp}{n+l}\right) + d_2\nabla^2 n, \tag{2}$$

where$a$ is capturing rate, d_1 and d_2 are diffusion coefficients. In [16], authors have studied the Turing and non-Turing patterns of model (2) with $l = 0$.

The point view of economic profit and harvesting of species are commonly happened by human in fishery, forestry, and wildlife management. This is inspired by introduce the harvesting of populations in the modelling prey-predator system and some interesting results on prey-predator model with harvesting term has been studied [17–19]. There are different types of harvesting strategy, such as constant-yield harvesting, constant-effort harvesting, age-selective harvesting, etc. and among them nonlinear harvesting is more realistic from economical as well as biological point of views rather than strategies. It takes the following form

$$H(n) = \frac{qEn}{m_1 E + m_2 n}$$

whereq stands the catchability coefficient, Erepresents the external effort applied to harvesting and m_1, m_2 are positive constants. As far as our knowledge, it is evident that prey harvesting has not yet been implemented in system (2) with diffusion term. This fact has motivated our present study.

Hence, we consider the modified Leslie-Gower predator-prey model with Beddington-DeAngelis functional response and nonlinear prey harvesting as follows:

$$\frac{\partial N}{\partial T} = RN\left(1 - \frac{N}{K}\right) - \frac{mPN}{N + AP} - H(N) + D_1 \nabla^2 N, \qquad x \in \Omega, t > 0, \qquad (3a)$$

$$\frac{\partial P}{\partial T} = CP\left(1 - \frac{DP}{N + L}\right) + D_2 \nabla^2 P, \qquad x \in \Omega, t > 0. \qquad (3b)$$

Now, we make the following non-dimensional scheme $N \to Kn, P \to \frac{RK}{m}, T \to \frac{1}{R}t$, and let$\alpha = \frac{AR}{m}, d_1 = \frac{D_1}{R},\ g = \frac{qE}{m_2 KR},\ h = \frac{m_1 E}{m_2 K},\ \beta = \frac{C}{R},\ \gamma = \frac{DR}{m},\ \delta = \frac{L}{K},\ d_2 = \frac{D_2}{R}$. Then the system (3) becomes

$$\frac{\partial n}{\partial t} = n\left(1 - n - \frac{np}{n + \alpha p} - \frac{g}{h + n}\right) + d_1 \nabla^2 n, \qquad x \in \Omega, t > 0, \qquad (4a)$$

$$\frac{\partial p}{\partial t} = \beta p\left(1 - \frac{\gamma p}{n + \delta}\right) + d_2 \nabla^2 p, \quad x \in \Omega, t > 0, \qquad (4b)$$

$$n(x,0) = n_0 \geq 0, \qquad p(x,0) = p_0 \geq 0, \qquad x \in \Omega, \qquad t > 0, \qquad (4c)$$

$$\partial_* n = \partial_* p = 0, \qquad x \in \partial\Omega, \qquad t > 0, \qquad (4d)$$

where the positive constants d_1 and d_2 are the diffusion coefficients of prey and predator, respectively, ∇^2 is usual two dimensional Laplacian operator in variable $X = (x, y) \in \Omega \subset \mathbb{R}$ and Ω is bounded domain in $\mathbb{R}^2$ with smooth boundary $\partial\Omega$. ∂_*Indicates the outward unit normal vector of the boundary $\partial\Omega$. Thus the prey and predator density at location X and time t is denoted by $n(X,t)$ and $p(X,t)$ respectively. The initial data$n(x,0) \geq\ 0$ and$v(x,0) \geq\ 0$ are continuous functions. The zero flux boundary conditions assure that there is no fluxes of populations through the boundary, i.e., no external input is imposed from outside.

II. PERMANENCE

In this section, we concern the permanence analysis of the system (4). Before going to prove permanence we introduce the following definition and lemma:

Definition 1.*System (4) is said to be permanent if there exist positive constants $\underline{c}$ and $\overline{c}$ such that $\left(n(x,t), p(x,t)\right)$ of (4) with $u_0 \geq 0$ and $v_0 \geq 0$ satisfies*

$0 < \underline{c} \leq \lim_{t\to\infty} \inf \min_{x\in\Omega} n(x,t) \leq \lim_{t\to\infty} \sup \max_{x\in\Omega} n(x,t) \leq \overline{c},$

$0 < \underline{c} \leq \lim_{t\to\infty} \inf \min_{x\in\Omega} p(x,t) \leq \lim_{t\to\infty} \sup \max_{x\in\Omega} p(x,t) \leq \overline{c}.$

Lemma 1.*Suppose that $u(x,t)$ satisfies*

$\frac{\partial u}{\partial t} - d\Delta u = u(1 - u), x \in \Omega, t > 0,$

$u(x,0) = u_0(x) \geq 0, x \in \Omega,$

$\partial_* u = 0, x \in \partial\Omega, t > 0.$

Then $\lim_{t\to\infty} u(x,t) = 1$ for any $x \in \Omega$.

Theorem 1.*Let $\left(n(x,t), p(x,t)\right)$ be the solution of (4) with $n_0 \geq 0$ and $p_0 \geq 0$.It holds*

1. $n(x,t) \geq 0, p(x,t) \geq 0, \forall t > 0, x \in \Omega.$
2. $\lim_{t\to\infty} \sup \max_{x\in\Omega} n(x,t) \leq 1$ and $\lim_{t\to\infty} \sup \max_{x\in\Omega} p(x,t) \leq \frac{1+\delta}{\gamma}$.

Proof. It follows from (4a), u satifies

$\frac{\partial n}{\partial t} - d_1 \Delta n \leq n(1 - n), x \in \Omega, t > 0.$

Then, by Lemma 1, for any $\epsilon > 0$ there exist $t_1 > 0$ such that

$n(x,t) \leq 1 + \epsilon, x \in \Omega, t \geq t_1,$ (5)

whichimplies

$\lim_{t\to\infty} \sup \max_{x\in\Omega} n(x,t) \leq 1.$

Similarly, from (4b)

$\frac{\partial p}{\partial t} - d_2 \Delta p = \beta p\left(1 - \frac{\gamma p}{u + \delta}\right), x \in \Omega, t > 0.$

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Hence, there exist a constant $t_2 > t_1$ such that

$$p(x,t) \leq \frac{(1+\epsilon+\delta)}{\gamma} + \epsilon, t \geq t_2.$$

Therefore, arbitrariness of ϵ, we complete proof.

Theorem 2.*Let $\big(n(x,t), p(x,t)\big)$ be the solution of (4) with $n_0 \geq 0$ and $p_0 \geq 0$ if $\alpha h > \alpha g + h$, we have*

$$\lim_{t\to\infty} \inf \min_{x\in\Omega} n(x,t) \geq \frac{\alpha(h-g)-h}{\alpha h}$$

$$\lim_{t\to\infty} \inf \min_{x\in\Omega} p(x,t) \geq \frac{\alpha(h-g)-h+\delta\alpha h}{\gamma\alpha h}. \qquad (6)$$

Proof. According to equation (3a) and Theorem 1, we have

$$\frac{\partial n}{\partial t} - d_1 \Delta n \geq n\left(1 - n - \frac{1}{\alpha} - \frac{g}{h}\right), t > t_2.$$

Since $\alpha h > \alpha g + h$ holds, then there exists a constant $t_3 > t_2$ such that

$$n \geq \underline{n} - \frac{\alpha(h-g)-h}{\alpha h} - \epsilon > 0, t \geq t_3. \qquad (7)$$

It follows from (4b) and (7) that

$$\frac{\partial p}{\partial t} - d_2 \Delta p \geq \beta p\left(1 - \frac{\gamma p}{\underline{u}+\delta}\right), t > t_3.$$

So there exist a constant $t_4 \geq t_3$ such that

$$p \geq \underline{p} = \frac{\underline{u}+\delta}{\gamma} - \epsilon > 0, t > t_4. \qquad (8)$$

for$\epsilon > 0$ small enough. From (7) and (8), we obtain (6).

Theorem 3.*According to Theorem 1 and 2, system (4) is permanent.*

Proof. Let $\underline{c} = \min\left\{\underline{n}, \underline{p}\right\}$ and $c = \max\{1, \gamma^{-1}(1+\delta)\}$. Hence, we complete the proof.

III. EQUILIBRIA AND STABILITY ANALYSIS

3.1 Constant equilibria

The equilibria of system (4) given by

$$n\left(1 - n - \frac{p}{n+\alpha p} - \frac{g}{h+n}\right) = 0,$$

$$\beta p\left(1 - \frac{\gamma p}{\underline{u}+\delta}\right) = 0.$$

Soling the above equations, we get the following equilibrium points:

i. The trivial equilibrium point $E^0(0,0)$.
ii. The predator free axial equilibrium point $E^1\left(\frac{1-h}{2} + \frac{1}{2}\sqrt{(1-h)^2 - 4(g-h)}, 0\right)$.
iii. The steady state of coexistence (interior equilibrium point) $E^*(n^*, p^*)$, where $P^* = \frac{n^*+\delta}{\gamma}$ with n^* is a root of the following cubic equation in z

$$az^3 + bz^2 + cz + d = 0, \qquad (9)$$

where

$a = -(\gamma+\alpha)$
$b = -\gamma\delta + (1-h)(\gamma+\alpha) - 1$
$c = (1-h)\alpha\delta + (h-g)(\alpha+\gamma) - (h+\delta)$
$d = (h-g)\alpha\delta - \delta h.$

Remark 1.*The equilibriumE^0 and E^1 always exists. If $g < h$ then only E^2 exists. It is easily to observe that from (9), leading coefficient α is always negative and d is positive if*
$(h-g)\alpha\delta > \delta h (10)$
holds. Hence, if (10) is satisfied, according to Descartes rule of sign assures that the above equation (9) possesses at least one positive root. Further, equation (9) has unique positive root say u^* if (10) holds along with any one the following conditions:

H1$\alpha\delta + 1 > (1-h)(\gamma+\alpha)$ *and* $(h+\delta) > (1-h)\alpha\delta + (h-g)(\alpha+\gamma)$
H2$\alpha\delta + 1 > (1-h)(\gamma+\alpha)$ *and* $(h+\delta) < (1-h)\alpha\delta + (h-g)(\alpha+\gamma)$
H3$\alpha\delta + 1 < (1-h)(\gamma+\alpha)$*and* $(h+\delta) > (1-h)\alpha\delta + (h-g)(\alpha+\gamma)$.

Hereafter, we will always assume that the system (4) satisfied one of the above conditions.

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3.2 Local stability analysis
In this subsection, we study the local stability analysis about positive equilibrium point $E^*(n^*, p^*)$.For the notation simplicity, we let the following notations,

$$\boldsymbol{z} = (n,p)^T, D = diag\{d_1, d_2\}, G(\boldsymbol{z}) = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} n\left(1 - n - \dfrac{p}{n+\alpha p} - \dfrac{g}{h+n}\right) \\ \beta p\left(1 - \dfrac{\gamma p}{\underline{u}+\delta}\right) \end{bmatrix}.$$

Then the system (4) can be rewritten as the vectorform

$$\frac{\partial \boldsymbol{z}}{\partial t} = D\Delta \boldsymbol{z} + G(\boldsymbol{z}). \qquad (11)$$

The linearization of (11) at any point $E(u,v)$ is

$$\frac{\partial \boldsymbol{z}}{\partial t} - D\Delta \boldsymbol{z} = G_{\boldsymbol{z}}(E)\boldsymbol{z}. \qquad (12)$$

where

$$G_z(E) = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} = \begin{pmatrix} 1 - 2n - \dfrac{\alpha p^2}{(n+\alpha p)^2} - \dfrac{gh}{(h+n)^2} & -\dfrac{n^2}{(n+\alpha p)^2} \\ \dfrac{\beta\gamma p^2}{(n+\delta)^2} & \beta - \dfrac{2\beta\gamma p}{(n+\delta)} \end{pmatrix}.$$

The Jacobian matrix (12) at E^* takes the form

$$G_z(E^*) = \begin{pmatrix} g_{11}^* & g_{12}^* \\ g_{21}^* & g_{22}^* \end{pmatrix} = \begin{pmatrix} \beta^* & -\dfrac{n^{*2}}{(n^*+\alpha p^*)^2} \\ \dfrac{\beta}{\gamma} & -\beta \end{pmatrix},$$

where $\beta^* = \frac{n^*p^*}{(n^*+\alpha p^*)^2} + \frac{gn^*}{(h+n^*)^2} - n^*$. Let $\{\mu_i, \varphi_i\ \}$ be an eigenpair of the operator $-\Delta$ on Ω with Neumann boundary condition, where $0 = \mu_1 < \mu_2 < \cdots E(\mu_i)$ is the eigenspace corresponding to μ_i in $C^1(\Omega)$. and $\varphi_{ij}, j = 1,2,\cdots \dim E(\mu_i)$ is an orthonormal basis $E(\mu_i)$. Let

$W = \{(n,p)^T \in [C^2(\Omega) \cap C^1(\Omega)]^2 | \partial n = \partial p = 0\}$(13)

And$W_{ij} = \left\{c\varphi_{ij} \middle| c \in R^2\right\}$.Consider the following decomposition

$W = \bigoplus_{i=1}^{\infty} W_i$(14)

where$W_i = \bigoplus_{j=1}^{\dim E(\mu_i)} W_{ij}$ and W_{ij} is the eigenspace corresponding to μ_i.

Remark 2.*The local stability can be concluded from the eigenvalues of Jacobianmatrix at equilibrium points that E^0 and E^1 are always unstable.*

Theorem 4.*Assume that*

$$1 > \frac{p^*}{(n\wedge + \alpha p^*)^2} - \frac{g}{(h+n^*)^2}. \quad (15)$$

Then E^* is locally asymptotically stable.

Proof. For each $i \geq 1$, is invariant under the operator $L = D\Delta + G_z(E^*)$, and λ is an eigenvalue of L on X, and only if it is an eigenvalue of the matrix $-\mu_i D + G_z(E^*)$. Denote

$$A(\mu_i, E^*) = -\mu_i D + G_z(E^*). \qquad (16)$$

The characteristic equation of $A(\mu_i, E^*)$ is

$$\lambda^2 - tr[A(\mu_i, E^*)]\lambda = \det A(\mu_i, E^*) = 0 \ (17)$$

where

$$trA(\mu_i, E^*) = -\mu_i(d_1 + d_2) + g_{11}^* + g_{22}^* = -\mu_i(d_1 + d_2) + \beta^* - \beta$$

$$\det A(\mu_i, E^*) = d_1 d_2 \mu_i^2 - (d_1 g_{22}^* + d_2 g_{11}^*)\mu_i + \det G_z(E^*)$$

$$= d_1 d_2 \mu_i^2 - (-d_1\beta + d_2\beta^*)\mu_i - \beta\beta^* - \frac{\beta n^{*2}}{\gamma(n^* + \alpha p^*)^2}.$$

In the view of conditions (15), it is easy to check that $\det A(\mu_i E^*) > 0)$ and $trA(\mu_i E^*) < 0$, for $i \geq 1$. So, two characteristic eigenvalues $\lambda_{1i}, \lambda_{2i}0$ of $A(\mu_i E^*)$ have negative real parts for $i \geq 1$. Hence the proof is complete.

3.3 Hopf bifurcation
Here, we derive the conditions for Hopf bifurcation near E^*

Theorem 5.*Assume that $\beta^* = \beta$ and*

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$\frac{{n^*}^2}{\gamma(n^*+\alpha p^*)^2} > \beta^* (18a)$
and
$d_1 > d_2.(18b)$
Then the system (4) exhibits Hopf bifurcation near E^*.
*Proof.*If $\beta^* = \beta$, it is evident that $trA(\mu_1, E^*) = 0, tr[A(\mu_i, E^*)] < 0, i \geq 2$ and from (18a) $\det A(\mu_i, E^*) > 0$. Transitivity condition $\frac{dtrA\,(\mu_1,E^*)}{b\beta} = -1 \neq 0$. This assures the existence of a Hopf bifurcation near E^*. Hence the proof.

3.4 Global stability analysis

In this subsection, we derive the condition for global stability of equilibrium point E^*.In this regard we construct following Lyapunov function

$$V(n,p) = \int_\Omega \left\{n - n^* - n^* \log\left(\frac{n}{n^*}\right) + \frac{1}{\beta_\gamma}\left(p - p^* - p^* \log\left(\frac{p}{p^*}\right)\right)\right\} d\Omega, \qquad (19)$$

where$(n(x,t), p(x,t))$ be any solution of the system (4). For simplicity, we denote

$$P = \frac{\alpha v^*}{\delta(1+\alpha u^* + \beta v^*)} + \frac{(1+\alpha u^*)^2}{4(1+\alpha u^*+\beta v^*)^2} - \frac{1+\alpha u^*}{(1+\alpha u^*+\beta v^*)\gamma^2(1+\delta)\big(\gamma+\alpha\gamma+\beta(1+\delta)\big)}.$$

Theorem 6.*Assume that*

$$\frac{\alpha v^*}{1+\alpha u^*+\beta v^*} + \frac{g}{h(h+u^*)} \leq 1 + \frac{1}{1+\delta} (20)$$

and

$$\frac{1}{1+\delta} - \frac{1}{4\gamma^2\delta^2} \geq P + \frac{g}{\delta h(h+u^*)}. (21)$$

Then the E^* is globally asymptotically stable.
Proof. Now, taking the derivative of V with respect to t along the trajectory of the system (4), we have

$$\frac{dV}{dt} = I_1 + I_2$$
$$= \int_\Omega \left\{d_1\left(\frac{n-n^*}{n}\right)\Delta n + \frac{d_2}{\beta\gamma}\left(\frac{p-p^*}{p}\right)\Delta p\right\} d\Omega$$
$$+ \int_\Omega \left\{(n-n^*)\left(1 - n - \frac{p}{n+\alpha p} - \frac{g}{h+n}\right) + \frac{(p-p^*)}{\gamma}\left(1 - \frac{\gamma p}{n+\delta}\right)\right\} d\Omega.$$

It is easy to see that from Green's first identity

$$I_1 \leq -\int_\Omega \left\{\frac{d_1 n^* |\Delta n|^2}{n^2} + \frac{d_2 p^* |\Delta p|^2}{\beta\gamma p^2}\right\} d\Omega \leq 0.$$

Furthermore,

$$I_2 = \int_\Omega (n-n^*)^2 \left(\frac{p^*}{(n^*+\alpha p^*)(n+\alpha p)} + \frac{g}{(h+n^*)(h+n)} - 1\right) d\Omega - \int_\Omega (p-p^*)^2 \frac{1}{n+\delta} d\Omega$$
$$+ \int_\Omega (n-n^*)(p-p^*)\left(\frac{-n^*}{(n^*+\alpha p^*)(n+\alpha p)} + \frac{1}{\gamma(n+\delta)}\right) d\Omega.$$

The above equation can be rewritten as follows

$$I_2(t) = -\int_\Omega \left\{(n-n^*, p-p^*)\begin{pmatrix} k(n,p) & l(n,p) \\ * & m(n,p)\end{pmatrix}\begin{pmatrix} n-n^* \\ p-p^*\end{pmatrix}\right\} d\Omega\,(22)$$

where

$$k(n,p) = 1 - \frac{p^*}{(n^*+\alpha p^*)(n+\alpha p)} - \frac{g}{(h+n^*)(h+n)}$$
$$l(n,p) = \frac{1}{2}\frac{n^*}{(1+\alpha u^*+\beta v^*)(1+\alpha u+\beta v)} - \frac{1}{2}\frac{1}{\gamma(u+\delta)}$$
$$m(n,p) = \frac{1}{n+\delta}.$$

It is obvious that $\frac{dV}{dt} < 0$ if and only if the matrix integrand of (22) is positive define, which is equivalent to $\phi_1(n,p) = k + m > 0$ and $\phi_2(n,p) = mk - l^2 > 0$, where

$$\phi_1 = 1 + \frac{1}{n+\delta} - \frac{p^*}{(n^*+\alpha p^*)(n+\alpha p)} - \frac{g}{(h+n^*)(h+n)}, \qquad (23)$$

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$$\phi_2 = \frac{1}{n+\delta} - \frac{p^*}{(n+\delta)(n^*+\alpha p^*)(n+\alpha p)} - \frac{g}{(n+\delta)(h+n^*)(h+n)} - \frac{n^{*2}}{4(n^*+\alpha p^*)^2(n+\alpha p)^2}$$
$$-\frac{1}{4\gamma^2(u+\delta)^2} + \frac{1}{2}\frac{n^*}{(n^*+\alpha p^*)(n+\alpha p)\gamma(n+\delta)}. \qquad (24)$$

By the conditions (20) and (21), we conclude that $\phi_1(n,p) > 0$ and $\phi_2(n,p) > 0$.Hence, the equilibrium point E^* is globally asymptotically stable.

IV. THE EXISTENCE AND NON-EXISTENCE OF NON-CONSTANT POSITIVE EQUILIBRIA

In this section, our aim is to study the existence and nonexistence of non-constant positive solution of steady state problem (25). Consider the steady-state problem of (4) as follows

$$-d_1\nabla^2 n = n\left(1 - n - \frac{p}{n+\alpha p} - \frac{g}{h+u}\right), \qquad x \in \Omega \qquad (25a)$$
$$-d_2\nabla^2 p = \beta p\left(1 - \frac{\gamma v}{u+\delta}\right), \qquad x \in \Omega, \qquad (25b)$$
$$\partial_* n = \partial_* p = 0, \qquad x \in \partial\Omega (25c)$$

4.1 A priori estimate

Now, we introduce some lemmas which used in this section.

Lemma 2.*(Maximum Principle). Let $g(x,w) \in C(\Omega \times \mathbb{R}^1)$ and $b_j(x) \in C(\omega), j = 1,2,\cdots,N$.*

1. if $w(x) \in C^2(\Omega) \cap C^1(\bar{\Omega})$ satisfies $\Delta w(x) + \sum_{j=1}^{N} b_j(x) w_{x_j} + g(x, w(x)) \geq 0$ in $\Omega, \frac{\partial w}{\partial n} \leq 0$, and $w(x_0) = max_{\bar{\Omega}}\, w$ then $g(x_0, w(x_0)) \geq 0$.

2. if $w(x) \in C^2(\Omega) \cap C^1(\bar{\Omega})$ satisfies $\Delta w(x) + \sum_{j=1}^{N} b_j(x) w_{x_j} + g(x, w(x)) \leq 0$ in $\Omega, \frac{\partial w}{\partial n} \geq 0$, and $w(x_0) = min_{\bar{\Omega}}\, w$ then $g(x_0, w(x_0)) \leq 0$.

Lemma 3. *(Harnack Inequality). Let $c(x) \in C(\bar{\Omega})$, and $w(x) \in C^2(\Omega) \cap C^1(\Omega)$ be positive solution to $\Delta w(x) + c(x)w(x) = 0$ in Ω subject to homogeneous Neumann boundary condition. There exists a positive constant $C = C(N, \Omega, \|c(x)\|_\infty)$ such that $max_{\bar{\Omega}}\, w(x) \leq C\, min_{\bar{\Omega}}\, w(x)$.*

Theorem 7.*(Upper bounds). For any positive solution of (25)*

$$\max_{x\in\Omega} n(x) \leq 1 and \max_{x\in\Omega} p(x) \leq \frac{1+\delta}{\gamma}. \qquad (26)$$

Proof. By Theorem 1 and comparison argument to (25), we easily obtain (26).

Theorem 8.*(Lower bounds). Let d be a fixed positive constant. Then for $d_1, d_2 > d$, there exists a positive constant $\underline{C} = \underline{C}(d)$ such that*

$$\min_{x\in\Omega} n(x) \geq C and \min_{x\in\Omega} p(x) \geq C. \qquad (27)$$

*Proof.*Let

$$n(x_1) = \min_{x\in\Omega} n(x), n(x_2) = \max_{x\in\Omega} n(x), p(y_1) = \min_{y\in\Omega} p(y), p(y_2) = \max_{y\in\Omega} p(y).$$

By Lemma (2), we have

$$1 \leq n(x_2) + \frac{p(x_1)}{n(x_1)+\alpha p(x_1)} + \frac{g}{h+n(x_1)} (28a)$$
$$1 \leq \frac{\gamma p(y_1)}{n(y_1)+\delta} (28b)$$
$$1 \geq \frac{\gamma p(y_2)}{n(y_2)+\delta}. \qquad (28c)$$

From (28b) and (28c), we respectively get

$$\frac{\delta + n(x_1)}{\gamma} \leq \frac{\delta + n(y_1)}{\gamma} \leq p(y_1), \qquad (29)$$

and

$$\frac{\delta + n(x_2)}{\gamma} \leq \frac{\delta + n(y_2)}{\gamma} \leq p(y_2), \qquad (30)$$

Since $0 < \frac{1}{n(x_1)+\alpha p(x_1)} < 1$ and (30), it follows that

$$1 \leq n(x_1) + \frac{p(x_1)}{n(x_1)+\alpha p(x_1)} + \frac{g}{h+n(x_1)}$$
$$\leq n(x_1) + p(x_1) + \frac{g}{h}$$
$$\leq n(x_1) + p(y_2) + \frac{g}{h}$$

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$\le n(x_1) + \dfrac{\delta + n(x_2)}{\gamma} + \dfrac{g}{h}$
$\le n(x_1) + Mn(x_2)$, (31)
where$Q$ is a large enough positive constant, such that $\gamma^{-1}(\delta + n(x_2)) + gh^{-1} \le Qn(x_2)$. Define
$c(x) = d_1^{-1}\left(1 - n - \dfrac{p}{n + \alpha p} - \dfrac{g}{h + n}\right),$
then by Lemma 3, it is known that
$n(x_2) \le C_1 n(x_1)$(32)
whereC_1 depends on $\|c(x)\|$. From (31), we obtain

$$\min_{x\in\Omega} n(x) = n(x_1) \ge (MC_1 + 1)^{-1} \coloneqq C_2 . \quad (33)$$

From (29)

$$\min_{x\in\Omega} p(x) = p(y_1) \ge \gamma^{-1}\delta + \gamma^{-1}(MC_1 + 1)^{-1} \coloneqq C_3 . \quad (34)$$

Finally, let $\underline{C} = min\{C_2, C_3\}$which completes the proof.

4.2 Nonexistence of nonconstant positive equilibrium

Theorem 9. There exist two constants d_1^*, d_2^* and if $d_1 > d_1^*$and $d_2 > d_2^*$ then the system (25) has no constant solutions.

*Proof.*Suppose that (n, p) is a positive solution of (25), and let $\overline{n} = \frac{1}{\Omega}\int_\Omega n(x)\, dx$and
$\overline{p} = \frac{1}{\Omega}\int_\Omega n(x)\, dx$. By multiplying $n - \overline{n}$to (25a), then integrating on $\Omega$and noting no-flux boundary condition, we arrive that

$$d_1 \int_\Omega |\Delta(n - \overline{n})|^2\, dx = \int_\Omega g_1(n,p)(n - \overline{n})\, dx = \int_\Omega (n - \overline{n})\big(g_1(n,p) - g_1(\overline{n},\overline{p})\big)\, dx$$
$$= \int_\Omega (n - \overline{n})\left(n - n^2 - \frac{np}{n + \alpha p} - \frac{gn}{h + n} - \overline{n} - \overline{n}^2 - \frac{\overline{n}p}{\overline{n} + \alpha\overline{p}} - \frac{g\overline{n}}{h + \overline{n}}\right) dx$$
$$= \int_\Omega \left\{\left(1 - n - \overline{n} - \frac{\alpha p\overline{p}}{(n + \alpha p)(\overline{n} + \alpha\overline{p})} - \frac{gh}{(h + u)(h + \overline{u})}\right)\left(u - \overline{u}^2\right) - \frac{n\overline{n}(n - \overline{n})(p - \overline{p})}{(n + \alpha p)(\overline{n} + \alpha\overline{p})}\right\} dx.$$

According to Theorem 7 and 8, it follows that

$$d_1 \int_\Omega |\Delta(n - \overline{n})|^2\, dx \le \int_\Omega \left(1 - \frac{\alpha\gamma^2\underline{C}^2}{\big(\gamma + \alpha(1 + \delta)\big)^2} - \frac{gh}{(h + 1)^2}\right)\left(n - \overline{n}^2\right) dx$$
$$- \int_\Omega \left(\frac{\gamma^2\underline{C}^2}{\big(\gamma + \alpha(1 + \delta)\big)^2}\right)(n - \overline{n})(p - \overline{p})\, dx . \quad (35)$$

By the same way, we can get the following

$$d_2 \int_\Omega |\Delta(p - \overline{p})|^2\, dx = \int_\Omega g_2(n,p)(p - \overline{p})\, dx = \int_\Omega (p - \overline{p})\big(g_2(n,p) - g_2(\overline{n},\overline{p})\big)\, dx$$
$$= \beta \int_\Omega (p - \overline{p})\left(p - \frac{\gamma p^2}{n + \delta} - \overline{p} - \frac{\gamma\overline{p}^2}{\overline{n} + \delta}\right) dx$$
$$= \int_\Omega \left\{\left(1 - \frac{\gamma(p + \overline{p})}{n + \delta}\right)(p - \overline{p})^2 dx + \frac{\gamma\overline{p}^2(n - \overline{n})(p - \overline{p})}{(n + \delta)(\overline{n} + \delta)}\right\} dx$$
$$= \int_\Omega \left(\beta - \frac{2\beta\gamma\underline{C}}{1 + \delta}\right)(p - \overline{p})^2 dx + \int_\Omega \frac{\beta(1 + \delta)^2}{\gamma\big(\underline{C} + \delta\big)^2}(n - \overline{n})(p - \overline{p})dx.$$

Thus, by Young's inequality, we obtain

$$d_1 \int_\Omega |\Delta(n - \overline{n})|^2\, dx \;+\; d_2 \int_\Omega |\Delta(p - \overline{p})|^2\, dx$$
$$\le \int_\Omega \left(1 - \frac{\alpha\gamma^2\underline{C}^2}{\big(\gamma + \alpha(1 + \delta)\big)^2} - \frac{gh}{(h + 1)^2}\right)(u - u^2)\, dx + \int_\Omega \beta\left(1 - \frac{2\gamma\underline{C}}{1 + \delta}\right)(p - \overline{p})^2 dx$$
$$+ \int_\Omega \left(\frac{\beta(1 + \delta)^2}{\gamma\big(\underline{C} + \delta\big)^2} - \frac{\gamma^2\underline{C}^2}{\big(\gamma + \alpha(1 + \delta)\big)^2}\right)(n - \overline{n})(p - \overline{p})\, dx ,$$
$$\le \int_\Omega \left\{1 - \frac{\alpha\gamma^2\underline{C}^2}{\big(\gamma + \alpha(1 + \delta)\big)^2} - \frac{gh}{(h + 1)^2} + \frac{1}{4\epsilon}\left(\frac{\beta(1 + \delta)^2}{\gamma\big(\underline{C} + \delta\big)^2} - \frac{\gamma^2\underline{C}^2}{\big(\gamma + \alpha(1 + \delta)\big)^2}\right)\right\}\left(n - \overline{n}^2\right) dx$$

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$$+\int_{\Omega}\left\{\beta-\frac{2\beta\gamma\underline{C}}{1+\delta}+\epsilon\left(\frac{\beta(1+\delta)^2}{\gamma(\underline{C}+\delta)^2}-\frac{\gamma^2\underline{C}^2}{(\gamma+\alpha(1+\delta))^2}\right)\right\}(p-\overline{p})^2dx$$

whereϵ is an enough small positive value. Using the Poincare inequality, we obtain

$$\int_{\Omega}(d_1\mu_2|(n-\overline{n})|^2\,dx\;+\int_{\Omega}d_2\mu|(p-\overline{p})|^2)dx\le\int_{\Omega}(d_1|\Delta(n-\overline{n})|^2\,dx\;+\int_{\Omega}d_2|\Delta(p-\overline{p})|^2)\,dx.$$

Let

$$d_1^*=\frac{1}{\mu_2}\left\{1-\frac{\alpha\gamma^2\underline{C}^2}{(\gamma+\alpha(1+\delta))^2}-\frac{gh}{(h+1)^2}+\frac{1}{4\epsilon}\left(\frac{\beta(1+\delta)^2}{\gamma(\underline{C}+\delta)^2}-\frac{\gamma^2\underline{C}^2}{(\gamma+\alpha(1+\delta))^2}\right)\right\}$$

$$d_2^*=\frac{1}{\mu_2}\left\{\beta-\frac{2\beta\gamma\underline{C}}{1+\delta}+\epsilon\left(\frac{\beta(1+\delta)^2}{\gamma(\underline{C}+\delta)^2}-\frac{\gamma^2\underline{C}^2}{(\gamma+\alpha(1+\delta))^2}\right)\right\}.$$

If $d_1>d_1^*$and $d_2>d_2^*$ then we can obtain $n=\overline{n}$and $p=\overline{p}$. Hence, the proof is complete.

4.3 Existence of non-constant positive equilibria

Now, we will study the existence of non-constant solutions of (25). From now on, the diffusion coefficients d_1,d_2 vary, while other parameters are kept fixed. Based on Theorem 4, the necessary conditions of no non-constant solution of (25) are $1<\frac{p^*}{(n^*+\alpha p^*)^2}+\frac{g}{(h+n^*)^2}$and $(h-g)\alpha\delta<\delta h$, so we assume these conditions throughout this section. By Theorem 6 and 7, there exists a constant C and define

$B(C):=\{(n,p)\in X|C^{-1}<n,p<C\}.$ (37)

Equation (25) can be rewritten as follows

$-\Delta\boldsymbol{z}=D^{-1}G(\boldsymbol{z})$ (38)

$\frac{\partial\boldsymbol{z}}{\partial n}=0.$ (39)

Thus $\boldsymbol{z}$ is positive solution to (25) if and only if

$F(d_1,d_2,\boldsymbol{z}):=\boldsymbol{z}-(I-\Delta)^{-1}[D^{-1}G(\boldsymbol{z})+\boldsymbol{z}]=0,\qquad \text{on } X,$ (40)

where$I$ is an identity operator and $(I-\Delta)^{-1}$ is the inverse of $(I-\Delta)$. Since F is compact perturbation of the identity operator, the Leray-Schauder degree deg(F,0,B(C)) is well-defined if $G\ne 0$ on ∂B. Furthermore, we note that

$D_{\boldsymbol{z}}(d_1,d_2,E^*):=I-(I-\Delta)^{-1}(D$^(−1) G_$\boldsymbol{z}$ $(E$^ ∗ $)+I),$ (41)

we remember that if $D_{\boldsymbol{z}}F(d_1,d_2,E^*)$ is invertible, the index $G(d_1,d_2,\boldsymbol{z})$ at the isolated fixed point E^* is defined as index $F(d_1,d_2,E^*)=(-1)^r$, where r is the no. of eigenvalues of $D_{\boldsymbol{z}}F(d_1,d_2E^*)$ with –ve real parts. If $G\ne 0$on $\partial B(C)$, then the degree $\deg(F(d_1,d_2,\boldsymbol{z}),0,B(C))$ is equal to the sum of the indexes over all isolated solutions to $F(d_1,d_2,\boldsymbol{z}):=0$ in $B(C)$.

By using decomposition (14), we will discuss the eigenvalues of $D_{\boldsymbol{z}}F(d_1,d_2,E^*)$. First, we known W_{ij} is invariant under $D_{\boldsymbol{z}}F(d_1,d_2E^*)$ for each integer $i\ge 1$ and each $1\le j\le\dim E(\mu_i)$.Thusλ is an eigenvalue of $D_{\boldsymbol{z}}F(d_1,d_2E^*)$ on W, if and only if it is an eigenvalue of the matrix

$1-\frac{1}{1+\mu_i}\{D^{-1}G_{\boldsymbol{z}}(E^*)+I\}=\frac{1}{1+\mu_i}\{\mu_iI-D^{-1}G_{\boldsymbol{z}}(E^*)\}.$ (42)

Thus, $D_{\boldsymbol{z}}F(d_1,d_2,E^*)$ is invertible if and only if

$1-\frac{1}{1+\mu_i}\{D^{-1}G_{\boldsymbol{z}}(E^*)+I\}$ (43)

is non-singular. Thus λ is an eigenvalue of $D_{\boldsymbol{z}}F(d_1,d_2,E^*)$ on W_i if and only if $\lambda(1+\mu_i)$ is an eigenvalue of M_i

$M_i:=\mu_iI-D^{-1}G_{\boldsymbol{z}}(E^*)=\begin{pmatrix}\mu_i-d_1^{-1}g_{11}(E^*) & -d_2^{-1}g_{12}(E^*)\\ -d_2^{-1}g_{21}(E^*) & \mu_i-d_2^{-1}g_{22}(E^*)\end{pmatrix}.$ (44)

Obviously,

$\det M_i=d_1^{-1}d_2^{-1}\{d_1d_2\mu_i^2-[d_2^{-1}g_{11}(E^*)+d_1^{-1}g_{22}(E^*)]\mu_i+\det G_{\boldsymbol{z}}(E^*)\}$ (45)

The trace of M_i is

$tr\,M_i=2\mu_i-d_1^{-1}g_{11}(E^*)-d_2^{-1}g_{22}(E^*)$ (46)

Let $H(d_1,d_2,\mu)=d_1d_2\mu-[d_2^{-1}g_{11}(E^*)+d_1^{-1}g_{22}(E^*)]\mu+\det G_{\boldsymbol{z}}(E^*)$, then $H(d_1,d_2,\mu_i)=d_1d_2\det M_i$, If $[d_2^{-1}f_{11}(E^*)+d_1^{-1}f_{22}(E^*)]^2>4d_1d_2\det G_{\boldsymbol{z}}(E^*).$ (47)

Then $H(d_1,d_2,\mu)=0$ has no real parts:

$\mu_+(d_1,d_2,\mu)=\frac{d_1g_{22}(E^*)+d_2g_{11}(E^*)+\sqrt{[d_2^{-1}f_{11}(E^*)+d_1^{-1}f_{22}(E^*)]^2-4d_1d_2\det G_{\boldsymbol{z}}(E^*)}}{2d_1d_2},$ (48)

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$$\mu_-(d_1,d_2,\mu)=\frac{d_1g_{22}(E^*)+d_2g_{11}(E^*)-\sqrt{[d_2^{-1}f_{11}(E^*)+d_1^{-1}f_{22}(E^*)]^2-4d_1d_2\det G_{\mathbf{z}}(E^*)}}{2d_1d_2}. \quad (49)$$

Let $A(d_1,d_2)=\{\mu|\mu\geq 0,\mu_+(d_1,d_2)<\mu<\mu_-(d_1,d_2)\}, S_p=\{\mu_1,\mu_2,\cdots\}$ and $m(\mu_i)$ be the multiplicity of the eigenvalue μ_i.

Lemma 4. *Suppose $H(d_1,d_2,\mu)\neq 0$, for all $\mu_i\in S_p$, then*

$$index\ (G(d,d_2,\cdot),z^*)=(-1)^{\sigma}, \quad (50)$$

where

$$\sigma=\begin{cases}\sum_{\mu_i\in A\cap S_p} m(\mu_i), & A\cap S_p\neq\Phi,\\ 0, & A\cap S_p=\Phi.\end{cases} \quad (51)$$

In particular, if $H(d_1,d_2,\mu)>0$ for any $\mu>0$, then $\sigma=0$.

Theorem 10. *Assume $1<\frac{p^*}{(n^*+\alpha p^*)^2}+\frac{g}{(h+n^*)^2}, (h-g)\alpha\delta<\delta h$ and $f_{11}/d_1\in(\mu_n,\mu_{n+1})$ for some n and $r_n=\sum_{i=1}^{n} m(\mu_i)$ is odd, then there exists $d^*>0$, such that (25) has atleast one non-constant positive solution when $d_2\geq d^*$.*

Proof. If (10) holds with conditions H1 or H2 or H3, then (25) possess unique positive constant equilibrium E^*. Furthermore, if $1<\frac{p^*}{(n^*+\alpha p^*)^2}+\frac{g}{(h+n^*)^2}$ then $g_{11}(E^*)>0$ and $f_{22}(E^*)>0$ It follows that if d_2 is large enough then (47) is easily obtained, and

$$0<\mu_-(d_1,d_2)<\mu_+(d_1,d_2)=0. \quad (52)$$

Furthermore,

$$\lim_{d_2\to\infty}\mu_+(d_1,d_2)=\frac{f_{11}}{d_1}, \lim_{d_2\to\infty}\mu_-(d_1,d_2)=0. \quad (53)$$

Since $f_{11}/d_1\in(\mu_n,\mu_{n+1})$, there exists $d_0>>1$ such that

$$\mu_+(d_1,d_2)\in(\mu_n,\mu_{n+1}), 0<\mu_-(d_1,d_2)<\mu_2, \text{for any} d_2\geq d_0. \quad (54)$$

According to Theorem 9, it is evident that there exists a large enough d_0 such that $d_1>d_0$ and (25) corresponding to $d_1=d, d_2\geq d$ has no non-constant positive solution. Moreover, we can take enough large d such that $0<f_{11}/d_1<\mu_2$, so there exists $d^*>d$ such that

$0<\mu_{-1}(d_1,d_2)<\mu_2$, for any$d_2\geq d^*$(55)

Now we are going to prove (25) has at least one non-constant positive solution by the namely, we assume that this assertion is not true for $d_2\geq d^*$. Then, a required contradiction is made by using a homotopy argument. Fix $d_2=d^*$ and define

$$D(t)=\begin{pmatrix} td_1+(1-t)d & 0\\ 0 & td_2+(1-t)d^*\end{pmatrix} (56)$$

and consider the following problem

$$-\Delta\mathbf{z}=D^{-1}G(\mathbf{z})$$
$$\frac{\partial\mathbf{z}}{\partial n}=0. \quad (57)$$

Then $\mathbf{z}$ is non-constant positive solution of (25) if and only if it is a non-constant positive solution of equation (57) when $t=1$. It is clear that E^* is unique constant positive solution of (57) for $t\in[0,1]$ We know that $\mathbf{z}$ is positive solution of (57) for $t\in[0,1]$ if and only if

$$G(t,\mathbf{z}):=\mathbf{z}-(I-\Delta)^{-1}[D^{-1}G(\mathbf{z})+\mathbf{z}]=0. \quad (58)$$

where$\mathbf{z}\in X_+$. Obviously, $F(1,\mathbf{z})=F(d_1,d_2,\mathbf{z})$, see equation (40). Via Theorem 9, it is well known that $F(0,\mathbf{z})=0$ has only one positive constant solution E^* in X_+.By direct computation, we obtain

$$D_{\mathbf{z}}F(t,E^*)=I-(I-\Delta)^{-1}(D^{-1}G_{\mathbf{z}}(E^*)+I). \quad (59)$$

In specific

$$D_{\mathbf{z}}F(0,E^*)=I-(I-\Delta)^{-1}(\widetilde{D}^{-1}G_{\mathbf{z}}(E^*)+I) \quad (60)$$

$$D_{\mathbf{z}}F(1,E^*)=I-(I-\Delta)^{-1}(D^{-1}G_{\mathbf{z}}(E^*)+I)=D_{\mathbf{z}}F(d_1,d_2,E^*), \quad (61)$$

where$\widetilde{D}^{-1}=diag\{d,d^*\}$ From (54) and (55), we have $A(d_1,d_2)\cap S_p$ and $A(d,d^*)\cap S_p$ Since r_n is odd, Lemma 4 yields

$$\text{index}(F(1,\cdot),E^*)=(-1)^{r_n}=-1, \text{index}(F(0,\cdot),E^*)=(-1)^0=-1. \quad (62)$$

From Theorem 7 and 8, there exists $\underline{C}(d,d_1,d^*,d_2^*,\Lambda)>0$ and $\overline{C}(d,d_1,d^*,d_2^*,\Lambda)>0$ such that the positive solution of (25) satisfies $\underline{C}<n(x),p(x)\overline{C}$ for all $t\in[0,1]$. Define $B^*=\{\mathbf{z}\in\{X\}^2,\underline{C}\leq n(x),p(x)\overline{C},x\in\Omega\}$ then $F(t,\mathbf{z})\neq 0$ for all $\mathbf{z}\in\partial\mathrm{B}^*$ and $t\in[0,1]$.By using homotopy invariance of the Leray-Schauder degree, we get

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$\deg(F(1,\cdot), B^*, 0) = \deg(F(0,\cdot), B^*, 0).$ (63)

Note that both equations $F(0, \boldsymbol{z}) = 0$ and $F(1, \boldsymbol{z}) = 0$ have unique positive solutions u^*in B^*, then

$\deg(F(0,\cdot), B^*, 0) = \mathrm{index}(F(0,\cdot), E^*) = 1,$ (64)

$\deg(F(0,\cdot), B^*, 0) = \mathrm{index}(F(1,\cdot), E^*) = -1,$ (65)

which contradict (63). The proof is complete.

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