



## Hydrological Drought Assessment: The Use of the ARRF Model for Monthly Streamflow Generation on Intermittent Rivers of the Northeast Brazil

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**ABSTRACT:** This paper presents the ARR model (Alternating Renewal Reward) for generating annual streamflow, coupled with the Fragment model to disaggregate the annual streamflow to monthly ones. This new coupled model, namely ARRF model, has been applied to three typical intermittent basins in the State of Ceará - Brazil. In order to better understand the phenomenology of the processes involved in modeling, several statistical tests were applied to flood and drought events of the typical hydrological streamflow series of this region. The ARRF model was able to preserve the analyzed statistical parameters of the historical streamflow series, as well as to reproduce the persistence (long periods of low and high flow, i.e. drought and flood periods) encountered in the historical flow series, which are fundamental for hydrological drought assessment and for the design and operation optimization of multi-purpose reservoirs system.

**KEYWORDS:** Streamflow Generation and Simulation, Semiarid Regions, Reservoir Operation Optimization  
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### I. INTRODUCTION

The Brazilian semiarid region has an area of about 1 million km<sup>2</sup>. It is characterized by strong temporal and spatial rainfall variability (400 to 1800 mm/year) and a high evaporation rate (above 2000 mm/year), associated with geological restrictive conditions (crystalline basement of reduced hydrological potential), that causes river intermittency. The construction of artificial dams along the major rivers of the region throughout the last century, was indispensable for water supply, especially during drought periods. Since 2011, this entire region has been suffering from a severe drought event [1]. In addition, [2], [3] and [4] demonstrated that this region will suffer even more due to climate change.

For hydrological drought assessment and for the design and operation of multi-purpose and multi-use surface reservoirs systems (water supply, irrigation, energy production, etc.) deterministic rainfall-runoff models and stochastic streamflow generation models are usually employed [5], depending on data availability, which is generally scarce in this region.

Several autoregressive models have been presented in the literature for generating synthetic streamflow in temperate regions. For semiarid regions, however, such models cannot reproduce satisfactorily the typical characteristics of intermittency. For this purpose, [6] has modified and applied nine autoregressive models to monthly flow generation in intermittent rivers of four basins in the semiarid region of Brazil. The analyzed models showed, in most cases, an overestimate of the average monthly flow, probably due to their Markovian character, as attested also by [7], among others. The objective of this paper is therefore to analyze and discuss the performance of the ARRF model for streamflow generation model, that could be applied to hydrological drought assessment in semi-arid regions.

### II. HYDROLOGIC DROUGHT DEFINITION

According to [8] four basic considerations must be evaluated for the definition of droughts, that are: 1) what is the greatest interest in the analysis, i.e. what is the nature of the deficit of water to be investigated (meteorological, hydrological or agricultural); 2) what is the time series discretization used in the analysis (annual, semiannual, monthly, etc.); 3) what is the threshold level of separation between flood and drought events; and 4) the choice of the regionalization and standardization methods to be adopted.

To analyze the characteristic parameters of drought and flood events an interval of discretization of one year was adopted in this study, due to the character of the annual events of drought in northeastern Brazil. Furthermore, only the hydrological aspects have been treated herein, i.e. the mathematical modeling of the streamflow series. The mean flow was chosen as a threshold for distinction between flood and drought events, resulting in an average severity for the same flood and drought length periods, within a complete alternating process cycle [9]. Due to the small sample of the annual series, two procedures have been applied for the identification of the probability distribution of the flood and drought duration: decimation and standardization.

A hydrological drought can be defined as a one or more sequenced years, when the average annual flow remains below the long term mean annual flow, considering all the existing series [8]. A drought event may thus be characterized by three parameters, namely the duration *D* in years; the accumulated deficit or severity *S* and the magnitude *M*, which represents the average cumulative deficit below the mean annual flow.

Thus, we may define *S*, the drought severity as:

$$S = \int_0^D Y(t) dt \rightarrow (1)$$

Where *Y*(*t*) = the deficit value of a *D*-year drought for the year *t*.

Considering flow rates, we may state:

$$S = \int_0^D [\bar{Q} - Q(t)] dt \rightarrow (2)$$

where *Q*(*t*) = not regularizable flow rate for year *t* of a *D*-year duration drought.

Then we may estimate the magnitude *M*, as follows:

$$M = \frac{1}{D} \int_0^D Y(t) dt \rightarrow (3)$$

or

$$M = S / D \rightarrow (4)$$

Hence, the magnitude may be considered a secondary parameter, since it may be expressed in terms of the duration and the severity, that are considered primary parameters. According to [10], the severity of a *D*-years duration drought may be described as:

$$S_d = Y_1^D + Y_2^D + \dots + Y_D^D \rightarrow (5)$$

### III. CHARACTERISTIC PARAMETERS OF DROUGHT AND FLOOD EVENTS AND STATISTICAL TESTS

In order to better understand the phenomenology of the processes involved in modeling, the following statistical analyzes of flood and drought events were performed:

- I) Stationarity test (trend);
- II) Stochastic test (lag-1 correlation coefficient);
- III) Correlation test between the parameters;
- IV) Cross correlation between the parameters of successive flood and drought events or inversely.

#### 3.1 Stationary (trend)

Stationarity degrees for flood and drought events were determined by comparing the slope of the regression line adjusted for each series with the *t*-statistic. The lowest values for the significance level  $\alpha$  associated with the rejection of the null hypothesis were estimated (the slope of the regression line equals zero). In this table, NS means that the series in question is very instationary ( $\alpha < 0.01$ ) and S means that the analyzed series is very stationary ( $\alpha > 0.20$ ). The signals correspond to the signs of the slope of the regression lines.

**Table 1:** Test *t*;  $\alpha$  values for non-stationary flood and drought events

Station	Flood			Drought		
	Duration	Severity	Magnitud	Duration	Severity	Magnitud
Faz. Cajazeiras	+S	+S	-S	-S	-S	-S
Pacajus	+S	+S	-S	-S	-S	+S
P. Sarasate	-S	+S	+S	-S	-S	-S

- 1) The signals correspond to the slope of the regression lines.
- 2) NS (non-stationary) for  $\alpha < 0.01$ ; S (stationary) for  $\alpha > 0.20$ .

Based on the results in Table 1, the following qualitative findings can be made:

- I) An increase in the duration of a flood period usually corresponds to an increase in severity and a decrease in the magnitude of flood periods (Figure 1);
- II) A reduction in drought duration leads to a decrease in drought severity and an increase in drought magnitude;
- III) Neither the characteristic parameters of floods nor drought periods present significant instabilities. This, however, may be due to the fact, that the test used does not have enough power to detect this from a small sample [11].

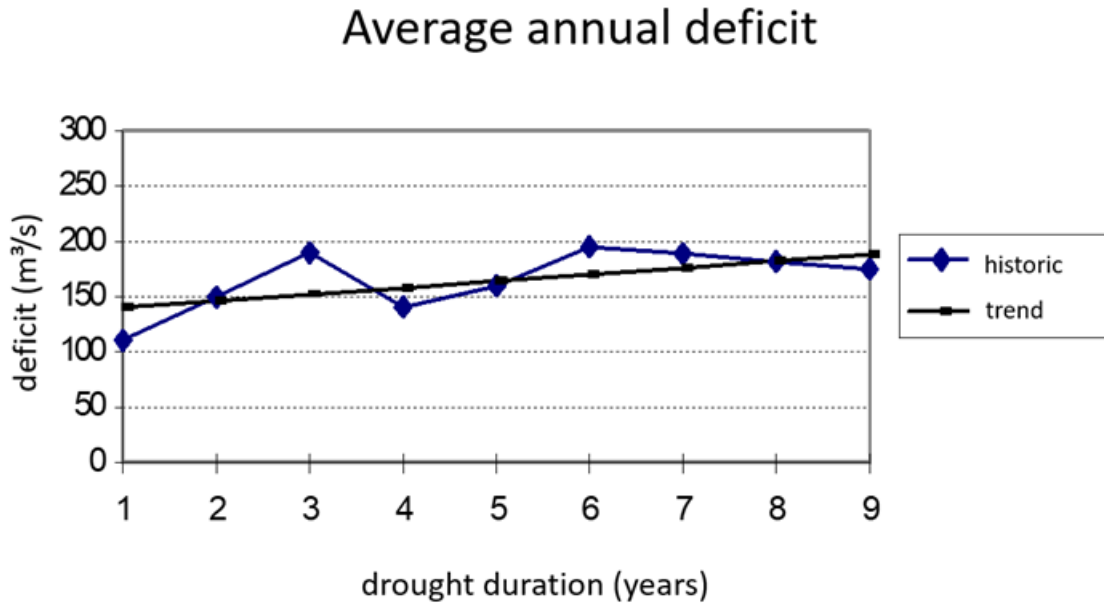


Figure 1: Expected value for the average deficit as a function of drought duration (Faz. Cajazeiras Station)

### 3.2 Stochasticity

The degree of stochasticity of the series was evaluated by the Anderson test [12] using the lag-1 correlation coefficient, assuming the cyclic series. The lowest significance level  $\alpha$ , associated with the null hypothesis,  $\rho_1 = 0$ , using the z statistic, is shown in Table 2. From this table comes the following qualitative conclusions:

- I) Negative correlation coefficients of flood durations are linked to negative correlation coefficients of flood severity and magnitude;
- II) Negative correlation coefficients of drought durations are associated with negative correlation coefficients of drought severity;
- III) Duration is the most random parameter and the most nonrandom severity;
- IV) All characteristic parameters of the flood periods of the tested series presented negative correlation coefficients;

Table 2: Test z;  $\alpha$  values for non-random behavior of flood and drought events

Station	Flood			Drought		
	Duration	Severity	Magnitud	Duration	Severity	Magnitud
Faz. Cajazeiras	-R	-R	-R	+0.05	+0.05	+R
Pacajus	-R	-0.10	-0.20	-R	-R	-R
Paulo Sarasate	-R	-0.10	-0.05	+0.05	+0.05	-R

- 1) The signals correspond to the slope of the regression lines.
- 2) NR (non-stochastic) for  $\alpha < 0.01$ ; R (stochastic) for  $\alpha > 0.20$ .

The duration and severity of drought events were then through studied envelopes. Estimates of flow maxima were determined by Tschebycheff inequality regardless of probability distribution, as follows:

$$\max X \approx u_x + (n)^{1/2} \sigma_x \rightarrow (6)$$

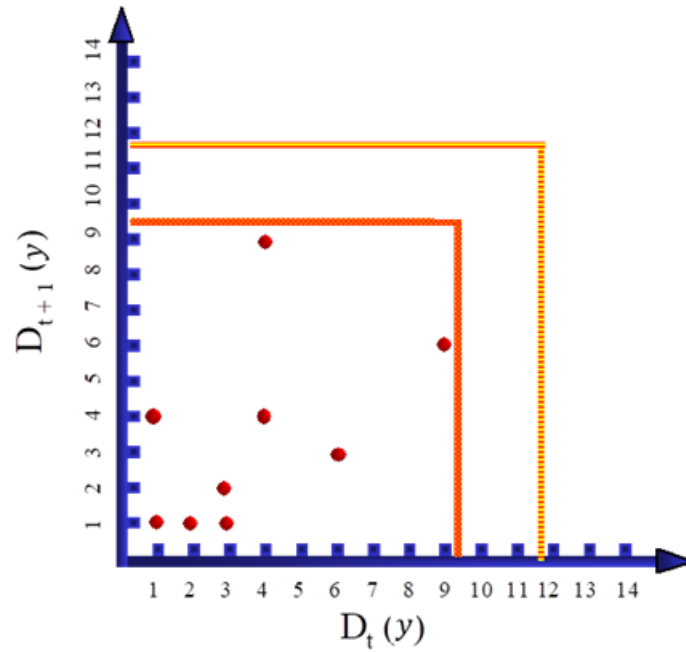
$u_x$  = mean;

$\sigma_x$  = standard deviation;

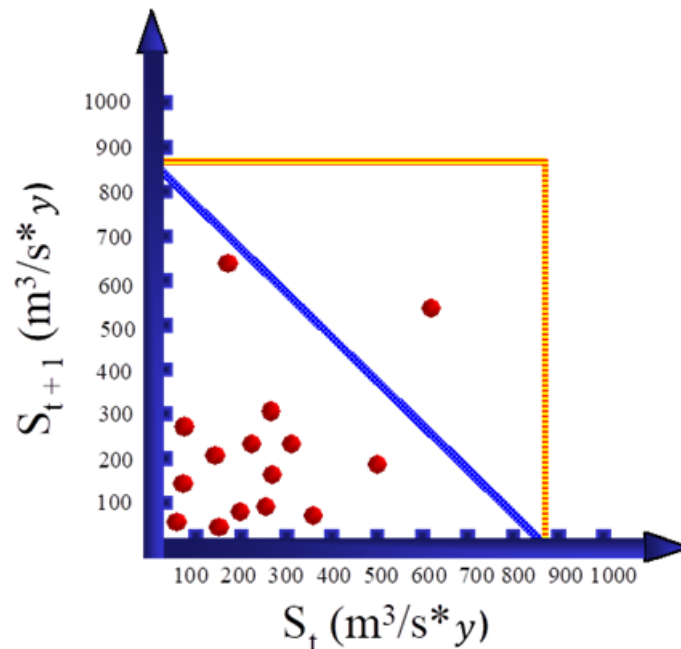
n = sample size.

For random parameters such as duration, [13] proposed to exchange  $n_1 / 2$  for  $(n / 2)^{1/2}$  in the above equation, which is renamed the modified Tschebycheff maximum. Figure 2 shows the dispersion diagram and the envelope for the duration of drought periods for Faz Cajazeiras Station (Acarau River) in the state of Ceará.

For non-random parameters, as appears to be the case with severity, severity values are usually restricted to the lower left triangle. This means that periods of extreme drought are only rarely followed by other periods of extreme drought. The envelope thus expresses the maximum response of the watershed to the observed events (figure 3). The upper right triangle therefore has a return interval longer than the length of the historical series [13].



**Figure 2:** Dispersion diagram, modified Tschebycheff maximum and envelope for the duration of drought periods for Faz. Cajazeiras Station (Acarau River)



**Figure 3:** Dispersion diagram, modified Tschebycheff maximum and envelope for the severity of drought periods for Faz. Cajazeiras Station (Acarau River)

**3.3 Simple correlation and cross correlation**

The relationship between the three characteristic parameters of hydrological droughts and floods can likewise be studied. We analyzed not only the relationship between each pair of parameters (simple correlation), but also the relationship between flood and drought events or between drought and flood events (cross correlation). This analysis allows a quantification of the internal and external structures of the drought and hydrological flood periods.

Tables 3 and 4 list the significance level values associated with the rejection hypothesis ( $r = 0$ ). Still, ID means that the correlation is not significant ( $\alpha > 0.20$ ) and D means that the correlation is very significant ( $\alpha < 0.01$ ). The signs correspond to the signs of the coefficients of simple and cross-correlations.

From tables 3 and 4 can be concluded that:

I) Duration and magnitude are the least correlated parameters, while duration and severity are the most correlated.

This internal dependency can be expressed by the following inequality:

$$C_v(M_i) < C_v(D_i) < C_v(S_i)$$

$C_v$  = variation coefficient.

**Table 3:** Test t; Values of  $\alpha$  for simple correlation of flood and drought events.

Station	Flood			Drought		
	D vs. M	D vs. S	M vs. S	D vs. M	D vs. S	M vs. S
Faz. Cajazeiras	+0.10	+D	+D	+0.20	+D	+0.05
Pacajus	+ID	+D	+0.01	-ID	+D	+ID
Paulo Sarasate	+0.05	+D	+D	+ID	+D	+0.10

- 1) The signals correspond to the slope of the regression lines.
- 2) ID (uncorrelated) for  $\alpha > 0.20$ ; D (correlated) for  $\alpha < 0.01$ .

**Table 4:** Test t; Values of  $\alpha$  for the cross-correlation of flood and drought events.

Station	Flood and Drought			Drought and Flood		
	Duration	Severity	Magnitud	Duration	Severity	Magnitud
Faz. Cajazeiras	-ID	-0.10	-0.20	-0.10	-0.20	-ID
Pacajus	+ID	+ID	+ID	+0.20	+0.05	-ID
Paulo Sarasate	-0.20	-0.20	-0.10	-0.10	-ID	+ID

- 1) The signs correspond to the slope of the cross-regression lines.
- 2) ID (uncorrelated) for  $\alpha > 0.20$ ; D (correlated) for  $\alpha < 0.01$ .

From the results of the above tests applied to the flow series one can adopt two procedures for modeling the flow series, namely:

- (I) the treatment of duration and magnitude under the assumption of independence;
- II) The treatment of duration and severity under the assumption of dependence.

One way of applying this first treatment was presented by [14] by joining a Markov chain for duration with an autoregressive model for magnitude. [8] and [13] demonstrated, however, that the likelihood of extreme droughts is undersized through a Markov process, so that droughts of greater duration and severity than those found in the historical series are rarely generated by this process. An example of a model that follows the second treatment, that is, using the assumption of dependency between duration and severity is the Alternating Renewal-Reward model - ARR, presented by [10]. Here this model was applied to the Brazilian semiarid, associated with the Fragment Method [15], for synthetic monthly streamflow generation, in order to perform hydrological drought assessment.

**IV. ALTERNATING RENEWAL REWARD / FRAGMENTS (ARRF)**

The model employed is the coupling of the Alternating Renewal Reward model (ARR), described by [10], used in the generation of annual streamflow values, with the Method of Fragment [15], for the disaggregation in coupled monthly ones.

Annual modeling: Alternating Renewal Reward - ARR

A basic assumption on modeling process of annual flows through the ARR model is that drought events come from different populations, ie, the deficit  $Y_i$  (deficit in year  $i$ ) is uniformly distributed and

independent, dependent, however, on duration. For the annual flow generation two stages are performed: 1) drought flood modeling process, 2) flow modeling within drought or flood periods. The model can therefore be found in one of two possible stages. If, for example, the system is assumed a priori to be flood, then  $DH_1$  years of flood are generated. The following step adopted is to assume the system is to be in drought condition, and  $DL_1$  years of drought are generated, and so forth.  $DH_n$  and  $DL_n$  are intended to have independent and uniformly distributed probabilities.

Thus, the problem consists only in identifying the probability distribution functions for the two-stage model. For the flood/drought modeling process, geometric distributions have been used and for the modeling of flood and drought severity, two-parameter gamma distribution have been applied. A limiting factor for the adjusting of duration (length) and severity distributions is the small sample. During a about 80 years data period, for example, there are approximately only 15 drought periods. In order to overcome this deficiency two procedures have been employed: decimation and standardization [16].

The decimation procedure [17] was used to obtain n (number of months) streamflow series, through the use of standardized flow values of the straight years, for each drought (flood), to each of the n series. The drought and flood periods severities are thus evaluated. This procedure simulates a regionalization through n flows sets of n different positions of a homogeneous region, subject to the same climatic conditions.

To simulate flood and drought duration geometric distributions was used, as follows:

$$f_x(x) = pq^{x-1} \rightarrow (7)$$

For the severity two-parameter gamma distributions for each duration (time length), was employed, as [10]:

$$f(Y_D) = \frac{\lambda e^{-\lambda Y_D} (\lambda Y_D)^{r-1}}{\Gamma(r)} \rightarrow (8)$$

with

$\Gamma$  = gamma function; r = shape parameter;  $\lambda$  = scale parameter.

Monthly modeling: Method of Fragment

The Method of Fragments from [15] is based on the disaggregation of annual flows generated by some annually model (in this case, the ARR model) into monthly flows (or shorter time interval). The model is characterized by estimating, for each month j and each year i of the historical flow series, the so-called fragments, given by:

$$f_{i,j} = \frac{Q_{i,j}}{\sum_{j=1}^n Q_{i,j}} \rightarrow (9)$$

n = number of months (n = 12);  $Q_{i,j}$  = streamflow in month j of year i.

The fragments  $f_{i,j}$  correspond to the percentage of annual streamflow (the denominator of the equation above) in year i. Following the historic annual flow values are placed in ascending order and separated into classes. The limits of the class intervals are formed by the mean values of successive flows. The total number of classes is equal to the number of years within the flow series measurement. The first class has zero as the lower limit and the last class upper limit of the last class is infinite. The annual streamflow generated are then distributed according to the class intervals and fragmented into monthly values.

## V. MODEL APPLICATION

In order to verify the applicability of the new model to the Brazilian semiarid intermittent rivers, the ARRF model was applied to three basins in the State of Ceará – Brazil, corresponding to the following reservoirs: Paulo Sarasate, Aires de Souza and Carão (Table 5).

**Table 5** - Characteristics of the analyzed basins

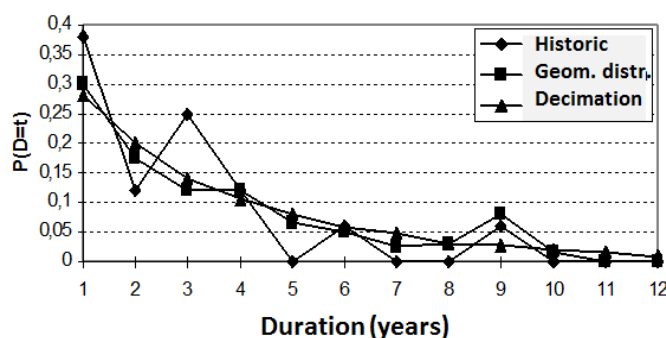
Station	Reservoir volume (million m <sup>3</sup> )	Basin area (km <sup>2</sup> )	Average annual flow (m <sup>3</sup> /s)	Period
Paulo Sarasate	891	3501	242.0	1912-88
Aires de Souza	104	1092	91.3	1912-88
Carão	20	305	10.9	1912-88

Table 6 presents the estimated values of the parameter p of the geometric distributions adjusted to the three analyzed basins for the drought and flood duration. Figure 4 shows the drought duration curve resulting from adjusting of the geometric distribution, after decimation process for the Paulo Sarasate reservoir basin.

**Table 6 - Parameters of the geometric distribution**

Station	p-flood (%)	p-drought (%)
Paulo Sarasate	52.5	27.5
Aires de Souza	49.5	35.0
Carão	66.0	22.5

**Drought duration curve**



**Figure 4 – Drought duration curve for the Paulo Sarasate station**

Table 7 contains the gamma distribution estimated parameters for the various flood and drought durations for the Paulo Sarasate station.

For each basin one thousand monthly synthetic streamflow series were generated. Figure 5 shows its mean values. For each month minimum, maximum, median and the historical value have been also shown. Table 8 show the relative average deviations (bias) and the root mean squared errors (RMSE) for the three analyzed basins.

**Table 7 - Gamma distribution parameters (Paulo Sarasate station)**

Duration (year)	Flood		Drought	
	$r$	$\lambda$	$r$	$\lambda$
1	3.212	0.018	38.199	0.306
2	4.653	0.026	35.526	0.299
3	7.052	0.044	27.858	0.236
4	4.101	0.029	47.147	0.373
5	-	-	27.424	0.240
6	-	-	16.418	0.146
7	-	-	14.994	0.138
8	-	-	49.307	0.437
9	-	-	39.137	0.324



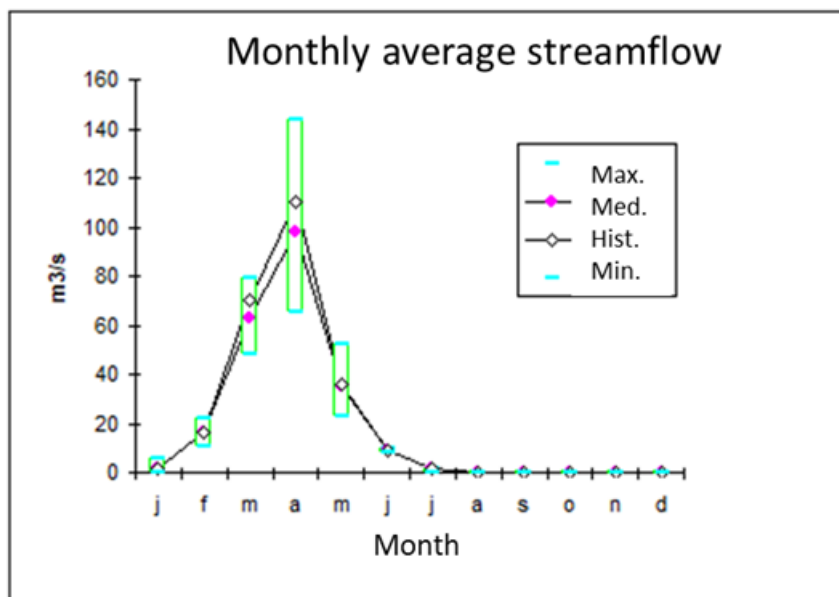


Figure 5 - Generated and historical series: statistical parameters (mean values) for the Paulo Sarasate station

Table 8 - Average adjustment errors (bias and RMSE)

Parameters*	Paulo Sarasate		Aires de Souza		Carão	
	bias	rmse	bias	rmse	bias	rmse
$\mu$	-0.024	0.406	-0.038	0.451	0.005	0.545
$\sigma$	-0.025	0.340	0.085	0.465	0.061	0.492
CV	0.053	0.326	0.094	0.395	0.103	0.229
$\gamma$	0.007	0.355	0.905	1.358	0.004	0.392
$\rho$	0.058	0.502	0.119	0.538	-0.009	0.281
$\rho$ lag-1	0.193	0.726	-0.571	0.605	0.340	1.193

\*  $\mu$  = mean;  $\sigma$  = standard deviation; CV = variation coefficient;  $\gamma$  = skewness coefficient;  $\rho$  = serial correlation coefficient;  $\rho$  lag-1 = lag-1 serial correlation coefficient

## VI. CONCLUSION

This paper presented the application of the ARRF model to generation of synthetic streamflow series in semi-arid regions to using on hydrological drought assessment. This new model, unlike autoregressive models, are based on the characteristics parameter (duration, severity and magnitude) of flood and drought periods, and when applied to different basins of the semi-arid region presents satisfactory results. Thus, it seems to be a useful tool for design and optimization of the operation of reservoir systems in semi-arid regions.

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## CONFLICT OF INTEREST

The authors declare no conflict of interest.

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