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Research Paper

Normal Form Combined In Pollution Elimination Model.

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ABSTRACT: *In this article we analyze international concerns about the environment, in particular the current policy of the United Nations, indicating in particular what Brazil is doing, but in order to achieve a healthy environment, not to polish and to achieve that the results reach the environment with concentrations of acceptable pollutants. Here we present a model by means of a system of differential equations that simulates the elimination of pollution, this system is simplified, if the behavior of the trajectories is studied and conclusions are drawn regarding the state with which the results come out to the environment for the critical case combined*
KEYWORDS: *Environment, pollution, mathematical model.*

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I. INTRODUCTION:

In the present work a critical case study is made when a null value and a pair of pure imaginary eigenvalues appear in the model of pollution elimination Batista E, Sánchez S, Lacrort M, Ferreira R, Ribeiro Z and Ruiz AI (2018); here we analyze the behavior of the trajectories of the system of differential equations and give conclusions regarding the state of the outcome.

Following the UN Conference on the Human Environment in June 1972 in Stockholm, the world focused on preserving the environment. Words like depollution, imbalance, favoritism, conservation, were heard throughout the planet. Various sectors, companies, governments, educandarios, organized civil society, among others, have discussed in depth the subject that has created a sense or unison feeling of discomfort, that is, has been the insomnia of many people in all planet and is not for less, since the effects of imbalance emerge everywhere imaginable, frightening, and bringing hopelessness.

Fiorillo (2012, p. 22) points out the moment when we live in a chaotic and alarming situation driven by scientific and technological advances, which enabled and stimulated new classes of consumption and new markets. Most of which occurred without that mechanisms for the preservation or recovery of the environment should be structured or contemplated, not only to improve the quality of life of the present generations, but mainly, in order to make possible the existence of future generations.

Pollution control is related to the ability of a group of people to remain in a given environment without causing harm to them and has in its core commitment to preserve the natural environment by ensuring the future of the environment and species, human. Thinking about the best solutions for the preservation of humankind in an intelligent and committed way, the discussion reaches Brazil through Rio 92 and almost 25 years later, debates permeate diverse sectors among which universities.

To cleanse waves for the development of environmental awareness,Wallace Meirelles (2012, p.229), points to the need for information sharing in society, according to the author the information will bring awareness that will point to the need to preserve and conserve ecosystems and biodiversity. It also indicates that rural and urban social empowerment, when coupled with the strengthening of environmental awareness, is of extreme importance for the relations between the human being and the environment. What is behind this thinking is the possibility of transforming the inhabitants of certain areas, such as the Amazon, into guardians of this immense patrimony, which are agents of preservation and conservation of natural resources and sustainable use.

The National Curricular Parameters / Presentation of the Transversal Themes, Secretariat of Fundamental Education-Ministry of Education (2000.p.33), brings the following contribution:
"Life grew and developed on earth as a plot, a great network of interconnected, interdependent beings. This network intertwines intensely and involves a set of living beings and physical elements. For every living being that inhabits the planet there is a space around it with all other elements and living beings that interact with it, through energy exchange relations: this set of elements, beings and relations constitute their environment. Explained in this way it may seem that, when it comes to the environment, one is talking only about physical and biological aspects. On the contrary, the human being is part of the environment and the relationships that are established - social, economic and cultural relations - are also part of this environment and, therefore, are objects of the environmental area ".

It is exactly in the middle of the energy exchange relations and in the set of elements that we register the involvement of the masses of population and scholars who seek to understand and transmit the idea of depollution; in this way involve the theme that will foster participation and co-responsibility for life collective and solidarity, making appropriate decisions that guarantee the quality of life and the environmental feeling.

Simone Messina (2010, p 2) in the work titled Ecological Literacy: discussion of philosophical and sociological aspects in environmental education makes the following considerations:
"Human civilization and its consumer culture, driven in recent years by the advent of technology, has led the planet to devastate its fundamental ecosystems and, consequently, to a crisis in the various sectors of society, such as economic, social, educational and why not philosophical. "

We draw attention to the terms "devastation" and "crisis" as they complement each other. There is no devastation without crisis and vice versa, however the world only came to understand this very recently. The picture worsens when the focus is consumer culture driven by scientific and technological advances. In fact, depollution must focus on environmental reeducation, or the SOS human being, because when we analyze parts of the letter of the Indians of America to the "white" chiefs we come to understand a little of the destruction that prevails and announces a tragic; not so distant, eminent, if it is not opposed by the agents of the pollution, before the increasing wave of imbalance that already acts in our environment.

Every process of pollution and depollution is directly related to the information held by society, whatever it may be. Antonio Cachapuz (2004, p.369) warns against understanding what one sees, in fact Cachapuz expresses the concern of being able to contribute positively:
"It was necessary to the avalanche of information of the most diverse types and by the most diverse means with which we are confronted to better understand that the information is but a necessary condition of the knowledge ... Perhaps the most perverse thing is that the construction of) knowledge is as easy as today's access to information through the simple press of a key. "

In fact, the speed of information forces the change of social thought, it requires effort, perseverance, commitment. Understanding these mechanisms involves breaking with common sense, breaking down barriers, analyzing, and even breaking down paradigms.

Nusdeo's approach (2012, p 5) focuses on environmental equilibrium and its relation to the issue of fairness, defines a depleted environment as a stage for meeting our current needs without compromising the ability of future generations to meet their needs. , in fact explains in its emergence, as a notoriously vague expression, a consensual term that allowed to accommodate the positions and the most diverse expectations of the different countries and of the multiple intellectual currents. The author also refers to the Stockholm Conference of 1972, as the one whose debates followed the Malthusian line, for the sake of clarity, pointed to catastrophic consequences for population growth and the increase of natural resources due to economic growth.

Furtado (2000, p. 21-30) points out the structural differences between developed and underdeveloped countries as regards the impact and the differentials brought about by the new way of exploiting the environment allied to the new initiative that focuses on the industrialization and development policy of economic and social structures, because inequality in the appropriation of resources between countries and between groups within countries are conflicting.

Faced with this rhetoric, the present work also proposes a model of depollution of any site, space, zone, etc. From then on it is possible to establish ordinary differential equations, which allow assigning variables in the decontamination processes. After tabulation, treatment of information, tests, collections and other procedures, it is possible to establish a mathematical model that will allow to gauge levels of pollution and depollution efficiently and effectively in the several values demonstrated and the verification of a model that involves a system of equations that will work in theory and practice, allowing to gauge levels and control of the pollution so widespread.

When it is desired to eliminate the pollution of certain outcomes, which could be in a liquid, solid or gaseous form; one should think about how to proceed to achieve our objectives rather to reach the environment, as one should not eliminate one form of contaminant and introduce another that could even be more aggressive.

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In general, these processes of depollution are presented in a combined form, since a certain matter is contaminated and one wishes to send to nature with the least possible affectation for the medium; these outcomes could be generated by sewage or the operation of one industry among others.

In this direction there are experiences such as the one practiced in the Tietê de São Paulo depollution where by means of certain plants the pollutants can be eliminated in a high percentage of the initial concentration. On the other hand, it is common to work using oxidation ponds to eliminate contaminants from the liquid endings left by the population, where by making use of chemical substances and sometimes natural products it is possible to depollute in large quantities the liquid part, and with the removal of solids these objectives are generally achieved.

The treatment that we will make in this case corresponds to other models presented in the researches of other diseases, especially the case of sicklemia, quite treated and with many models already developed, will only mention some of these works. In Sánchez, S., Fernández, A. A., Ruiz. A. I. (2012) and Sánchez, S., Fernández, G. A. A., Ruiz. A. I., & Carvalho, E. F (2016) is applied the qualitative study of differential equations to different models in an autonomous and non-autonomous form corresponding to the formation of polymers.

In Sánchez. Sa, Fernández A.a, Ribeiro. Zb, E. Lacort M.b, Ferreira R.b, Ruiz A. I.b (2018) applies the Qualitative Theory of differential equations to study a combined critical case; for a periodic non-autonomous system with respect to time.

Development:

We present the case of two oxidation ponds, where a decontamination procedure is applied in each of them. Here it is considered as the compartment a first lagoon; compartment two is the second pond and compartment three is given by the exit of the material to the environment; this procedure is being used in the city of Leticia capital of the state of the Colombian Amazon.

Initially we will give some basic principles that we will take into account in the writing of the model; let's denote by $\overline{x_1}$, $\overline{x_2}$ and $\overline{x_3}$ the permissible values of the concentration of pollutants in compartments one, two and three; we will indicate the following other variables to consider:

- $\tilde{x}_1(t)$ is the concentration of pollutant in compartment a at time t.
- $\tilde{x}_2(t)$ is the concentration of pollutant in compartment two at time t.
- $\tilde{x}_3(t)$ is the concentration of pollutant in compartment three at time t.

In the system we will consider the variables x_1, x_2 and x_3 defined as follows $x_1 = \tilde{x}_1(t) - \overline{x_1}$, $x_2 = \tilde{x}_2(t) - \overline{x_2}$ and $x_3 = \tilde{x}_3(t) - \overline{x_3}$ so when $(x_1, x_2, x_3) \to (0,0,0)$ so $\tilde{x}_1(t) \to \overline{x_1}$, $\tilde{x}_2(t) \to \overline{x_2}$ and $\tilde{x}\ (t) \to \overline{x_3}$.

It is good to realize that in our model will appear the pollutants that will be placed in both ponds, in addition, if it is considered that there is no other supply of contaminated material from that initial moment, so we have that a possible model would have the next way

$$\begin{cases} x_1' = -a_{12}x_1 + X_1(x_1, x_2, x_3) \\ x_2' = a_{12}x_1 - a_{23}x_2 + a_{32}x_2 + X_2(x_1, x_2, x_3) \\ x_3' = a_{23}x_2 - a_{32}x_3 + X_3(x_1, x_2, x_3) \end{cases} \quad (1)$$

On here X_1, X_2 and X_3 represent functions of depollution, that is to say the action of the chemical or natural products supplied for the elimination of pollution, if it supposes these functions contain only non-linear terms and the powers will depend on the speed with which the pollution is eliminated; $a_{ij}\, x_i\,(t)$ represents the coefficient with the transfer variable of the compartment i to the compartment j **,** where the output and the input are indicated. In this way the problem of Cauchy, given by the system (1) with the initial conditions, will be defined, $x_1(0) = N$, $x_2(0) = 0$, $x_3(0) = 0$; in this case it is considered that in the initial moment all the contamination is concentrated in the first lagoon. If we consider that $X(t)$ represents the total concentration of the pollution, if it would have that in any moment t will be valid the following mass balance,

$$X(t) = x_1(t) + x_2(t) + x_3(t) = N\ .$$

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With respect to the variation of the concentration of pollutants, making use of the equations of the system,

$$\frac{dX}{dt} = \frac{dx_1}{dt} + \frac{dx_2}{dt} + \frac{dx_3}{dt} = X_1 + X_2 + X_3$$

This indicates that if the signal from the right side of the previous expression is negative the treatment will eliminate the concentration of the pollutants, and depending on the effectiveness of the process our objectives could be reached.But the continuation will see different critical cases, for which the Analytical Theory of Differential Equations will be applied.

It may be the case that the system (1) is such that the matrix of the linear part has a pair of pure imaginary eigenvalues $\pm\sigma i$ at where σ is irrational, and that the other eigenvalue has a real negative part this would be a critical case. Without loss of generality, the system (1) can be written in the form,

$$\begin{cases} y_1' = i\sigma y_1 + Y_1(y_1, y_2, y_3) \\ y_2' = -i\sigma y_2 + Y_2(y_1, y_2, y_3) \\ y_3' = \lambda y_3 + Y_3(y_1, y_2, y_3) \end{cases} \quad \textbf{(2)}$$

Being $y_2 = \overline{y_1}$ e $Y_2 = \overline{Y_1}$ (conjugated).

Using the Analytical Theory of Differential Equations the system (2) can be reduced to the almost normal form (FCN), which in this case can be written in the form,

$$\begin{cases} u_1' = i\sigma u_1 + u_1 P(u_1 u_2) \\ u_2' = -i\sigma u_2 + u_2 \overline{P}(u_1 u_2) \\ u_3' = \lambda u_3 + U(u_1, u_2, u_3) \end{cases} \quad \textbf{(3)}$$

At where $U(u_1, u_2, 0) = 0$.

The series can be written as follows,

$$P(u_2 u_3) = \sum_{n=k}^{\infty} a_n (u_2 u_3)^n + i \sum_{n=l}^{\infty} b_n (u_2 u_3)$$

If k is such that $a_k < 0$, then the equilibrium position of the system (3) is asymptotically stable.

For this purpose, the positive definite Liapunov function is used,

$$V(u_1, u_2, u_3) = u_1 u_2 + u_3^2$$

Since, $u_1 = \overline{u_2}$. The derivative of the function along the trajectories of the system (3) is,

$$V'(u_1(t), u_2(t), u_3(t)) = 2a_k |u_1|^{k+1} + 2\lambda u_3^2 + R(u_1, u_2, u_3)$$

This function V' is defined as negative due to the signs of the coefficients, because $a_k < 0$ and $\lambda < 0$. Furthermore, in R if they have the higher powers of $k+1$ respect to $|u_1|$, and above the second degree with respect to u_3.

In this case we do not do a formal demonstration analyzing all the details, because in the following case the process will be done step by step what would be done in a similar way here.

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Combined Critical Case:

Suppose that the matrix of the linear part of the system (1) has a pair of pure imaginary eigenvalues and a null eigenvalue, in this case, our proposal is to reduce the system to the combined normal form to arrive at conclusions with respect to the behavior of the trajectories of the system.

Suppose that the eigenvalues of the matrix of the linear part of the system are: $0, i\sigma, -i\sigma$; non-degenerate linear transformation $x = Sy$, transforms the system (1) into the next system,

$$\begin{cases} y_1' = Y_1(y_1, y_2, y_3) \\ y_2' = i\sigma y_2 + Y_2(y_1, y_2, y_3) \\ y_3' = -i\sigma y_3 + Y_3(y_1, y_2, y_3) \end{cases} \quad \textbf{(4)}$$

Since the second members of the system (1) are analytic functions, the series of powers $Y_i(y_1 y_2, y_3), (i = 1,2,3)$ are convergent.

Theorem 1: There is a change of variables,

$$\begin{cases} y_1 = z_1 + h_1(z_1) + \overline{h_1}(z_1, z_2, z_3) \\ y_i = z_i + h_i(z_1), (i = 2,3) \end{cases} \quad \textbf{(5)}$$

which reduces the system (4) to the system,

$$\begin{cases} z_1' = Z_1(z_1) \\ z_2' = i\sigma z_2 + \tilde{Z}_2(z_1, z_2, z_3) \\ z_3' = -i\sigma z_3 + \tilde{Z}_3(z_1, z_2, z_3) \end{cases} \quad \textbf{(6)}$$

Where the series $h_i, (i = 1,2,3)$ and $\overline{h_1}$ are determined in a unique way and

$\tilde{Z}_i(z_1,0,0) = 0, (i = 2,3)$. Thus, for the $z_i = 0$ $(i = 2,3)$ one has the normal shape

$z_1' = Z_1(z_1)$.

Demonstration: By deriving the transformation (5) along the trajectories of systems (4) and (6) we obtain the system of equations,

$$\begin{cases} Y_1 - \dfrac{dh_1}{dz_1} Z_1 - \dfrac{\partial \overline{h_1}}{\partial z_1} Z_1 - \dfrac{\partial \overline{h_1}}{\partial z_2} \tilde{Z}_2 - \dfrac{\partial \overline{h_1}}{\partial z_3} \tilde{Z}_3 = Z_1 + (p_1 - p_2) i\sigma \overline{h_1} \\ Y_2 - \dfrac{dh_2}{dz_1} Z_1 = \tilde{Z}_2 + i\sigma h_2 \\ Y_3 - \dfrac{dh_3}{dz_1} Z_1 = \tilde{Z}_3 - i\sigma h_3 \end{cases} \quad \textbf{(7)}$$

To determine the series involved in the systems and transformation, let's separate the coefficients of the power of degree $p = (p_1, p_2, p_3)$ following two cases:

Case I) Considering in the system (7) $z_i = 0, (i = 2,3)$, that is to say $p = (p_1,0,0)$, resulting the following system,

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$$\begin{cases} Y_1 - \dfrac{dh_1}{dz_1} Z_1 = Z_1 \\ Y_2 - \dfrac{dh_2}{dz_1} Z_1 = i\sigma h_2 \\ Y_3 - \dfrac{dh_3}{dz_1} Z_1 = -i\sigma h_3 \end{cases} \quad \textbf{(8)}$$

The system (8) allows determining the coefficients of the series Z_1, and the corresponding series $h_i, (i = 2,3)$, which are obtained in a unique way, since
h_1 which would be arbitrary if it is considered equal to zero to achieve uniqueness.

Case II) In the case where $p = (p_1, p_2, p_3)$, being p_2 or p_3 not null, the system (7) deduces the following system,

$$\begin{cases} Y_1 - \dfrac{\partial \overline{h_1}}{\partial z_1} Z_1 - \dfrac{\partial \overline{h_1}}{\partial z_2} \tilde{Z}_2 - \dfrac{\partial \overline{h_1}}{\partial z_3} \tilde{Z}_3 = (p_1 - p_2) i\sigma \overline{h_1} \\ Y_2 = \tilde{Z}_2 \\ Y_3 = \tilde{Z}_3 \end{cases} \quad \textbf{(9)}$$

The system (9) allows to calculate the coefficients of the series,
$\tilde{Z}_i(z_1,0,0) = 0, (i = 2,3)$ and $\overline{h_1}$, since the remaining series appearing in the system (9) are known expressions; this completes the proof of the theorem.

Theorem 2: The change of variables,

$$\begin{cases} z_1 = u_1 \\ z_i = u_i + h_i(z_1, z_2), (i = 2,3) \end{cases} \quad \textbf{(10)}$$

transforms the system (6) into the combined normal form,

$$\begin{cases} u_1' = U_1(u_1) \\ u_2' = i\sigma u_2 + u_2 P(u_2 u_3) \\ u_3' = -i\sigma u_3 + u_3 \overline{P}(u_2 u_3) \end{cases} \quad \textbf{(11)}$$

Where the series $h_i, (i = 2,3)$ are determined uniquely.

Demonstration: By deriving the exchange of variables (10) along the trajectories of systems (6) and (11) we obtain the system of equations,

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$$\begin{cases} U_1 = Z_1 \\ u_2 P + (p_1 - p_2 - 1) i\sigma h_2 = \widetilde{Z}_2 - \dfrac{\partial h_2}{\partial u_2} u_2 P - \dfrac{\partial h_2}{\partial u_3} u_3 \overline{P} \\ u_3 P + (p_1 - p_2 + 1) i\sigma h_3 = \widetilde{Z}_3 - \dfrac{\partial h_3}{\partial u_2} u_2 P - \dfrac{\partial h_3}{\partial u_3} u_3 \overline{P} \end{cases} \quad \textbf{(12)}$$

The system (12) allows the determination of the coefficients of the series $h_i, (i = 2,3)$ as well as those of P and $\overline{P}$. In this case, the resonance equations for the second equation are $p_1 - p_2 - 1 = 0$, and for the third, $p_1 - p_2 + 1 = 0$, this characterizes the way P, and similarly for $\overline{P}$ in the third equation.

In the system (11) the function P and $U_1(u_1)$ have the following forms:

$$P(u_2 u_3) = \sum_{n=k}^{\infty} a_n (u_2 u_3)^n + i \sum_{n=l}^{\infty} b_n (u_2 u_3)$$

and

$$U_1(u_1) = \alpha u_1^s + \ldots$$

Theorem 3: If $\alpha < 0$, s odd and $a_k < 0$ then the trajectories of the system (11) are asymptotically stable, otherwise they are unstable.

Demonstration:
Consider the positive definite Lyapunov function,

$$V(u_1, u_2, u_3) = u_1^2 + u_2 u_3$$

Deriving function V with respect to t along the trajectories of the system (11), the following expression is obtained:

$$V'(u_1(t), u_2(t), u_3(t)) = 2\alpha u_1^{s+1} + 2a_k |u_2|^{k+1} + R(u_1, u_2, u_3)$$

Where in R we have the higher-order powers of $k+1$ respect to $|u_2|$, and higher than s+1 respect to u_1, like this V' is defined as negative, which completes the proof of the theorem.

II. CONCLUSIONS:

1. By the characteristics of the problem considered it is natural for the combined critical case to appear, that is to say when the matrix of the linear part has a unique null value and a pair of pure imaginary eigenvalues.
2. The combined normal form allows for a qualitative study of the trajectories of the system.
3. Theorems (1) and (2) give the following procedure for simplifying the original system in order to find a more effective treatment of the problem.
4. If $\alpha < 0$, s is odd and $a_k < 0$ then the null solution of the system is asymptotically stable which ensures that the substances will leave the environment with an acceptable amount of contamination, otherwise it will be necessary to review the process followed in the treatment of the pollutants to achieve the proposed objectives.

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