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Research Paper

Calculating the e/m Ratio of an Electron and Determining the Cyclotron Frequency of Certain Magnetic Fields

Abhinav Agarwal
(Grade 12, Modern School, Barakhamba Road, New Delhi)
Corresponding Author: Abhinav Agarwal

ABSTRACT: Electrons were discovered by J.J. Thomson in 1897 in the form of cathode rays, which were beams of particles showing behavior similar to negative charge. The beams always had the same properties, irrespective of the gas in the discharge tube, and thus all the particles in the beam had a fixed charge to mass (e/m) ratio.
This experiment aims at calculating the charge to mass ratio of an electron and calculating the cyclotron frequency of the maximum magnetic field produced in an experiment. This experiment was conducted under the guidance of Professor Timothy Halpin-Healy at Columbia University.
KEYWORDS: Electrons, e/m ratio, magnetic fields

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I. PRE-REQUISITES

When in an electric field, charged particles experience a force which can accelerate them to high velocities. If q is the charge of the particle and E is the magnitude of the electric field, then force F is given as:

F=qE

A potential difference V exists across the two ends of the field, and the work done on the particle when accelerated across the field would be:

W=qV

By work energy theorem, work equals change in kinetic energy.

A charged particle of charge q moving with a velocity v in a magnetic field of strength B also experiences a force. This force, FB is given by:

FB=q (v x B)

It is important to understand that the magnetic field can never change the magnitude of the velocity of a substance. It can only change the direction.

The kinetic energy and centripetal force of an object of mass m and moving with velocity v in a circle of radius r are given by:

(1/2) mv2 and (mv2)/r respectively. [1]

II. EXPERIMENT DESCRIPTION

This experiment involved emitting electrons at a low velocity from a heated filament, and then accelerating them through a potential difference V generated by an electric field to give them a velocity v. Then, they were passed through a perpendicular magnetic field B, which applied a centripetal force on them to make them revolve in a circle of radius r.

In the apparatus used for the experiment, the tube in which electrons were accelerated was evacuated and some mercury vapor was introduced in it. When the electrons had sufficiently high energy, they collided and ionized them. Mercury ions then recombined near the site of ionization and emitted a purple light, making their path visible in a dark room.

The apparatus comprised of Helmholtz Coils, the magnetic field produced by which is proportional to the current through them. To derive the expression of the magnetic field, a single circular loop of radius R was assumed. The aim was to calculate the magnetic field at its axis at distance 'a' from the center of the loop.

Let the y-axis be along the axis of the coil, x-axis be parallel to one of the radii, and the point at distance 'a' from center be the origin. The x components of magnetic fields at origin due to opposite current elements on the loop cancel out and what remain are y components. By taking the integral, magnetic field at distance 'a' from the center on the axis of coil for N turns is given by:

$B= (\mu_o NR^2 I)/2(R^2+a^2)^{3/2}$

In the experiment, the radius of the coils was 33cm, they had a total of 72 turns and the tube in which electrons were observed was at a distance of 16.5 cm from the center of the coil.

By inputting appropriate values to the above formula, the magnetic field B of the coils can be determined as:

$B_C = 1.96 \times 10^{-4}$ I, where is the current through the coils.

The potential difference through which the electrons were accelerated was also controlled, and the readings were taken on two voltages, 40V and 50V.

Even though the Earth's magnetic field might cause interference, it was taken to be negligible during the experiment.

By work energy theorem for an electron of charge e, mass m moving by a velocity v in a circle of radius r, after accelerating through a potential difference V:

eV= (1/2) mv2

v2=2eV/m (1)

Also, using circular motion and centripetal force exerted by magnetic field B acting perpendicular to v:

F_B= qvBsinθ = qvBsin90 = evB

mv^2/r = evB (centripetal force)

v=eBr/m (2)

From (1) and (2)

$e^2B^2r^2/m^2=2eV/m$

$1/r^2=eB^2/(2mV)$

So, we get the working formula as:

$1/r = (e/2mV)^{1/2}B$

Thus, by observing the variation of the radius of the electron beams as the magnetic field is varied for a fixed voltage, the e/m ratio can be calculated. This would be done by taking the slope of the 1/r vs B graph, squaring it and multiplying it by twice the voltage.

The radius was calculated by the markings on a rod in the tube. As the outer edge (maximum energy) of electron beam hit these markings, radius could be found. These markings were at: 6.48cm, 7.75cm, 9.02cm, 10.30cm and 11.54cm. This data was given in the lab handout. [2]

III. OBSERVATIONS SET 1 (V =50 VOLTS)

V = 50 Volts

r(m)	1/r(1/m)	I (A)	B(T)
0.0324	30.86	3.8	7.4×10^{-4}
0.0388	25.77	3.2	6.2×10^{-4}
0.0451	22.17	2.8	5.4×10^{-4}
0.0515	19.42	2.4	4.7×10^{-4}
0.0577	17.33	2.2	4.3×10^{-4}

Table 1: Readings taken from the apparatus

Using the values in Table 1, a graph was plotted on numbers and the slope was taken from the line of best fit as shown on Figure 1.

**Corresponding Author: Abhinav Agarwal*
(Grade 12, Modern School, Barakhamba Road, New Delhi)

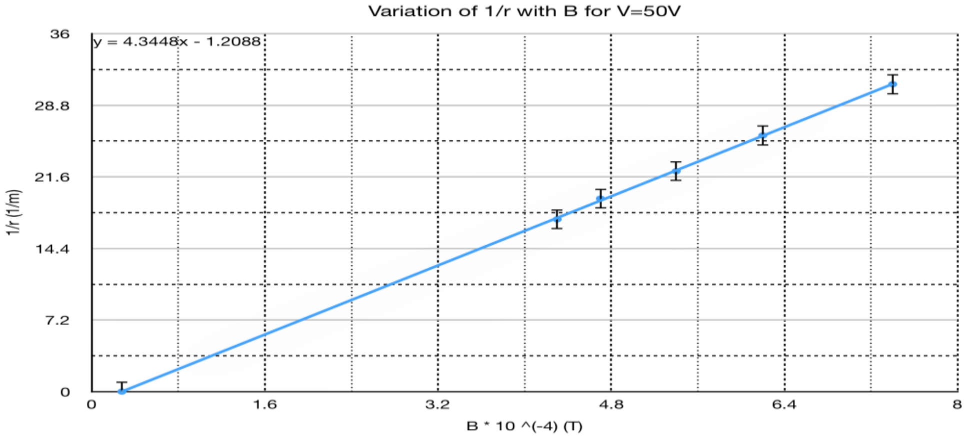

Figure 1: Graph for Set 1 readings

IV. CALCULATIONS FOR OBSERVATIONS SET 1

4.1 Calculating the Charge to Mass Ratio

V = 50 volts

Slope = 4.345×10^4 (Calculated from line of best fit)

$\sqrt{\frac{e/m}{100}} = 4.345 \times 10^4$

e/m = 1.88×10^{11} c/kg

Slope $_{max}$ = 4.709×10^4 (Calculated Using Positive Error Bar of Max Reading and Negative Error Bar of Min Reading)

$\sqrt{\frac{e/m}{100}} = 4.709 \times 10^4$

e/m $_{max}$ = 1.96×10^{11} c/kg

Slope $_{min}$ = 4.1×10^4 (Calculated Using Negative Error Bar of Max Reading and Max Error Bar of Min Reading)

$\sqrt{\frac{e/m}{100}} = 4.1 \times 10^4$

e/m $_{min}$ = 1.74 10^{11} c/kg

$$\frac{e/m_{max} - e/m_{min}}{2} = \frac{1.96 \times 10^{11} - 1.74 \times 10^{11}}{2}$$

= 0.11×10^{11} C/kg

So, calculated e/m at V = 50 volts is $1.88 \times 10^{11} \pm 0.11 \times 10^{11}$ C/kg

Deviation from actual value of 1.758×10^{11} C/kg = 6 %

**Corresponding Author: Abhinav Agarwal*
(Grade 12, Modern School, Barakhamba Road, New Delhi)

4.2 Calculating the Cyclotron Frequency for a Field of 7.4 x 10^{-4} T

|e| = 1.6 x 10^{-19} C

$$m = \frac{1}{e/m} \times e$$

= 0.85 x 10^{-30}

≈ 8.5 x 10^{-31} kg

$$v = \sqrt{2 \times (e/m) \times V}$$

= 4.33 x 10^{6} m/s

$$\beta = \frac{v}{c}$$

= 0.014

At B = 7.4 x 10^{-4} Ts

Radius = 0.0324 m

$$v_e = 4.33 \times 10^6 \, m/s$$

Circumference = 2πr = 0.203 m

$$Time\ for\ one\ revolution = \frac{Circumference}{v_e}$$

= 4.7 x 10^{-8} s

w =Cyclotron frequency of a magnetic field of 7.4 x 10^{-4} T

= 1/ Time Period = 2.126 x 10^{7} Hz

V. OBSERVATIONS SET 2 (V =40 VOLTS)

V = 40 Volts

r(m)	1/r(1/m)	I (A)	B(T)
0.0324	30.86	3.4	6.67 x 10^{-4}
0.0388	25.77	2.9	5.69 x 10^{-4}
0.0451	22.17	2.5	4.9 x 10^{-4}
0.0515	19.42	2.1	4.12 x 10^{-4}
0.0577	17.33	2	3.92 x 10^{-4}

Table 2: Readings taken from the apparatus

Using the values in Table 2, a graph was plotted on numbers and the slope was taken from the line of best fit as shown on Figure 2.

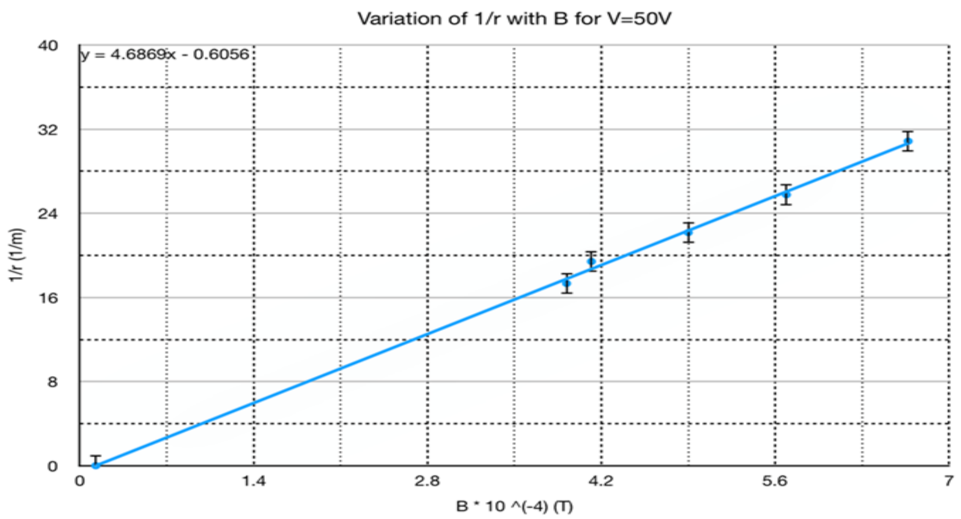

Figure 2: Graph for Set 2 readings

VI. CALCULATIONS FOR OBSERVATIONS SET 2

5.1 Calculating the Charge to Mass Ratio

V = 40 volts

Slope = 4.686 x 10^4 (Calculated from the Line of Best Fit)

$\sqrt{\frac{e/m}{80}}$ = 4.686 x 10^4

e/m = 1.756 x 10^{11} C/kg

Slope $_{max}$ = 4.69 x 10^4 (Calculated Using Positive Error Bar of Max Reading and Negative Error Bar of Min Reading)

$\sqrt{\frac{e/m}{80}}$ = 4.69 x 10^4

e/m $_{max}$ = 1.77 x 10^{11} c/kg

Slope $_{min}$ = 4.19 x 10^4 (Calculated Using Negative Error Bar of Max Reading and Positive Error Bar of Min Reading)

$\sqrt{\frac{e/m}{80}}$ = 4.19 x 10^4

e/m $_{min}$ = 1.41 x 10^4 C/kg

$$\frac{e/m_{max} - e/m_{min}}{2}$$

$$= \frac{1.77 \times 10^{11} - 1.41 \times 10^{11}}{2}$$

= 0.18 x 10^{11} c/kg

So, e/m at V = 40 volts is 1.756 x 10^{11} $\pm$ 0.18 x 10^{11} C/kg

Deviation from Actual Value of 1.758 x 10^{11} C/kg = .11%.

|e| = 1.6 x 10^{-19} c

$m = \frac{1}{e/m} \times e$

= 0.91 x 10^{-30}

≈ 9.1 x 10^{-31} kg

$v = \sqrt{2 \times (e/m) \times V}$

= 3.74 x 10^6 m/s

$\beta = \frac{v}{c}$

= 0.012

At B = 6.67 x 10^{-4} T

Radius = 0.0324 m

$v_e = 3.74 \times 10^6\ m/s$

Circumference = 2πr = 0.203 m

Time for one revolution = Time Period

$$Time\ for\ one\ revolution = \frac{Circumference}{v_e}$$

= 5.8 x 10^{-8} s

w = Cyclotron frequency of a magnetic field of 6.67 x 10^{-4} T

=1/ Time Period = 1.72 x 10^7 Hz

VII. CONCLUSION

Using two voltages 40V and 50V and varying the current, which in turn varied the magnetic field, we could calculate the charge to mass ratio by observing the radius of the rotating electron beam. Using this charge to mass ratio, we calculated the cyclotron frequency of a given magnetic field, which is independent of the velocity or radius of revolution and varies according to the magnetic field for a fixed charge.

**Corresponding Author: Abhinav Agarwal*
(Grade 12, Modern School, Barakhamba Road, New Delhi)

The values of e/m calculated were:

1. $1.88 \times 10^{11} \pm 0.11 \times 10^{11}$ C/kg for V=50V
2. $1.756 \times 10^{11} \pm 0.18 \times 10^{11}$ C/kg for V=40V

The cyclotron frequencies were as follows:
1. 2.126×10^{7} Hz for a field of 7.4×10^{-4} T
2. 1.72×10^{7} Hz for a field of 6.67×10^{-4} T

Sources of error may include ignoring the effect of the Earth's magnetic field, misalignment of the electron tube at the center of the coils. Some readings in the first observation set might have been taken from the inner end of the electron beam, leading to a higher error of 6% compared to .11% in set 2.

Overall, the experiment successfully calculated the charge to mass ratio of an electron and determined the cyclotron frequency for two magnetic fields.

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