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**Research Paper**

# Some Fuzzy Basic Concepts And Features

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***ABSTRACT:*** *In this study, some basic concepts and features such as fuzzy set, fuzzy point, fuzzy coincident and fuzzy hausdorff space were studied in studies done by Wong [5], Ming [2], Srivastava [1] and Yalvaç [4]. Some of the theorems and propositions given only in these works have been proven in these studies.*
***KEYWORDS:*** *Fuzzy set, fuzzy point, fuzzy quasi-coincident, fuzzy membership, fuzzy concepts*

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## I. INTRODUCTION

The concept of fuzzy clusters, first proposed by Zadeh in 1965, has led to the mathematical testing of the fuzzy concept existing in our world and the creation of new branches of mathematics. The concept of fuzzy sets, which correspond to unexplained physical states, is now being used in a number of useful ways, such as computing, engineering, language arts, space, aviation and statistics. Since the introduction of the fuzzy set concept, many researches have been made and it has been researched whether many concepts and theorems in mathematics are applied to fuzzy theory. In 1968, Fuzzy topological space was first defined by Chang. In his research, Chang also described some important concepts such as continuity, convergence and compactness in fuzzy topological spaces. In 1981, Azad did useful work on fuzzy continuity types. In 1987 Yalvaç made various studies on fuzzy clusters and fuzzy functions in fuzzy topology. After Zadeh's work on fuzzy clusters in 1965 and Chan's definition of fuzzy topological space in 1968, many researchers have discovered many of the concepts in general topology in fuzzy topological spaces. Using the definition of fuzzy set, we have studied various fields such as fuzzy topological spaces, fuzzy vector spaces, fuzzy groups, fuzzy integrals, fuzzy dual space and fuzzy measures.

## II. SOME BASIC CONCEPTS

In this section, some basic concepts defined in fuzzy topological space and theorems and proposals related to these concepts are given.

**Definition 2.1.** X={x} is a subset of A, X, with a set of non-empty points. The denominator A denoted by $\mu_A$ membership function defined in X den I = [0, 1] closed interval is called the fuzzy set in X [6]. Here, the membership function of the fuzzy set of the $\mu_A$ function and the membership function of the fuzzy set of the x-point to the $\mu_A$ (x) value of $x \in X$ are called the membership grade value.

**Definition 2.2.** Let's get two fuzzy sets A and B of X. The membership functions of the A and B fuzzy sub-clusters are denoted by $\mu_A(x)$ and $\mu_B(x)$ for $x \in X$, respectively.
**a)** If the membership functions of A and B at each point of X are equal to each other, then the A and B fuzzy sets are equal and written as A = B briefly;

$$\mathbf{A = B \Leftrightarrow \mu_A(x) = \mu_B(x), \text{ for all } x \in X} \quad [6]. \qquad \mathbf{(1)}$$

**b)** for all $x \in X$, $\mu_A(x) \le \mu_B(x)$ is called the fuzzy subset of A or B and written $A \subset B$; Briefly;

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* This study was produced from the doctoral dissertation of the author in 1996.

$$A \subset B \Leftrightarrow \mu_A(x) \le \mu_B(x), \text{ for all } x \in X \text{ [6].} \qquad (2)$$

**c)** $$C = A \cup B \Leftrightarrow \mu_C(x) = \underset{x \in X}{\mathrm{Max}} \{\mu_A(x), \mu_B(x)\}, \text{ for all } x \in X \qquad (3)$$

This expression is called the association of fuzzy clusters A and B and is defined by the membership function $\mu_{A \cup B}(x)$ [6].

**d)** $$D = A \cap B \Leftrightarrow \mu_D(x) = \underset{x \in X}{\mathrm{Min}} \{\mu_A(x), \mu_B(x)\}, \text{ for all } x \in X \qquad (4)$$

This expression is called the intersection of the fuzzy clusters A and B and is denoted by the function $\mu_{A \cap B}(x)$ [6].

**Definition 2.3.** for $p \in X$ ,

$$\mu_p(x) = \begin{cases} \lambda, & x = x_p \quad (0 < \lambda < 1 \\ 0, & x \neq x_p \end{cases} \qquad (5)$$

A special fuzzy set defined by the membership function is also called a fuzzy point for X [5], [1]. Here p is a fuzzy point. p is called the support of p at the $x_p$ point at which the fuzzy point gets value (Support) and is represented by Dayp = $x_p$ (Supp p = $x_p$). $\lambda$ is called the value of p [5].

A is a fuzzy set, where p is a fuzzy point, and $\mu_p(x_p) < \mu_A(x_p)$ is the element of p if A and is denoted by $p \in A$ [5].

Ming [2] defines the definition of a fuzzy point and a fuzzy set of such points as follows

**Definition 2.4.** for $p \in X$ ,

$$\mu_p(x) = \begin{cases} \lambda, & x = x_p \quad (0 < \lambda < 1 \\ 0, & x \neq x_p \end{cases} \qquad (6)$$

$\mu_p : X \to [0,1]$, which is defined by the membership function defined by X, is called the fuzzy point of X in p fuzzy set. $x_p$ is the p nin durability. $\lambda$ is called the value of p [2].

A is a fuzzy set, p is a fuzzy point, if $\mu_p(x_p) \le \mu_A(x_p)$, p is the element of the A and is denoted by $p \in A$ [2].

The definition of $p \in_1 A$ is denoted by Definition 2.3 and the definition of $p \in_2 A$ by Definition 2.4. Also, if you say $p \in A$, it is assumed to be valid in both definitions [4].

**Theorem 2.1.** Let A and B be two fuzzy subsets of X. In that case,

$$A \subset B \Leftrightarrow (p \in_1 A \Rightarrow p \in_1 B), \text{ for all } x \in X \text{ [4].} \qquad (7)$$

**Theorem 2.2.** Let A and B be two fuzzy subsets of X. In that case,

**a)** $$A \subset B \Leftrightarrow (p \in A \Rightarrow \mu_p(x_p) \le \mu_B(x_p)), \text{ for all } p \in X \qquad (8)$$

**b)** $$A \subset B \Leftrightarrow (p \in_2 A \Rightarrow p \in_2 B), \text{ for all } p \in_2 X \text{ [4].}$$

**Proof: a)** $\Rightarrow$ **:** If $A \subset B$ then $\mu_A(x) \le \mu_B(x)$ is written for each $x \in X$ accordimg to equation (2). If $p \in A$, then for each $x_p \in X$ accordimg to equation (5) ve (6),

$\mu_p(x_p) \le \mu_A(x_p)$ written. From here, for all $x_p \in X$

$\mu_p(x_p) \le \mu_A(x_p) \le \mu_B(x_p) \Rightarrow \mu_p(x_p) \le \mu_B(x_p)$, we obtain.

$\Leftarrow$ **:**Let $A \not\subset B$ be. In that case, there is $x_p \in X$. So that $\mu_B(x_p) \le \mu_A(x_p)$ becomes.

$p \in A \Rightarrow \mu_p(x_p) \le \mu_A(x_p)$

$\Rightarrow \mu_B(x_p) \le \mu_P(x_p) \le \mu_A(x_p)$

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$\Rightarrow \mu_B(x_p) \leq \mu_P(x_p)$ written. This contradicts equation (5). In that case $A \subset B$ becomes.

**b)** $\Rightarrow$ **:** Let $A \subset B$ and $p \in_2 A$ be. In that case, let's show $p \in_2 B$. If $A \subset B$ then $\mu_A(x) \leq \mu_B(x)$ can be written for each $x \in X$ according to equation (2). Again, If $p \in_2 A$, then according to equation (6), $\mu_P(x_p) \leq \mu_A(x_p) \leq \mu_B(x_p)$ is written for each $x_p \in X$ için $\mu_P(x_p) \leq \mu_A(x_p) \leq \mu_B(x_p)$. Here, for each $x_p \in X$, $\mu_P(x_p) \leq \mu_B(x_p)$ is obtained. This indicates $p \in_2 B$.

$\Leftarrow$ **:** Assume that $p \in_2 A$ and $p \in_2 B$, but assume that it is not $A \not\subset B$. Accordingly, for some $x \in X$ becomes $\mu_B(x) < \mu_A(x)$. For $x \in X$ and Sup p = x, If $\mu_B(x) \leq \mu_P(x) \leq \mu_A(x)$ is satisfied, $p \in_2 A$ is satisfied but $p \notin_2 B$. Because $p \in_2 B$ means that $\mu_P(x) \leq \mu_B(x)$ is. It is then contrary to our acceptance that $p \in_2 A$ ve $p \notin_2 B$. So it becomes $A \subset B$.

**Theorem 2.3.** Let A and B be two fuzzy subsets of X. In that case;

$$\mathbf{A = B \Leftrightarrow \text{for all } p \in X \ (p \in A \Leftrightarrow p \in B)} \ [4]. \quad \mathbf{(9)}$$

**Proof:** Theorem 2.1. and Theorem 2.2. is a result obtained.

**Teorem 2.4.** Let A and B be two fuzzy subsets of X. In that case;

$$\mathbf{p \in_1 A \cap B \Leftrightarrow p \in_1 A \ \text{ and } \ p \in_1 B} \ [4]. \quad \mathbf{(10)}$$

**Teorem 2.5.** Let A and B be two fuzzy subsets of X. In that case;

$$p \in_2 A \cap B \Leftrightarrow p \in_2 A \ \text{ and } \ p \in_2 B \ [4]. \quad \mathbf{(11)}$$

**Proof :** $p \in_2 A \cap B \Leftrightarrow \mu_p(x_p) \leq \mu_{A \cap B}(x_p)$

$\Leftrightarrow \mu_p(x_p) \leq \text{İnf}\{\mu_A(x_p), \mu_B(x_p)$

$\Leftrightarrow \mu_p(x_p) \leq \mu_A(x_p)$ and $\mu_p(x_p) \leq \mu_B(x_p)$ is written. According to (6) equation is written.

$\Leftrightarrow p \in_2 A$ ve $p \in_2 B$,

Using the equation (11) and the proof of theorem 2.5, the following theorem 2.6 can easily be proved

**Theorem 2.6.** Let A and B be two fuzzy subsets of X. In that case;

$$p \in_1 A \cup B \Leftrightarrow p \in_1 A \ \text{ or } \ p \in_1 B \ [4]. \quad \mathbf{(12)}$$

**Proof :** $p \in_1 A \cup B \Leftrightarrow \mu_p(x_p) < \mu_{A \cup B}(x_p)$

$\Leftrightarrow \mu_p(x_p) < \text{Sup}\{\mu_A(x_p), \mu_B(x_p)\}$

$\Leftrightarrow \mu_p(x_p) < \mu_A(x_p)$ or $\mu_p(x_p) < \mu_B(x_p)$,

$\Leftrightarrow p \in_1 A$ or $p \in_1 B$ is obtained.

**Definition 2.5**. A, X is a fuzzy set and p is the fuzzy point, it is called pqA and is denoted as pqA if $\mu_p(x_p) + \mu_A(x_p) > 1$ or $\mu_p(x_p) > \mu_{A'}(x_p)$, ( p, coincited with A) [2]. **(13)**

**Definition 2.6.** For A and B fuzzy subsets, if there is a point $x \in X$ such that $\mu_A(x) + \mu_B(x) > 1$ or $\mu_A(x) > \mu_{B'}(x)$, it is called coincident A and B and is denoted by AqB. If A and B are not coincident of fuzzy clusters, they are expressed as $A \not{q} B$ [2]. **(14)**

**Definition 2.7.** Let $p, q \in X$. If Dayp $\neq$ Dayq then p and q are called different fuzzy points [1].

**Teorem 2.7.** Let A and B be two fuzzy subsets of X. In that case; **(15)**

**a)** It is necessary and sufficient condition for $A \subset B$ that A and B are not coincident

**b)** If fuzzy clusters A and B for $x \in X$ are coincident in X, then A and B fuzzy clusters mean that they cut each other.

**c)** It is necessary and sufficient condition to be $p \in_2 A$ that it does not coincident to p and A' [2].

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**Proposition 2.1.** $f : X \to Y$ is a function and $p \in X$, In that case;
**a)** For the fuzzy set $B \subset Y$, If f(p)qB is pq$f^{-1}$(B) **(16)**
**b)** For the fuzzy set $A \subset X$, If pqA is f(p) q f(A) [4].

**Proposition 2.2.** A is a fuzzy set, and if a puzzy point can be chosen as $\mu_P(x) = 1 - \lambda$ for any $\lambda$ such that $\mu_A(x) = t \neq 0$ for $x \in X$ $0 < \lambda < t$ p fuzzy point A fuzzy set becomes coincident with. **(17)**

**Proof :** Let $\mu_A(x) = t$ for all $x \in X$, where A is the fuzzy subset. In this case it will be shown that p and A are coincident. (that is, pqA ). If you choose a p fuzzy point for a A such that $\mu_P(x) = 1 - \lambda$, $(0 < \lambda < t)$, Let (14) denote $\mu_P(x) + \mu_A(x) > 1$ for each $x \in X$. For every $x \in X$, write $\mu_P(x) + \mu_A(x) = 1 - \lambda + t > 0$. If $0 < \lambda < t$ then $t - \lambda > 0$ and $-t < -\lambda < 0$. Thus,

$$-t < -\lambda < 0 \Rightarrow 1+t-t < -\lambda+1+t < 0+1+t$$

$\Rightarrow 1 < 1-\lambda+t < 1+t$ obtained. So, $1-\lambda+t > 1$ is obtained. This indicates that the p fuzzy point and the A fuzzy set are in accordance with the expression (13).

**Definition 2.8 :** For p and r different fuzzy points, $p \in_1 X$, $r \in_1 X$ are called $p \in_1 U$, $r \in_1 V$ and $U \cap V = \varnothing$, and if there are U and V fuzzy open clusters, X is called fuzzy Hausdorff space [1]. **(18)**

**Definition 2.9 :** For p and r different fuzzy points, $p \in_2 X$, $r \in_2 X$ are called pqU, rqV and $U \cap V = \varnothing$, and if there are U and V fuzzy open clusters, X is called fuzzy Hausdorff space [2]. **(19)**

Now, let [4] prove the following suggestion that only the expressions are given.

**Proposition 2.3.** For the fuzzy topological space X, the following expressions are identical. **(20)**
**a)** For p and r different fuzzy points with $p \in_1 X$, $r \in_1 X$, there are open clusters such as $p \in_1 U$, $r \in_1 V$ and $U \cap V = \varnothing$ U and V fuzzy.

**b)** For p and r different fuzzy points with $p \in_2 X$, $r \in_2 X$, there are open clusters such as pqU, rqV ve $U \cap V = \varnothing$ U and V fuzzy.

**c)** For p and r different fuzzy points with $p \in_1 X$, $r \in_1 X$, there are open clusters such as pqU, rqV an $U \cap V = \varnothing$ U and V fuzzy.

**İspat: a) ⇒ b:** Let $p \in_1 X$, $r \in_1 X$ and p, r be different fuzzy points. If $\mu_P(x_p) < 1$ ve $\mu_r(x_r) < 1$, then $\mu_{P'}(x_p) = 1 - \mu_P(x_p)$ ve $\mu_{r'}(x_r) = 1 - \mu_r(x_r)$ becomes. The p' and r' fuzzy points defined in this way are different fuzzy points. From (18), there are open clusters of U and V fuzzy for different fuzzy points of $p' \in_1 X$, $r' \in_1 X$. Let's now show that they are pqU and rqV by moving $p' \in_1 U$, $r' \in_1 V$. From (5), $p' \in_1 U \Rightarrow \mu_U(x_p) > \mu_{P'}(x_p) = 1 - \mu_P(x_p)$ ve $r' \in_1 V \Rightarrow \mu_V(x_r) > \mu_{r'}(x_r) = 1 - \mu_r(x_r)$, we obtain. Thus, $\mu_U(x_p) + \mu_P(x_p) > 1 - \mu_P(x_p) + \mu_P(x_p) = 1$ written. From here, $\mu_U(x_p) + \mu_P(x_p) > 1$, we obtain. This indicates that p and u are coincident according to (14). So it becomes pqU. Similarly, $\mu_V(x_r) + \mu_r(x_r) > 1 - \mu_r(x_r) + \mu_r(x_r) = 1$, written. From here, $\mu_V(x_r) + \mu_r(x_r) > 1$, we obtain. This means that the definition of 1.6 is that v and r coincidet. So rqV, we obtain. Also, $U \cap V = \varnothing$ from the expression (19).

**b ⇒ c:** Straightforward

**c ⇒ a:** Let p, r be different fuzzy points, $p \in_1 X$, $r \in_1 X$. in that case, By choosing p 'and r' fuzzy points $p' \in_1 X$, $r' \in_1 X$ with the equations of $\mu_{P'}(x_p) = 1 - \mu_P(x_p)$ ve $\mu_{r'}(x_r) = 1 - \mu_r(x_r)$ are different fuzzy points from each other. in that case from (19), p'qU, r'qV ve $U \cap V = \varnothing$ so that there are U and V fuzzy open clusters. Let P'qU and r'qV move to show that they are $p \in_1 U$ ve $r \in_1 V$. (13) in accordance with the expression,

$$p'qU \Rightarrow \mu_{P'}(x_p) + \mu_U(x_p) > 1, \quad \Rightarrow 1 - \mu_P(x_p) + \mu_U(x_p) > 1$$

$$\Rightarrow - \mu_P(x_p) + \mu_U(x_p) > 1, \quad \Rightarrow - \mu_P(x_p) + \mu_U(x_p) > 0$$

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$\Rightarrow - \mu_P(x_p) > - \mu_U(x_p)$ , $\Rightarrow \mu_P(x_p) < \mu_U(x_p)$, we obtain.
This shows that is $p \in_1 U$ according to the equation (5). Similarly,

$$r'qV \Rightarrow \mu_{r'}(x_r) + \mu_V(x_r) > 1, \quad \Rightarrow 1 - \mu_r(x_r) + \mu_V(x_r) > 1$$

$$\Rightarrow - \mu_r(x_r) + \mu_V(x_r) > 1-1, \quad \Rightarrow - \mu_r(x_r) + \mu_V(x_r) > 0$$

$\Rightarrow - \mu_r(x_r) > - \mu_V(x_r)$, $\Rightarrow \mu_r(x_r) < \mu_V(x_r)$, we obtain. Thus, Definition 2.3. it becomes $r \in_1 V$. This shows that $p \in_1 U$, $r \in_1 V$ ve $U \cap V = \varnothing$ so that U and V fuzzy open clusters are present.

## III. CONCLUSION

In this study, some theorems and propositions about these basic concepts have been proved by giving definitions of some basic concepts such as fuzzy set, fuzzy point, fuzzy coincident and fuzzy hausdorff space. It is known that these basic concepts have an important place in fuzzy topology as well as in classical mathematics. Because the concept of fuzzy set is the basis of fuzzy thinking. Applications related to fuzzy concepts are widely used in various fields especially in engineering field in many parts of the world. For this reason, it is thought that such studies will be a significant contribution to fuzzy applications.

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