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Research Paper

Critical Introduction of Solow Growth Theory

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ABSTRACT: *The Main starting point of this paper is the Solow – Swan model named after Robert (Bob) Solow and Trevor Swan, generally called the Solow model. These economists published a more valuable economic article in 1956, The growth Solow model is the starting point of all analyses in modern economic growth theories, thus understanding of the model is essential to understanding the theories of the Solow growth. This paper illustration of the complex economic conditions and explains to the process of growth or macroeconomics equilibriums. Moreover, try comparison different sectors or multiple social interactions (saving, consumption etc.), and production of the society, as well as this model, implies different real income accounted in different capital inputs., processes from the installed capital, labor, and technology, make the governing equations infinite dimensional in the growth economy.*
KEYWORDS: *Basic Model, dynamics Modem, TechnologicalProgress, PopulationEffect, Advantage and limitations*

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I. INTRODUCTION OF THE SOLOW MODEL

The starting point of this chapter is introducing the Solow – Swan model named after Robert (Bob) Solow and Trevor Swan, generally called Solow model. These economists published more valuable economic article in 1956 [1], and in first times, they introduced the Solow model. Understanding of the Solow model is essential to understanding the theories of the modern growth; therefore, Solow model is much essential model for growth economy. Moreover, this model compares to different sectors or multiple social interactions, (saving, consumption, population etc.), thus, this model implies as a model of production processes from the installed capital, labor, and technology.[2]

1.1 Basic Solow Model (1956)

Economic growth is the dynamic process between inputs (capital, labor, and technology) and output, but, the consumption and population behaviors are changed this dynamic result, this model explain these different conditions how to effect to the output. The Solow model can be evaluated in two separate assumptions, which could be either discrete or continuous; this both conditions are used in macroeconomics,

Household and Productions, Considering the closed economy, with a unique final good. The economy is discrete time is running to an infinite horizon, so that time is indexed by $= 0.1.2.3 \ldots$[3].In addition, this model uses the terms household individual and agent are interchangeable. Households are identical.so that the economy trivially admits a representative-household. Meaning that the demand and labor supply side of the economy can be represented as if it resulted from the behavior of a single household.

The other main agent in the economy is firms, such as consumers. These are highly heterogeneous in practices, even within narrowly defined in the economy two firms are not identical. Firms have access to same productions for the final output. in that point the aggregate productions functions can be given by,

$$Y(t) = F\big(K(t), A(t)L(t)\big),(1)$$

When, (Y) output, (K) capital stock, (L) labor or total employability, and (A) effectiveness of the labor, “knowledge” or level of technology are important inputs of this model. Moreover, L and A are assumed to grow

[1] Robert M. Solow. 1956 “A Contribution to the Theory of Economic Growth” The Quarterly Journal of Economics, Vol. 70, No. 1 (Feb., 1956), pp. 65-94
[2] Technological possibilities are represented by a production function and effect to the amount of the knowledge increases
[3] This time period can be counted either day, week or year

exogenously. These are combined to produce output and estimate for an aggregate production function in the economy.

The capital stock K(t) corresponding to the quality of the machine (or more specially, equipment and structures) used in production, simply, K is the physical capital in the economy, Thus, it is used in production process of more good.TechnologyA(t), impotent production input for the productions process.it effects to efficiency of the factor of production. For the example, technological capabilities and labor (AL) are contributed to as efficiency for the L and A, in this trend is known as “labor augmenting or Harrod neutral”[4] ,

Moreover, we can specify how A enters, with other assumptions of the model. Defining the capital-labor ratio, $\frac{K}{L}$ does not represent any statically variance in upward and downward trend over the extended period. In the standard model of Solow (1956), eventually that capital, labor ratio is constant makes the estimations. But in ‘Keynesian’’ and ‘‘Schumpeterian’’ policies are rejected this constant condition of the capital-labor ratio in the economy. Because, lower unemployment levels and inside long run growth do not continue to the constant capital-labor ratio in the economy, as same as the matching or mismatching between innovative exploration of new technologies and the conditions of demand generate this change in the economy. However, technological possibilities are represented by a production function combined with the capital, and labor, (Solow 1956)[5] .therefore in main critical assumptions of the Solow model depended on the properties of the production’s functions and evaluation of the all inputs process (Capital, Labor, and Knowledge).

1.1.2 Assumption based productions functions.

The specific assumptions of the productions function defining, it has constant retunes of the scale in capital and effective labor, because the model cannot get well without constant returns to scale. (Solow 1994)[6].what is the simply explanation about constant retunes of scale, example, doubling the value of the quantity of capital and the labor (K and L) with A fixed, Double the production in same as quantity changes in K and L. In generally, multiplying the K and L by any positive constants C causes output to change by the same quantity of the K and L.

$$\mathrm{F(cK, cAL) = cF(K, AL)} \text{For a } c < 0 \quad (2)$$

This assumption of the constant returns as depending on combinations of the two assumptions, one is that economy large enough that increases combinations form K and L.In the smallest economies, they are probably enough ability for the combination of the factors that doubling that quantity of the K and L more than double output, (Romer; 1996). Somehow, in economies bigger enough to specializations of the K and L (inputs) essentially, increasing the output same way as existing K and L (given value, A is fixed).

Second assumptions is K, L and A (knowledge) are proportionally unimportant, as same as, Solow model eliminates the land and other natural resources.[7] “Natural resources are important doubling capital and labor

[4] Assume an economy with one product (Y(t)) and two factors of production, capital and labor, K(t) and L(t) respectively, capital being an accumulated stock of Y. Assume neutral technological progress at a constant rate p. We assume further that Y(t) is subject to a linear homogeneous production function, that is, constant returns to scale. We called this technological process “Hicks neutral”

$$Y(t) = A(t)F[(K(t).L(t)]$$

. But if, knowledge enters in the form called capital augmenting

$$Y(t) = F[A(K(t), L(t)],$$

Ryuzo Sato.1964 “The Harrod-Domar Model vs. the Neo-Classical Growth Mode” The Economic Journal. 74, No. 294 (Jun., 1964), pp. 380-387

[5] Robert M. Solow. 1956 “A Contribution to the Theory of Economic Growth” The Quarterly Journal of Economics, Vol. 70, No. 1 (Feb., 1956), pp. 65-94

[6] Solow,1994 “Perspectives on Growth Theory” journal of Economic Perspectives—Volume 8, Number1(Winter 1994)—Pages 45–54

[7]Moreover, Solow model (1956) neglects physical investment rates, human capital investment rates, export shares, inward orientation, the strength of property rights, government consumption, population growth, and regulatory pressure. Permanent changes in these variables, at least according to some endogenous growth model, should lead to permanent changes in growth rates. But these all factors did not consider in this model.

could less than double out", (Romer, 1996). However, in the standard Solow model, the importance of natural resource does not consider an estimating of the growth.
Considering the productions functions in intensive form under the assumption of the constant returns, setting $c = 1/AL$ in equation (2) yields,

$$F\left(\frac{K}{AL}, 1\right) = \frac{1}{AL} F(K, AL) \tag{3}$$

K/L is the amount of the capital of the unit of the labor and $F(K, AL)/AL$ is Y/AL,output per labor Explain, $k = \frac{K}{AL} = y = \frac{Y}{AL}$ and $f(k) = F(k, 1)$.According to equations we can rewrite.

$$y = f(k) \tag{4}$$

Simply, we can illustrate output per unit of the labor as a function of capital per unit of labor.

The amount of the output per unit of labor is not depended on the overall size of the economy.it depends only on the quantity of the capital per units of labor. this is illustrated mathematically in the equation (4). Because, dividing to the economy at AL, each 1 unit of the economy into K/AL units of capital. Since productions, a function has constant retunes. Thus, small economics or undivided economy, generate productivity $1/AL$ in the large, (simply labor efficiency is higher in small economics). If we can estimate the total amount of the productivity, in opposite to the amount per labor, we can derive using the multiplier of the labor $Y = ALf(k)$.
$f(k)$ is the incentive form of the productions function.it is estimated to conditions of $f(0) = 0, f'(k) > 0 f''(k) < 0$,[8]$f'(k)$ is the marginal productivity of the capital, since , $F(K, AL) = ALf\left(\frac{K}{AL}\right) \partial K = ALf'\left(\frac{K}{AL}\right)\left(\frac{1}{AL}\right) = f'(k)$,However, in that it decreases as capital rice, wecan explain this Inada conditions in mathematically, $\lim_{k\to 0} f'(k) = \infty, \lim_{k\to\infty} f'(k) = 0$.[9]
According to the Inada (1964), state that $\lfloor f'(k) \text{the marginal productivity of capital} \rfloor$,is very large the when capital stock significantly small. Therefore, the production function can satisfy $f'(*) > 0 f''(*) < 0$.[10] In graphically, the Inada conditions is represented (Figure 1.1)

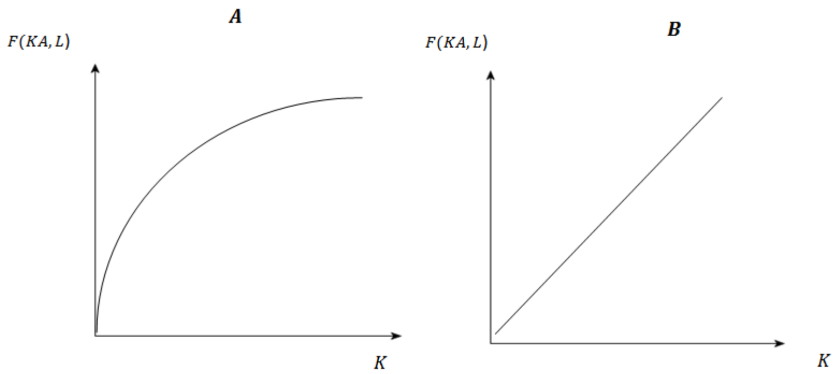

***Figure* (1.1)**: *Productions functions A satisfies the Inada conditions, While B does not ,*

Charles I, Jones. (1995) "Time Series Tests of Endogenous Growth Models" The Quarterly Journal of Economics, Vol. 110, No. 2 (May 1995), pp. 495-525
[8] $f'(*)$ first derivative of the $f(*)$, $f''(*)$ second deriviìative

Inada, 1963 "Two-Sector Model of Economic Growth: Comments and a Generalization" The Review of Economic Studies, Vol. 30, No. 2 (Jun., 1963), pp. 119-127

[9] This condition can explain again,

$$\lim_{k\to 0} f'(K, L, A) = \infty, \lim_{k\to\infty} f'(K, L, A) = 0 \text{ for all } L > 0 \textit{ and all } A$$
$$\lim_{L\to 0} f'(K, L, A) = \infty, \lim_{L\to\infty} f'(K, L, A) = 0 \text{ for all } K > 0 \textit{ and all } A,$$

Moreover $F(0, L, A)$ *for all L and A*

[10] Simply, the marginal productivity of both capital and labor are diminishing, that is,$f''(k) < 0$ *and* $f''(L) < 0$, so that more capital, holding everything else constant, increasing output less and less .(Daron ,2009)

When F exhibits, and under the constant retune to scale in K and L. In addition, capital and labor are sufficiently abundant; their Marginal products are close to zero. These conditions can represent, $\mathrm{F}(0,\mathrm{L},\mathrm{A})=0$ for all the A and L make capital essential inputs, According to the continuity, differentiability, positive and diminishing marginal product and constant retunes assumptions[11], Figure (1.1) ,Solow productions function $\mathrm{F}(\mathrm{K},\mathrm{L},\mathrm{A})$ as a function of K , for given L and K

A specific example, Solow's model takes two inputs, capital, and labor, which are paid their marginal products. We assume a Cobb-Douglas production function[12], so production at time t is given by,

$$\mathrm{F}(\mathrm{K},\mathrm{AL})=\mathrm{K}^{\alpha},(\mathrm{AL})^{1-\alpha}, \qquad 0<\alpha<1. \tag{5}$$

The Cobb- Dauglasfunction has constant retunes, multiplying by C value, yields,

$$\begin{aligned}\mathrm{F}(\mathrm{cK},\mathrm{cAL}) &= (\mathrm{cK})^{\alpha}(\mathrm{cAL})^{1-\alpha}\\ &= \mathrm{c}^{\alpha}\mathrm{c}^{1-\alpha}\mathrm{K}^{\alpha}(\mathrm{AL})^{1-\alpha}\\ &= \mathrm{cF}(\mathrm{K},\mathrm{AL}).\end{aligned} \tag{6}$$

In Cobb-Douglass productions L,K and Hicks neutral technological process are all definitely same, for the example rewrite (5) can define $\bar{\mathrm{A}}=\mathrm{A}^{1-\alpha}$ then $\mathrm{Y}=\bar{\mathrm{A}}(\mathrm{K}^{\alpha}\mathrm{L}^{1-\alpha})$. According to this condition, divide both inputs byAL, in incentive production functions, shown by

$$\begin{aligned}\mathrm{f}(\mathrm{k}) &\equiv \mathrm{F}\left(\frac{\mathrm{K}}{\mathrm{AL}},1\right)\\ &= \left[\frac{\mathrm{K}}{\mathrm{AL}}\right]^{\alpha}\\ &= \mathrm{k}^{\alpha}\end{aligned} \tag{7}$$

Because,$f^{'}(k)=\alpha k^{\alpha-1}$ expression is positive,$when\lim_{k\to\infty}(k)=0, lim_{k\to 0}(k)=\infty$. Finally,$f^{''(k)}=-(1-\alpha)\alpha k^{\alpha-2}$ is negative. We call these Cobb-Douglass intensive effects of the productions function.

1.1.3 The Evolution of the Inputs into Productions in Solow Model

The production at time t,K,L and A taken .and A and L grow at constant rates, The Cobb-Douglas production function, given by,

$$Y(t)=K(t)^{\alpha}(A(t)L(t))^{1-\alpha} \qquad 0<\alpha.<1.$$

Remember the assumptions of the Solow Model of K, L and A are assumed to grow exogenously at rates n and g

$$L(t) = L(0)e^{nt} \tag{8}$$

$$A(t) = A(0)e^{gt} \tag{9}$$

The number of effective labors,$(t)orL(t)$.growth at rate$n+g$. Where, n and$g$ are exogenous parameters and where evaluates an estimate respective to the time, because time, where the variables are defined only at specific dates (usually $t=1,2,3,\ldots$)[13] .Model has same implications in both discrete and continuers time, but is easier to analyze in continue time.[14] Under the conditions note that $L(t) = L(0)e^{nt}$ implies that $L(t)= L(0)e^{nt}n=nL(t)$ and that initial value L is, $L(0)e^{0}$ or $L(0)$ this is similar to the A , therefore growth of labor and knowledge value at given time 0, (8) and (9) imply $L(t) = L(0)e^{nt}$,$A(t) = A(0)e^{gt}$ and also this relationship can rewrite,

$$L^{\blacksquare}(t)=nL(t) \tag{10}$$

[11]this condition can explain in mathematically, (including the total productions factors, with labor capital and technological process)

$$F^{'}(K,L,A)=\frac{\partial F(K,L,A)}{\partial K}>0 \quad , \quad F^{'}(K,L,A)=\frac{\partial F(K,L,A)}{\partial L}>0$$

$$F^{''}(K,L,A)=\frac{\partial F(K,L,A)}{\partial K}<0 \quad , \quad F^{''}(K,L,A)=\frac{\partial F(K,L,A)}{\partial L}<0$$

[12] Charles Cobb and Paul Douglas (1928] this functional form in their analysis of US. Manufacturing. Interestingly, they argued that this production function, with a value for an of 1/4, fit the data very well without allowing for technological progress. Recall that if F(aK, aL] = aY for any number a > 1, then we say that the production function exhibits constant

[13] . The economy is discrete time is running to an infinite horizon, so that time is indexed by $=0.1.2.3\ldots$

[14] That is $\dot{\mathrm{X}}(t)$ is the similar conditions of the shorthand$dX(t)/dt$, Equate to the (1.8) and (1.9) that labor and knowledge growth exponentially.

$$A^{■}(t) = gA(t) \quad (11)$$

Equations (10) and (11) represent growth rate of labor, $L(t)$and growth rate of knowledge, $A(t)$.population growth rate given by the parameter of n and g[15].

1.2 The Fundamental law of Motion of the Solow model (Discrete Time conditions)

Total productions of the outputs can divide between consumptions and the investment, Where, $I(t)$ in investment at given time$t$,In assumptions, national income accounting for a closed economy, the total amount of final good in the economy must be either consume, C(t) or inverts $I(t)$.given bu time t, is shown

$$Y(t) = C(t) + I(t). \quad (12)$$

Moreover, since the economy closed (and there is no government spending), aggregate investment is equal to the savings.

$$C = I(t) = Y(t) - C(t), \quad (13)$$

The assumption that saving is the constant functions and exogenous,$s \in (0\ 1)$ and income can express as

$$S(t) = sY(t), \quad (14)$$

Which, in turn, implies that they consume the remaining $(1 - s)$ factions of their income, thus,

$$C(t) = (1 - s)Y(t). \quad (15)$$

The fraction of output devoted to $I(t)$ and $S(t)$ exogenous and s is constant, one unit of the output devoted to investment yields one unit of new capital, in addition,K capital depreciates exponentially at the rate of δ

$$\dot{K}^{■}(t) = sY(t) - \delta K(t). \quad (16)$$

As same, as there were no restrictions are working on n , g and δ individually, thus, sum of these values assumed to be positive. Finally, in summary conditions of the economic growth and Solow assumptions, low of the motion[16] of the capital stork, yielded by.

$$K(t+1) = (1-\delta)K(t) + I(t) \quad (17)$$

Traditional Solow model is mixture of an old Keynesian model and modern dynamic macro-economic models. Moreover, this model describes that relationship among a collection of the endogenous variables that is, among variables values are determined with in model itself. Thus, it is also involved parameters and exogenous variables, as well as household do not optimize when it comes to their saving and consumption. This behavior explained by (14) and (15). but in basic Solow model for a given sequence of $\{L(t), A(t)_{t=0}^{\infty}\}$and an initial capital stork$K(0)$, an equilibrium path is a sequence of the capital stork. However, the first Solow model, $Y = F(K, L) = K^{\alpha}, L^{1-\alpha}$ omitted the real world. Because, some of the futures are more important to the real economic growth, but it is natural to think these features of the model as defects.[17] The purpose of this model is

[15] It is convenient in describing the model to assume that the labor force participation rate is unity, example every members of the population is also worker, (Jones, 2002).

Jones.2002," introductions to economics growth".Noten and company (2002)-ISBN 0 -393-97745-5 ,PP ; 24-25

[16]Where $C(t)$is using (1),(12)and (17) ,any dynamic allocations in this economy must safety, shown by,

$K(t+1) = F(K(t), L(t), A(t), +(1-\delta)K(t) - C(t)$ and in terms, capital market clearing, (14) implying that the supply for the time $(t+1)$ resulting from household behaviors can be expressed as $K(t+1) = (1-\delta)K(t) + S(t) = (1-\delta)K(t) + sY(t)$

Setting supply and demand equal to each other and using (1) and (17) yields the fundamental law of the motions of the Solow model

$$K(t+1) = sF(K(t), L(t), A(t), +(1-\delta)K(t)$$

This equation described by low of the motion for $L(t)$ and $A(t)$.

[17]Since this is the first model of Solow, grossly simplified in the two $ways\ , for\ the\ example$,there is the only single good, government is absent, fluctuation is employment are ignored, productions is described by an aggregate productions function with three inputs, and the rate of the savings, depreciations ,populations growth, and technological progress are constant. This simple model omitted the real world, because some of the futures are more important to the real economic growth in the world. (Romer, 1996)

not realistic. The world realistic, the problem with this model is that too complicated to understand. A model purpose to provide insights about particular of the world (End of this chapter explain criticisms of this model),

1.3 The dynamics of the Solow Model

1.3.1 The dynamic of the K

At beginning of this section, we derived the main key equations of the Solow model in term of capital per labor$k$, rather than the unadjusted capital stork,K,since $k = K/AL$,we can use to estimate chain of rule to find,[18]

$$k^{■} = \frac{K^{■}}{A(t)L(t)} - \frac{K(t)}{(A(t)L(t))^2}[A(t)L^{■}((t) + L(t)A^{■}(t)] \quad (18)$$

$$= \frac{K^{■}(t)}{A(t)L(t)} - \frac{K(t)}{A(t)L(t)}\frac{L^{■}}{L(t)} - \frac{K(t)}{A(t)L(t)}\frac{A^{■}(t)}{A(t)}$$

$\frac{K}{AL} isthesimplek$,(10) and (11) is the $L^{■} andA^{■}$, $\frac{A'}{A} and \frac{L'}{L}$ are $gandn$,$K^{■}$ is given by (16) ,substituting the (18),

$$k^{■}(t) = \frac{sY(t)}{A(t)L(t)} - [k(t)n - k(t)g]$$

$$= s\frac{Y(t)}{A(t)L(t)} - \delta k(t) - nk(t) - gk(t)$$

Simply, these equations can explain, when an economy starts out with stock capital per labor, and given populations growth rate, depreciation rate, and investment rate, how does output per labor evolve over time in the economy.in other way this equations can rewrite $k^{■} = sY - (n + d)k$ but in given time $Y = k^{\alpha}$, [19]

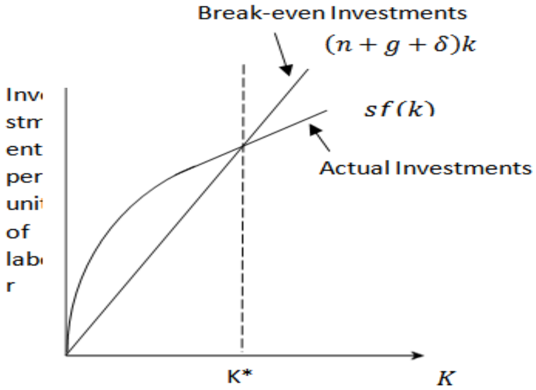

Figure* (1.2): Actual *and break even investment without population growth and* technological *changes

Using the factor of the $\frac{Y}{AL}$ is given by $f(k)$, we have

$$k^{■}(t) = sf\big(k(t)\big) - (n + g + \delta)k(t). \quad (19)$$

This is the key equations of the Solow model; in this equilibrium, the capital labor ratio remains constant. Since there is no population growth, this implies that the level of the capital stork will also remain constant. This behavior depend of the two different terms, the first $sf(k)$is the actual investment of the unit of the labor, also output of labor is the functions of k , $f(k)$,thus function of the output invested that s.simpaly,An Alternative visual representation show the study stat as intersection between a ray thought the origin with slope δ and the functions of the $sf(k)$. The second term, $(n + g + \delta)k$, is break-even investment, this is must be done keep k at its existing level. There are two reason that some investment in need to prevent k from falling. First, capital is depreciations. This is representing the δk at term in (19). Second, the quantity of the labor is

[18] That is, since, k is a function of $K\ nad\ A$,earch of which functions of t,

$$k^{■} = \frac{\partial k}{\partial K}K' + \frac{\partial k}{\partial L}L' + \frac{\partial k}{\partial A}\text{A}'$$

[19] $d = \delta$ depreciation rate of capital

growing, therefore doingenough investment and keeping capital stork constant K ,not enough to keep the $k$constant. Science quantity of labor is growing at rate $n + g$ the capital stock must grow at rate $(n + g)$to hold the k stedy. Because, the growth rate of the ratio two variables,X_1/X_2 is the difference of their growth rates[20], $\frac{X^{\blacksquare}{}_1}{X_1} - \frac{X^{\blacksquare}{}_2}{X_2}$. Thus, the growing rate of $= K/AL$, is $\frac{K^{\blacksquare}}{k} - \left(\frac{A^{\blacksquare}}{A} + \frac{L^{\blacksquare}}{L}\right)$.It follows that keeping k constant requires, $\frac{K^{\blacksquare}}{K} = n + g$.

Figure (1.2) In a steady-state equilibrium the capital–labor ratio remaining constant. Science is no population growth, and the capital stock is remaining constant. Mathematically, a steady-state equilibrium corresponds to a stationary point of the equilibrium. This figure (1.2) illustrates case for this simple model.

Figure (1.2) explain actual and break-even investment without population growth and technological changes. The $k^{\blacksquare}$ is the function of k.the Brake-even investment,$(n + g + \delta)k$, is the proposition of the k.$sf(k)$, is the Actual investment and it is constant to the times output per unit of labor. The ray through the origin with slope n represents the function$(n + g + \delta)k$. The other curve is the function $sf(k)$. It is here drawn to pass through the origin and convex upward: no output unless both inputs are positive, and diminishing the marginal productivity of capital, as would be the case, for example, with the Cobb-Douglas function. At the point of intersection$(n + g + \delta)k = sf(k)$, and $k = 0$. if the capital-labor ratio $K^{\blacksquare}$ should ever be established, it will be maintained, and capital and labor will grow thenceforward in proportion. By constant returns to scale, real output will also grow at the same relative rate$(n + g + \delta)$, and output per head of labor force will be constant.

On other words, small value of the , and actual investment is large than the Brake even investment,Inada conditions also imply that $f'(k)$,that point start the Brake-investment$f(0) = 0$ and equal to the$k = 0$[21], $f'(k)$ is large.$sf(k)$Line faster than the $(n + g + \delta)k$ line two must cross.Finally, $f''(k) < 0$ implies that the two lines intersect only once for $k > 0$. Therefore k' is the done value of the$k$, namely, Steady-state or point of actual investment and brake even investment is equal.

In Summary, which shows $k^{\blacksquare}$ is the faction of k.and if k generally less than the k*. thus, actual investment and brake even investment and $k^{\blacksquare}$ is positive therefore if $k$is rising and k exceeds k*,k' is negative. In end $k =$ k* and$k^{\blacksquare} = 0$, thus nevertheless where k starts, it converges to k*, in graphically it is given by (Figure 1.4),

[20] $\frac{X^{\blacksquare}{}_1}{X_1}$ is reference its proportional rate of change, and X is the growth rate of the variable.

[21] If $notk = 0$, where $k = \varepsilon$ for some $\epsilon > 0$ or the convention that the intersections at $k = 0$ is being ignored even though $f(0) = 0$) ,this we call Unique steady state condition. We see below this interpretation, even when exists, is an unstable point (Hakenes , Irmen. 2006)

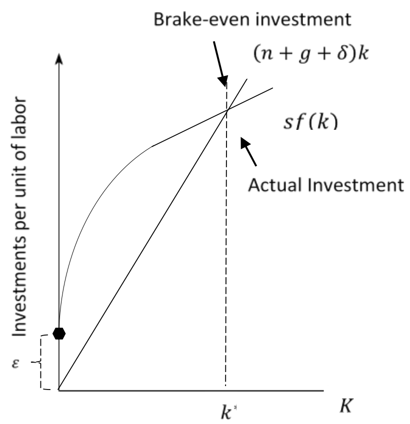

***Figure* (1.3)**: Unique steady state in the basic solow model when $f(0) = \varepsilon > 0$

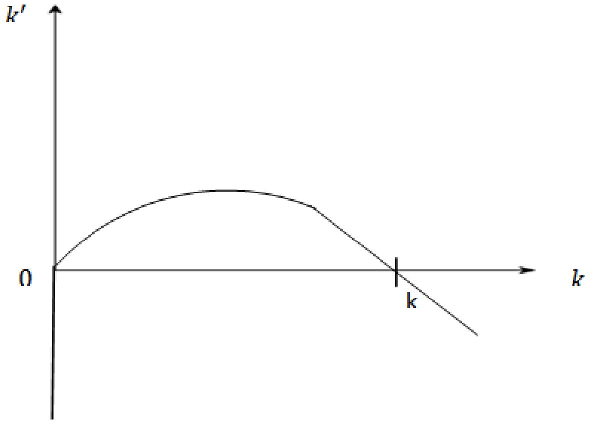

Figure **(1.4)**: The phase diagram for k in the Solow model

1.3.2 The Solow growth path

(Figure 1.5), Starting from the initial capital stock $k(0) > 0$, which is blow the study state level k*, economic grows forward k* and while capital labor increases. Both capital deepening comes growth per capital income. if instead the economy were to start with $k'(0)$ >k*, it would reach the steady state by dissimulating capital and constricting.

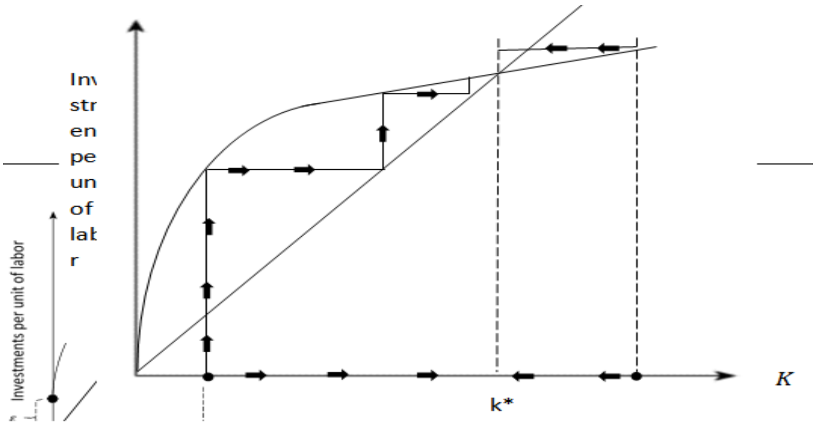

Figure **(1.5)** Dynamics in the basic Solow model

Recall that when the economy starts with too little capital relative to the labor supply, the capital labor ratio will increase. Thus, the marginal product of the capital will fall due to diminishing returns of the capital. Considering the balance growth, since , k converge to k*, it is natural to ask how of the model behave when k equals k*. According to the assumption $LandA$ are growing at rate $nandg$, respectively. The capital stock K equals to theALk, since k is constant at k* is growing at rate $(n + g)$.the assumption of constant returns implies that output,Y, Finally capital per labor , (K/L) and output per worker (Y/L) at growing rate g.and the balance of the growth part, the growth rate of output per worker is determined solely by the rate of the technological progress .(Caldor 1961).[22]

The analysis has established Solow growth has a number of properties, unique steady state, global stability, and finally, simple and intuitive comparative static. Yet so far it has no growth, the steady stat is the point at which there is no growth in the capital labor ratio, no more capital deepening, and no growth per capita. Consequently, the basic Solow model (without technological process) can only generate economic growth along the transition path of the steady state. However, this growth is not sustained.it slows down over time and eventually ends.

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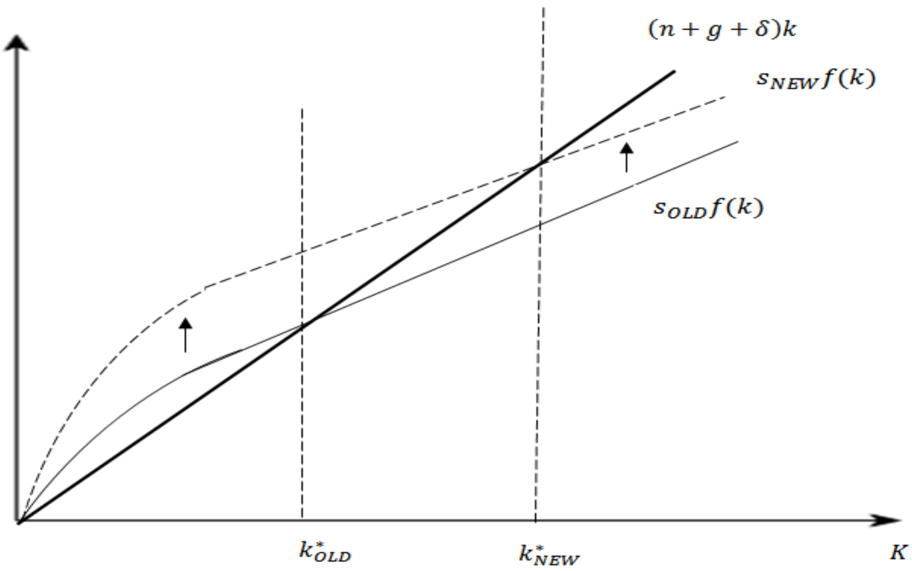
1.4 Impact of the Savings and investment for the Solow Model

Consider an economy that has arrived at its steady state value of output per worker. Suppose that the economy decides to increase the saving rate s.we will consider a Solow model economy that is a balance growth path.



Investments per unit of labor

The increase in s shifts the actucal investment line upward and so k* rises. This is shown (Figure 1.6.)k dose not immediately change to new vale of k*, however generally k is equal to the old k*.In that point, actual investment now exceeds to break even investment, thus more resources are being devoted to investment than are needed to hole k constant, and this point $k^{\blacksquare}$ is positive , moreover k being to rice, it continues rise until reach the new k*.

Therefore, countries with higher saving rate and technology will have higher capital labor ratio will be richer. Those with greater δ depreciation will tend to have lower capital labor ratio and will be poorer. In mathematically,Y/L equals $Af(k)$ .when k is constant,Y/L grows at rate$g$, the growth rate of$A$. When k increasing Y/L grows both because Ain increasing reason of k is increasing. Moreover growth A effect to growth of Y/L , and growth rate of Y/L retunes to g.this premenet increase of s generate temporally increase in growth rate of output per labor. Finally, Additional savings is maintaining the higher level of k in economy.

The golden rule saving rate was introduced by Edmund Phelps (1961). It is called the “golden rule” rate with reference to the biblical golden rule “do unto others as you would have them do unto you” this has shown that an increasing the savings rate rises the capital per labor and level of the steady state toward and finally output per labor and national productions are higher, moreover While the golden rule saving rate is of historical interest and useful for discussions of dynamic efficiency it has no intrinsic optimality property since it is not derived from well-defined preferences

On the other hand, this investment behavior of the growth rate per worker over time is displayed, (Figure 1.7),

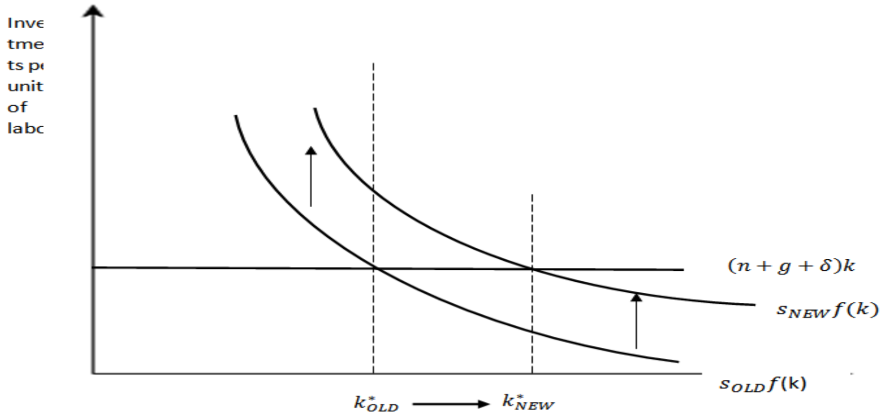


***Figure* (1.7)**:Impact of the increase in the saving rate for capital stock

In summary, a change in the saving rate has level effects but not a growth effects, its change the balance of the growth path, but not change the growth rate of the output per labor. However, model change the rate of the technology have growth effects, all other changes have only level effect.

1.5Impact of the consumptions for the Solow Model

Consider the basic Solow model, where the capital labor ratio k* $(0, \infty)$, capital output ratio given by

$$y^* = f(k^*) \tag{20}$$

In addition, per capital capita consumption is given by,

$$c^* = (1-s)f(k^*) \tag{21}$$

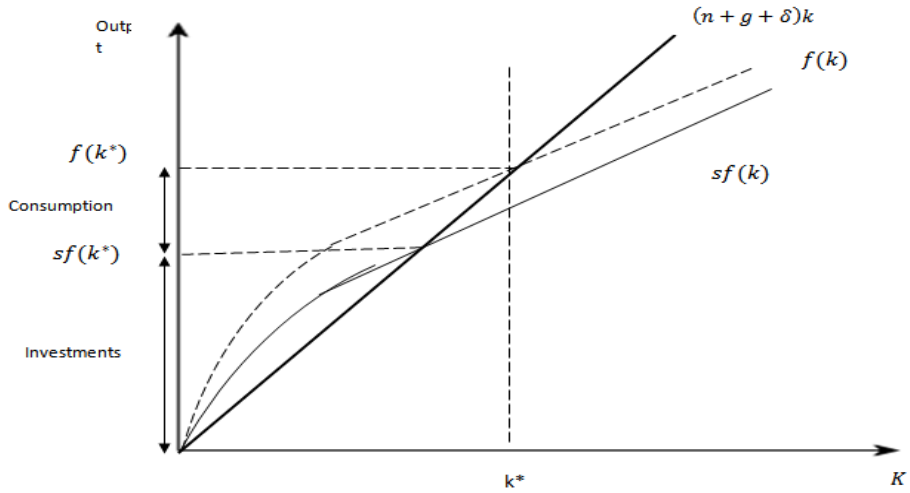


***Figure* (1.8)**:Impact of the consumption and on investment

Let c* denote consumptions per unit for labor on the balance of growth path, and c* equal output per unit of the labor
$f(k^*)$, minus investment per unit of the labor.$sf(k^*)$. on the balanced growth part, actual investment equal brake even investment $(n+g+\delta)k^*$.thuis,

$$c^* = f(k^*) - (n+g+\delta)k^* \tag{22}$$

k* is determined by s and the other parameters of the model. Therefore, we can rewrite $k^* = k^*(s, n, g, \delta)k^*$ thus (22) implies,

We know that increasing in s rise the k* (figure 1.6),thus whether the increase raises or lower consumption in the long run depended on the marginal productivity of the capital is more or less than $(n+g+\delta)$.If,$f^{'}(k^*)$,is less than $n+g+\delta$, then the additional output from the increased, capital is not enough to the maintain the capita stock, at its higher level.in that point, c* must fall to maintain the higher capital stork. Conversely, if $f^{'}(k^*)$,exceed$n+g+\delta$,there is more than enough additional output to maintaining the k at the higher level and so c* rise .if $f^{'}(k^{*})$, is less than the$n+g+\delta$, so an increase in saving rate lower consumption rate even when the economy has recharged the new balanced growth path. Finally,$f^{'}(k^{*})$,just equal to the $n+g+\delta$,that is the $f(k)$ and $(n+g+\delta)k$ line are parallel at $k=(k^{*})$, in this case marginal change in s has no effect on consumptions in the long run, and the consumptions is at its maximum possible level among balanced growth path. This value of the k^{*} is known golden rule level of the capital stock.

1.5.1 Output in long run.

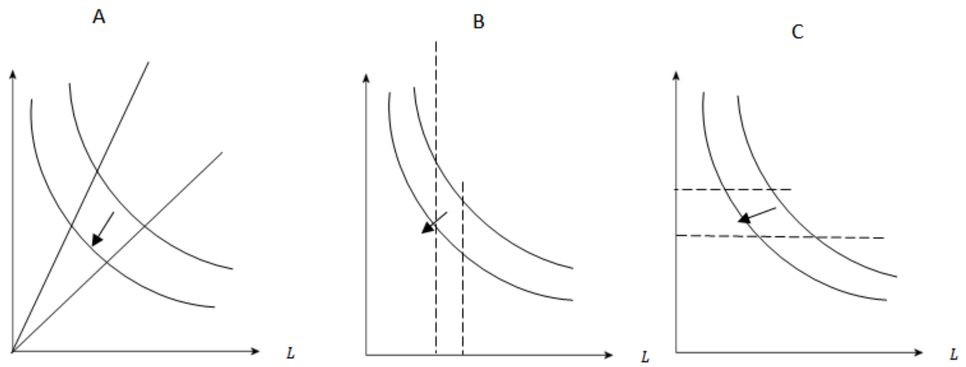
If market is competitive and there are no externalities, capital earns its marginal product. According to the given case,$k^{*}f'(k^{*})/f(k^{*})$[23] is the elasticity of output with respect to capital at $k=k^{*}$, given by ,

$$\frac{s}{y^{*}}\frac{\partial y^{*}}{\partial s}=\frac{ak(k^{*})}{1-ak(k^{*}} \qquad (23)$$

In this case, total amount of the received by capital (per unit of labor) on the balanced growth path is $k^{*}f'(k^{*})/f(k^{*}$ or $ak(k^{*})$, therefore in long run in most countries; the share of income paid to capital is about one third. If we use this estimate of $ak(k^{*})$,it follows that the elasticity of output with respect to the saving rate .in Significant changes in saving have only moderate effects on the level of output on the balanced growth. Intuitively a small vale of the $ak(k^{*})$ makes the impact of saving on output low for two reason, First, it implies that the actual investment curve,$sf(k)$, therefore in result, an upward shift of the curve moves its intersection with the brake even investment line relatively little. Thus the impact of change in k* on y* is small.

1.6Solow model with Technological Progress

The model analyzed so far did not consider the technological progress. Introduce changes in $A(t)$ to capture improvement in the technological knowledge in economy. Solow model generate sustained growth without technological progress. but only if some of the assumptions imposed so far relaxed.in that point given by,



$\textbf{Figuer (1.9)}$; (A)*Hicks Neutral*, (B)*Solow neutral and* (C) *Harrod neutral shift in isoquants*

Consider an aggregate production Function (special types of the production functions in balance of the growth),$\mathcal{F}$**.** and normal production function $F(K(t),L(t),A(t)$ is too general to achieve balance growth.$\mathcal{F}$, let us define different types of neutral technological progress. **A** first possibility is,

$$\mathcal{F}[K(t),L(t),A(t)=A(t)F(K(t),L(t)](24)$$

Simply, constant retunes of the production function Fimplies that the technology term $A(t)$ multiplicative of another production function F.This type of the technological progress called “Hicks-neutral”

Another alternative is to have capital augmenting or Solow neutral technological progress, in that form,

$$\mathcal{F}[K(t),L(t),A(t)=F(A(t)(K(t),L(t))](25)$$

Simply, which is referred to as capital augmenting progress. Because a higher $A(t)$ is equivalent to the economy having more capital. this type of the technological progress corresponding to the isoquants shifting inward as if the capital axis were being shrunk.

---

[23] The long run effect of a rise in saving on output is given by ,

$$\frac{\partial y^{*}}{\partial s}=f'(k^{*})(\frac{\partial k(s,n,g,\delta)}{\partial s}$$

And $y^{*}=f(k^{*})$ is level of output per unit of effective labor on the balance growth path. According to this conditions equate the hold for all value of s and where the arguments of k* are omitted for simplicity can finally obtained,

$sf(k^{*})=(n+g+\delta)k^{*}$ and substitute for s,this is given us

Finally, we can have labor augmenting or Harrod neutral technological progress given by panel$C$,

$$\mathcal{F}(K(t), L(t), A(t)) = F(A(t)(K(t), L(t))$$

While increasing the technological progress $A(t)$ increasing the output as if the economy had more labor and thus corresponds to an inward shift of the isoquant as if labor axis were being shrunk.

Using the Production function can illustrate this simply, let's consider,$\mathrm{y(t)} = \mathrm{A}(1+\lambda)^{\mathrm{t}}\mathrm{k}_{\mathrm{t}}^{\propto}$. since technological progress ,$\mathrm{A(t)}$ increases yearly at an exogenously, the λ is the technological growth rate per yearly, thus, $\lambda > 0$, this increase has the effect of shifting up world to the productions equal to the amount of the λ per yearly. This is given by,

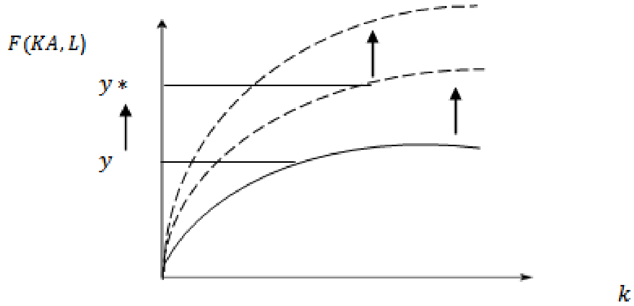


***Figure* (1.10)**: *Production growth with technical progress*

1.7 convergence

Solow model predict countries converge to their balanced growth paths Thus so extent that difference is output per worker arise from countries begin at different points relative their balance of the growth paths. Moreover, the Solow model implies that the rate of returns on capital is lower in countries with more capital per worker. Thus, there are incentives for capital to flow from rich to poor countries this will also tend to cause of convergence, (Baumol, 1986: Mddison, 1982)

In practices, eventual effects of the changes (such as saving rate) rapidly effects occur in long run equilibrium, but we are only interested eventual effects in short run. therefore, we miss to use approximations around the long run equilibrium to address this issue.

For mathematically, we most focus of the behavior of k rather than y . but our target is to estimate how rapidly k approaches k*. we knowk■ is the functions of the k.see (19). When k =k*,k■ is equal to zero. That is k■ is approximately equal to the product of the difference between k and k*. and the derivative of the k at k =k*.

In this process $\mathrm{sf(k^*)} = (\mathrm{n+g+\delta})\mathrm{k^*}$substitutes, in the vicinity of the balanced growth path, capital per unit of labor convergence toward k*. Simply, one can show the y approaches y* at the same rate that k approaches k* that is,$\mathrm{y(t)} - \mathrm{y^*} \cong \mathrm{e}^{-\lambda \mathrm{t}}[\mathrm{y(0)} - \mathrm{y^*}]$.[24]

1.8 Population Growth Effect of The Solow Model

Population growth is well known that the total fertility rate (number of birth per women) is much higher in past century .in this case n is the population growth rate of the economy , considering the population growth effects to the k and y in this economies .graphically, $(\mathrm{n+g+\delta})\mathrm{k}$ curve rotates up and to left to new curve $(\mathrm{n+g+\delta})\mathrm{k}'$, thus, the current steady state change, because, investment per labor is now no longer higher enough to keep the capital labor ratio constant in the face of the rising population ,therefore the capital labor ratio begins to fall .it is consistency falling gown until the point at new steady state $\mathrm{k^{**}}$, at that point ,the economy has less capital per labor than it began with and therefore poorer, per capita output is ultimately lower after the increase in population growth .

[24] That is defining $x(t) = k(t) - k^*$ and $\lambda = (1 - ak)(n + g + \delta)$ implies the growth rate of x is constant and equal $-\lambda$ .therefore the balace path given by $x(t) \cong x(0)e^{-\lambda t}$ where $x(0)$ instial value of x. in terms of k

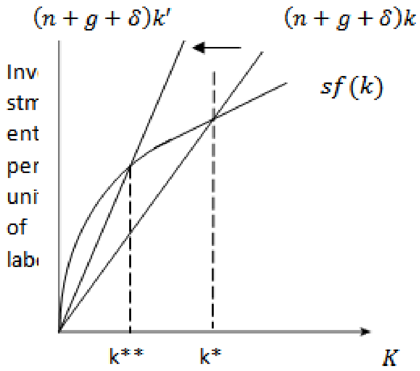


Figure **(1.11)**: Actual *and break even investment with population growth*

Population growth rate both positively and negatively affected to the economic growth, thus it makes deferent beehives in Solow model, those all detailed knowledge explains in next chapters in this thesis.

1.9 Advantage and the limitations of the Solow model

One of the most investigated problems in the study of the growth, models are the asymptotic stability of the equilibrium growth.in some believes, Considering this model it is a theoretical and practical significance, the growth models have been studied extensively (Accinelli and Brida 2007; Boucekkine et al. 1997; Deardorff 1970; Emmenegger and Stamova 2002; Fanti and Manfredi 2003; Ferrara 2011; Jensen and Larsen 1987; Guerrini 2006; Nerlove and Raut 1997; Raut and Srinivasan 1994; Sheshinski 1969; Szydowski and Krawiec 2004). It represents closer to the reality but in other way it gets feather away from it.

Advantages of this model notes the indigenization of the capital labor ratio such ratio varies in fact according to the level of capital per worker and parameterα of the production function, thus, this model introduces the technological concept correlation with the economic growth, moreover it explains a modest part of the variance of the growth. For the example growth of the output of labor in economies due to the decreasing retunes of the capital per labor, while this concept is not often estimated in real world or when it is estimated is mainly explained by the other variables.

Limitations of the model concern the assumptions for the example technological progress assumed exogenously. But this is depending of the decisions of the investment in education, research and development and, innovations, etc. Thus, in the analysis of the convergence, the model assumes the same technology for all countries .it does not assume domestic factors (deferent educations capabilities and abilities to absorb imported technologies and local level), which nations use for the deferent technological progresses. Moreover, this model ignores important factors which are discussed in resent other growth models. For the example it ignores fundamental growth factors such as human capital, international trade, social capital etc., but after the endogenous growth theories, these are important to note that all this modeling takes form the neoclassical models

II. CONCLUSION

in conclusion this paperillustrated Solow model short overview, but this model has not yet been assumed various factors that in the theatrical important are considered growth such as social capital, international trade income distribution etc. However, Solow progress is not so much theatrical problems allow to formations of mathematical view, but difficulties met in properties of production factions and variables. end of the chapter, my personal opinion, there is a long way to go this model with the worth comparing the main characteristics.

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