*Quest Journals*
*Journal of Research in Humanities and Social Science*
*Volume 9 ~ Issue 3 (2021)pp: 50-67*
*ISSN(Online):2321-9467*
www.questjournals.org

**Research Paper**

# "Analysis of the Productive Efficiency of Ivorian Firms by the Stochastic Frontier"

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***Abstract:***
*Our analysis sought to understand and explain the determinants of economic efficiency or inefficiency in Côte d'Ivoire. This study derives its justification more in favor of the post-electoral crisis which created socio-political instability and disorganized the production apparatus. The additional effect of the inefficient use of factors of production and of the skilled labor force that immigrated to other countries probably had an influence on economic efficiency. Also: is it possible to argue that Ivorian companies are economically efficient? What then are the determinants of this efficiency or inefficiency?*
*Therefore, we used data from the INS Financial Data Bank relating to 4443 companies observed over the period from 2008 to 2012. The estimation of a stochastic production frontier and the analysis of the efficiency have shown that the production system is not economically efficient. The results indicate that the gap between observed output and potential output is 18%. This large gap between the efficiency levels of the 12 industries shows that there is enormous potential to increase their efficiency. The estimation of a Tobit model then made it possible to identify the explanatory factors of economic inefficiency. These include the size of companies, the institutional environment, national savings, financial debts and value added.*
***Keywords: technical, allocative, economic efficiency, stochastic frontier***

*Received 02 Mar, 2021; Revised: 12 Mar, 2021; Accepted 14 Mar, 2021 © The author(s) 2021.*
*Published with open access at* www.questjournals.org

## I. INTRODUCTION

In this paper, we propose to reflect on economic efficiency in Côte d'Ivoire. The concept of efficiency can be defined by dissociating what is technical from what is due to a poor choice, in terms of input combination, from input prices. According to Farrell (1957) technical efficiency measures how a firm chooses the quantities of inputs that enter the production process when proposals for the use of factors are given.

Technical efficiency differs from price efficiency or allocative efficiency which assesses how the firm chooses the proportions of different inputs in relation to the supposed competitive market price. Economic efficiency is determined by the combination of technical efficiency and allocative efficiency. It refers to the concepts of productivity, performance, quality and performance on the one hand and reduction of the number of employees and costs on the other. The concept of economic efficiency will be associated with the value test. Thus, a change to increase value is seen as an effective and ineffective change if not.

The aim of this study is to identify the explanatory factors of economic efficiency or inefficiency in Côte d'Ivoire. Specifically, we want to analyse the production organization, study the impact of the institutional environment on productivity and identify the level of performance of the production sectors of the economy.

The answer to these questions will lead to the identification of the sources and determinants of the efficiency or inefficiency of the Ivorian economy.

In recent years, socio-political instability has led to a disorganization of the production apparatus and an unraged use of production factors (Ouattara 2008). This instability has also led to the loss of skilled labour that has migrated to other countries. We therefore postulate for the main assumption that this situation has had a negative effect on the efficiency of the Ivorian economy. This study, the results of which can be used by governments to act on the sources of inefficiency, is intended to contribute to improving the overall efficiency of the economy. To assess this effectiveness, we follow the approach of production borders. We have a database provided by the Financial Data Bank (BDF) of the National Statistical Institute (INS). The study sample covers a total of 4443 companies. This database is observed over the period 2008 to 2012.

## II. LITERATURE REVIEW: MEASURE OF PRODUCTIVE EFFICIENCY

In its presentation of the economy as a competitive regime, the free play of market inflows and exits carries with it the mechanisms for restoring the competitiveness of enterprises. Condemned to succeed on the battlefield of competition at the risk of losing market share or leaving the market entirely to the benefit of others, companies ensure the allocation of their resource or the efficient use of factors of production. In short, companies that hold up well in the market are those that in essence produce efficiently and therefore best combine the factors of production.

Already, the classics had addressed this problem thanks to Adam Smith's "invisible hand" theory. The market bears the seeds of productive efficiency, so it is not really appropriate to overcome failures within companies. Some authors have brought criticisms to this theory, which is meant to be universal.

Indeed, presenting the economy as a pure and perfect integral competition regime is far from reality. Markets are in imperfect competition in practice, and the risks of permanent inefficiency are not excluded.

Hirshman (1992) wrote that no economic system can guarantee that companies will always act in such an efficient way as to behave as effectively as they are expected to behave. To avoid any general economic dysfunction, resulting from the deviant and cumulative behaviours of a significant proportion of companies representing the economy, the need for an analytical framework arises acutely to describe and explain the reason for the failures and/or successes of companies. The current economic literature provides an appropriate methodological framework based on modern microeconomics.

Despite the multitude of modern microeconomic theories, we present here only Harvey Leibensteinr's theory of efficiency X (1966) to illustrate productive efficiency.

Either $Q = f(K, L)$ production function of a firm with Q output, K and L respectively capital and labor. By definition, it will be said that the firm is technically efficient when it is on the frontier of production opportunities, in other words, with a specified amount of factors, it obtains the highest level of achievable production.

The concept of efficiency or inefficiency X was developed in 1966 by Leibenstein to essentially say that for one reason or another, work in the company is not as effectively to lead the firm towards its goal of maximizing. For Leibenstein, inefficiency X is the type of inefficiency resulting from the misuse of resources within production organizations.

In a broader context, Leibenstein contrasts overall economic efficiency with the overall process of allocating resources within the company. As a result, if we assume the allocation of the constant factors, the organization is able to generate surpluses through the increase of its productive efficiency (X efficiency). Thus, the X-efficiency results from the fact that organizations do not make the most of their resources. Thus, with the same technology and production factors, companies can achieve different results in terms of productivity. Therefore, not all companies are on the "efficient border" of the production package from the moment on or not all of them value the existence of an X input, distinct from the traditional factors (capital and labour), and which reflects the overall quality of resource management within the organization.

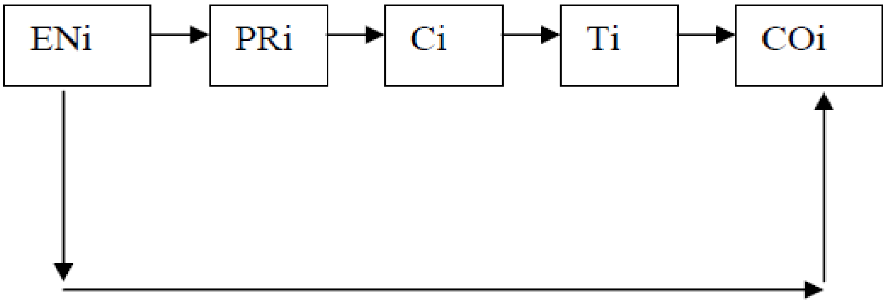
Among the facts that generate this surplus, Leibenstein stresses the motivating factors related to the general working conditions. For example, the fact that a company fails, at a given input level, to obtain the desired volume of outputs can be easily explained by considering that "employment contracts are incomplete...some complex machines whose specifications (in terms of production) are fixed, generate variable performance depending on their respective use. it is, moreover, exceedingly rare that all elements of performance are considered by the employment contract." (Leibenstein, 1966, op.cit).

This theory differs from the views of conventional economies that for a given amount of resource, the company achieves optimal production, or by setting its level of production, it uses the minimum amount of resource to achieve it.

Proponents of this theory argue that a company's productive inefficiency can be explained by several factors, but the main variable is the effort that depends on the degree of motivation in place in the company. Indeed, the discretionary nature of the individuals' effort can lead to a divergence between individual and corporate objectives. The stress can be increased however either by internal pressure or by external pressure. If the internal pressure is that exerted by the supervisor within the company, the external pressure is that resulting from the competition.

Leibenstein materializes this pioneering work through the causal relationship presented below which is in fact the essence of the theory of efficiency or inefficiency X.

**Chart 1: H. Leibenstein's causal relationship**




With $EN_i$,:Environment in which the company i evolves that can be either competitive or monopolistic;
$PR_i$ : Variable representative of the level of pressure in the company i;
$C_i$ : Choice of effort within the company;
$T_i$: Variable representing the specific technique or conversion of
Inputs in outputs;
$CO_i$ : The unit cost of output production.

This causal relationship states that the market is the basis of productive efficiency. Through the pressure it puts in place, each company chooses a productive behavior (effort in production) resulting in the acquisition of more elaborate or more adapted technology thus resulting in the reduction of production costs.

When it comes to contracts in a production economy, in which agents have production assets, they are intended not only to optimize the exchange but also to organize production to improve efficiency. They appear as the articulation of several essential mechanisms designed to organize technical coordination, guarantee the fulfillment of promises, share risk or encourage effort.

Indeed, to measure the efficiency scores of companies, much empirical work has used the notion of efficiency boundary to estimate the function of production:

Shapiro and Muller (1977) demonstrate that the lack of optimist behaviour in the agricultural sector and in particular sufficient motivations, cotton producers in Tanzania are failing in promoting effective technology, despite the availability of market knowledge.

Shen (1984) concludes in his paper, which examines the relationship between the level of development and the efficiency that development decreases with development. It divides a sample of 18 different countries of development level into five relatively homogeneous groups on the basis of the per capita income criterion.

Caves and Sarton (1990) test the different specifications of the production function, measuring the different factors of production of the manufacturing sector in the USA by exploring the literature of analysis and measures of technical efficiency. They find that the different factors of production explain the differences in efficiency between firms and in relation to their characteristics and deduce the implications in terms of economic policies. In the same vein Caves (1992) in its study of manufacturing sectors in five countries, namely Japan, Australia, South Korea, Canada and the USA. Empirical results showed that efficiency scores were sensitive to the choice of estimation methods, distribution assumptions, choice of production function form, observations, and study period.

Corelli (1995) incorporates in its estimate the socio-economic factors that are supposed to influence the level of change in efficiency. It identifies two variables whose model does not reject the hypothesis of a significant impact on the performance of farms, namely health status and social cohesion.

Mr. Mbangala. M and Perleman (1997) tested the relationship between efficiency scores and a number of environmentally representative variables in which the nine railway companies in sub-Saharan Africa operate. Using DEA's approach, they conclude that companies that have expanded their passenger operations have achieved the highest performance rates.

In the case of the Ivorian manufacturing sector, Plane and Lesueur (1998) estimate econometrically a stochastic function boundary on business data. They measured the dispersion of efficiency scores for each industry. The methodological basis of their study lies with the agency's normative theory within the informational asymmetry model of efficiency pay theory.

However, they make an amendment to the standard efficiency pay model by assuming an effort function in which wage incentive and hierarchical control are substitutable to neutralize the rent-collecting behaviour of enforcement personnel. The econometric results show that the arguments of the effort function have had a particularly significant effect on technical efficiency in capital goods and intermediate goods companies. On the other hand, for agri-food and consumer goods activities, only wages have a statistically significant effect on efficiency. They also previously confirmed that Baumol and Lee (1991),Chambs and Lesueur (1992) had already concluded that the influence and intensity of the efficiency relationship achieved by each company was conditioned by the degree of counter-stability faced in the product market; in other words, by the level of trade protection. When the activities of equipment and intermediate goods are exposed to international competition, they find it difficult to adapt a human resources management strategy focus on the search of productivity gains that is one of competitiveness.

Guarda. Roubabah. (1999) studied the efficiency and performance of 136 banks in six countries: France, Belgium, Luxemburg, Germany, the United Kingdom and Switzerland. The results of estimating a stochastic cost border, indicate very pronounced differences in efficiency by bank size and country. The average efficiency of large banks is significantly greater than that of small banks. Luxemburg's banks are distinguished by higher average efficiency.

Barr R.S and al. (1999) use the DEA method to assess the productivity of commercial banks in the US from 1986 to 1998. They find a strong relationship between efficiency and inputs and outputs, which is independent of performance measures. They suggest that the impact of changes in economic conditions is due to a certain extent by the relative efficiencies of banks operating under these conditions.

Rao. S and Tang. J (2000) analysed the performance of manufacturing companies. Estimates indicate that Canadian-controlled companies are on average 19/100 less productive than their foreign-controlled rivals, tests carried out to account for differences in the age of the company, the quality of work, unionization, export orientation and business size revealed that these factors had not beencontributed to the productivity gap.

Ann.PBartel (2000) studied the relationship between the performance of the banking service sector and the human resource management environment. It incorporates the effect of the HRM variable (human resource management) into the sales equation. Econometric analysis shows that this variable has a significant effect on the performance of the industrial industries. The HRM variable is measured by the incentive system and the quality of communication between the manager and the staff. Hibbs and Locking (2000) analysed the impact of wage dispersion on the productivity of businesses and industries in Sweden during the 1968-1993 and 1972-93 periods. Their results based on a decomposition of variance and a production equation such as Akerlof and Yellen (1990) do not support the hypothesis that the performance of a firm or industry is greater in the presence of a wage structure. Therefore, they do not corroborate with theories based on "cooperation and equity" (Akerlof and Yellen, 1990; Levine, 1991). In the same vein, Lallemandand al (2004 a) studied the impact of wage dispersion on performance. Belgian companies. Intra-firm wage inequality is calculated unconditionally by three indicators: standard deviation, the coefficient of variation and the max-min ratio of gross individual hourly wages. A firm's performance is measured by gross operating overs (profit indicator) and value added (productivity indicator). Their results corroborate the predictions of 'tournament' theories.

Bingly and Eriksson (2001) study the impact of wage dispersion and wage asymmetry on worker effort and firm productivity in Denmark. To do so, they used longitudinal aggregate data covering the period 1992-1995. In order to control for simultaneity bias, they instrument wage dispersion with personal income tax data (in particular by using the variability in municipal taxes paid by individuals employed in the same firm).

Fernando and al. (2001) used the stochastic production boundaries they apply to a panel of 17 major industries in Brazil over the period 1995-1998. They find that productivity is growing faster in multinational companies than at national ones and that technological change is thedetermining factor in this increase. Similarly, they find that inefficiencies are very low for multinational firms than for national firms.

Doughherty and Mc Guckin (2002) looked at the effects of privatization and decentralization on efficiency in 20992 large and medium-sized Chinese enterprises. They find privatization to have a positive effect on efficiency.

Toufik.S (2002) used the parametric approach to estimate the technical efficiencies of Moroccan and foreign companies. He finds that foreign companies have higher degrees of technical efficiency than Moroccan companies. This performance in the manufacturing industry can be explained by the structure and behaviour of foreign companies in the manufacturing industries in Morocco. Foreign firms, have advanced technologies, pay very high salaries to salaries, use the best manufacturing processes and management techniques and quality control. This explains this superiority in terms of efficiency. Their results are obtained by the ordinary square lower doubles method, support tournament theory, as well as equity-based theory.

Frick and al. (2003) have a positive relationship between salary dispersion and the performance of basketball and hockey teams. On the other hand, they find that the wage squeeze improves the performance of football teams. The others attribute the heterogeneity of their results to different needs for cooperation in the four sports leagues. These conclusions are obtained from studies based on panel data for the four major sports disciplines in the United States. Lallemand et al. (2004 b, 2005) analysed the relationship between wage dispersion and the performance of large Belgian companies using data from 1995. Using the methodology of Winter-Ebmer and Zweimuller (1999), the authors show that there is a concave positive relationship between wage dispersion and corporate performance (measured by profits and added value). They add that the wage incentive further stimulates performance: there are companies that are mainly blue-collar and companies where the effort of workers is under intense control.

In conclusion, we note that variables such as wage incentives, hierarchical control and the economic environment in production processes can boost a firm's performance. The economic literature does not offer an unequivocal answer as to the sign of this assignment. At the theoretical level, two main currents of thought are at odds. According to tournament theory, salary dispersion positively influences the performance of a smoker (Lazear and Rosen, 1981). On the other hand, theories based on cooperation and equity (Akerlof and Yellen, 1990; Levine 1991) point out that a compressed wage structure promotes cooperation and cohesion among workers. However, these two factors are considered essential for a firm to perform well. Indeed, Yang (1996) shows that in an agency relationship, the recruitment of supervisor always improves the position of the principal.

At the empirical level, efficiency scores are highly sensitive to the choice of estimation methods, distribution assumptions, form of function and study periods. In addition, the size of the frills and the type of ownership and variables such as wage incentive and hierarchical control can also affect the firm's effectiveness.

Wautabouna Ouattara[1] (2009) presented a study by identifying the determinants of economic efficiency or inefficiency in Côte d'Ivoire. From a stochastic boundary of production, the factors responsible for inefficiency were highlighted by estimating a Tobit model to account for the truncated character (0 or 1) of the dependent variable (efficiency).

Ngbo Aké[2] (1994) worked on the productive effectiveness of French scops. He made an estimate and simulation from a stochastic production boundary. It assesses the productive efficiency of French production cooperatives in nine sectors of activity. It uses a Cobb-Douglas-type stochastic production boundary, including participation indices with non-cylinder panel data (1987, 1988 and 1989). In this study, the stochastic nature as well as the specification in terms of borders were confirmed by statistical tests.

The results of the simulations show that conditional mathematical expectation is a good measure of productive effectiveness. Indeed, the values estimated by the maximum likelihood method are very close to sampling means. The specific nature of French production cooperatives (a mixture of private and collective ownership) is not enough to explain the differences in productive efficiency within the SCOPs studied. Therefore the comparison of efficiency between firms of different natures in order to assess the role of the nature of the property should be done with caution, it would be desirable to conduct a very disaggregated analysis in order to eliminate as much as possible the other effects.

2-1 Overview of the concepts of production boundary

To measure the level of technical efficiency of any operation, one must first estimate the production or cost boundary, which represents points indicating the maximum amount of products or the minimum cost that can be obtained for a given volume of inputs. Thus, the production or cost boundary is obtained by all the points describing the optimal decisions of companies. In the case of the production boundary, each point of the production-input space is coordinated with the volume of the product in question and the volume of inputs (capital, labour) used by the operator.

Once the production line is set, the sample of operators is located either below this border it is said that the firm is technically ineffective either on it but never beyond because the technology used by the firm does not allow this. Those located on the border function are the most effective and their efficiency score is equal to one. On the other hand, others will move away from it and, for the latter, their efficiency score is between zero and one. The second task is to assess the level of technical efficiency. This level is measured in two different ways:

If the analysis is done on the production side, it is assessed as the difference between the observed level of production and the maximum level of production determined by the production boundary,

If the analysis is done on the input side, the most technically efficient operator is the one that obtains the same volume of production with a lower volume of inputs and the input efficiency index is the ratio between the optimal volume of inputs and the volume of inputs actually used.

[1]Consultant PED N° 11/2008

[2]Author had thanked Bom J.-J Laffont Lovell Perelman and the Anonymous reporters for their comments and suggestions as well as the CIR and the Public Economy Service of PestieauUniversité de Liège and AUPELF for its research grant The revision of his article was completed at the C.R.D.E University of Montréal)

**2-2 Method by deterministic frontier**

According to Amara and Romain (2000) Farrell was at the origin of the deterministic and parametric approach. He proposed the approximation of the efficient production function by a functional form known at first sight. Thus, an easier specification and a better analysis of the different algebraic properties of this function become possible. He used the Cobb-Douglas functional form to illustrate the use of this approach on agricultural data from 48 US states, while imposing constant yields at scale. Coelliand al (1998) presents the method of determining this type of border based on a study by Aigner and Chu (1968) of a sample of N firms, the model of which is as follows:

$\mathrm{Ln}\,(Y_i) = \beta_0 + \sum_{n=1}^{N} \beta_i \ln X_{it} - u_i$ with $u_i \geq 0$ and $i = 1, 2, \ldots N$ : (1)

With Ln ($Y_i$) the logarithm of the firm's production $i$, $X_i$: is a vector line of (K+1) elements of which the first takes the value 1 and the others, logarithms of each quantity of K inputs used, $\beta_i = (\beta_1\ \beta_2 \ldots\ldots\ldots\ldots \beta_k)$ a column vector of (K+1) elements that are the parameters to be estimated, $u_i$: is a non-negative random variable that reflects the firm's technical efficiency in terms of production i. The ratio between the observed production ($Y_i = \exp(x_i\beta - u_i)$) and the estimated production on the border of a perfectly efficient firm using the same input vector, gives an estimate of technical efficiency. The level of technical efficiency (TE), between 0 and 1, is given by:

$\mathrm{TE} = \frac{Y_i}{\exp\,(x_i\beta)} = \frac{\exp\,(x_i\beta - u_i))}{\exp\,(x_i\beta)} = \exp(-u_i)$ : (2)

Where $Y_i$ is the observed production of the firm « *i*» and $\exp(x_i\beta)$ is the estimated border production. According to Amara and Romain (2000), several other authors will draw on this specification of Aigner and Chu (1968) to make various modifications and refine the model. Timmer (1971), cited by Amara and al (2000), proposed the probabilistic model based on the sensitivity of the border function to extreme observations. This three-step iterative method consists of first estimating the boundary function for the entire sample, gradually reducing the sample of a number of firms, chosen at first, among those closest to the border, and estimating a new one to result in much more stable coefficients attached to the production function. This technique has been successfully implemented in the agricultural sector by Bravo-Ureta et al (1997). The deterministic function approach has been used several times, including Richmond (1974), Greene (1980), Taylor and al. (1986), Bravo-Ureta and Rieger (1990) cited by Nkunzimana (2005), which have made some modifications, the main objective being always to strive for models with the best precision and efficient estimators.

Despite this infatuation with its use, the parametric and deterministic approach has not ceased to be met with serious criticism, so that it is becoming less and less popular with researchers. Indeed, in addition to its limits dictated by the deterministic nature of the production boundary, the parametric approach is subject to two other criticisms. First, it is very sensitive to extreme observations and, second, the attribution of a functional form to the border function is restrictive, in the sense that each functional form implicitly reflects a number of assumptions (Fried and al., 1993). Thus, this notion of a deterministic boundary overlooks the possibility that a firm's performance may be affected by several factors beyond its control, such as climatic hazards, poor machine performance or input shortages, the effect of which is as important as the factors controllable by the firm. These arguments are at the root of the development of the stochastic approach or compound error.

**2-3 Method by the stochastic frontier**

According to Coelli and al (1998), Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977) proposed the stochastic boundary of production, which presents a non-negative random variable, added to the equation (1) of the previous deterministic case.

$\mathrm{Ln}(Y_i) = \beta_0 + \sum_{n=1}^{N} \beta_i \ln X_{it} + v_{it} - u_i$ :(3)

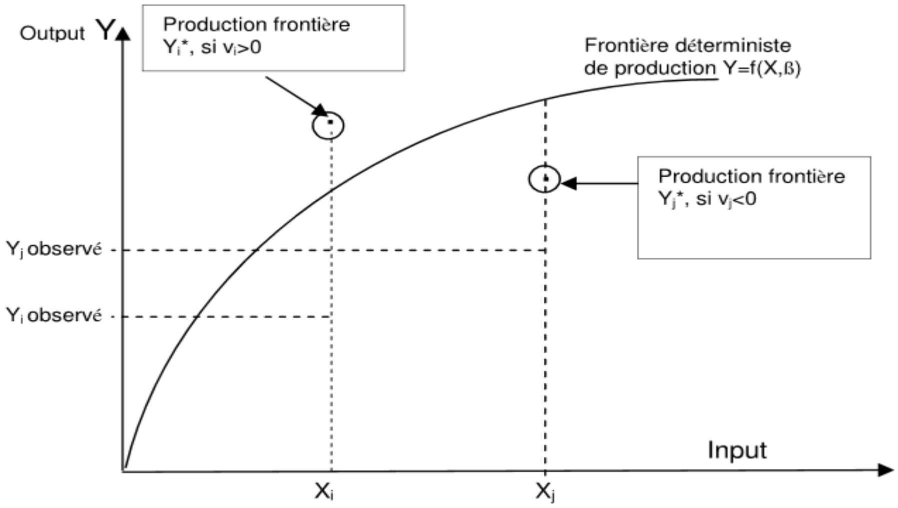
with 1,2,......N

Random error takes into account measurement errors and other random factors such as climate effects, random phenomena on the value of the production variable, etc., combined with the effects of unspecified input variables. The $v_i$ are assumed to represent random variables with an independent and identical normal distribution with a zero average and a constant variance $\sigma_v^2$ independent of $u_i$ which are assumed to follow an identical and independent exponential distribution or a semi-normal random distribution. . This model is called stochastic production border because output values are limited to the random stochastic $\exp(x_i\beta + v_{it})$**.** Random errors can be positive or negative.

The Figure below illustrates the differences between the deterministic production boundary and the stochastic production boundary in the case of two firms i and j. It is assumed that these two firms use input quantities $X_i$ and $X_j$, to produce, respectively, the outputs $Y_i$ and $Y_j$ From the figure, we can see that, for the firm i, the level of production $Y_j$, which corresponds to the exp stochastic border $\exp(x_i\beta + v_{it})$ is higher than the level of production of the deterministic border

$$y_i = \exp(x_i\beta)$$

**Figure 2: The stochastic boundary of production**




This is probably due to the fact that the firm *i* did not face adverse conditions beyond its control; which means that the term random error $v_i$ispositive. On the other hand, for the firm j, the level of stochastic production$Y_j$ is lower than the deterministic level of production$Y_j$ so$v_j$ is negative. It should be noted that the two levels of stochastic border production are not observed because the terms of errors$v_i$and$v_j$ are not observable (Amara and Romain, 2000). The maximum likelihood (MV) method estimates this border function and separates the error component reflecting technical inefficiency *u* from the purely random component v (Amara, 2000; Coelli, Xioxang 2000). This approach, in its initial version, does not make it possible to differentiate between the two components for each observation. It only allows to calculate the average efficiency level for the whole sample. It will be further developed by Jondrowand al (1982) in order to determine the technical efficiency of each firm in the sample.

These authors have shown that by assigning known distributions to the two components of the term error, it is possible to differentiate them and obtain a measure of effectiveness for each observation.

**2-4-Cost frontier**

The stochastic cost border determines economic efficiency and consequently allocative production efficiency. According to the model presented by Ogundari and Odjo (2006), the cost line is specified as follows:

$C_i = g(Y_i, P_i, \alpha_i) + \varepsilon_i$ with $i = 1,2 \ldots . n$(4) Where $C_i$ is the total cost of production

$Y_i$: represents the output

$P_i$: the cost of inputs

$\alpha_i$: The parameters of the cost function and,

$\varepsilon_i$: The term error consisting of two elements ($\varepsilon_i = v_i + u_i$ )

U and V have the same characteristics as in the case of the stochastic border. However, since inefficiency is supposed to increase costs, these error components show positive signs. According to Coelli et al (1998), the $U_i$provide information on the cost efficiency level or economic efficiency ($EE_i$) of the firm i. This efficiency is calculated by the ratio between the minimum cost on the border ($U_i$ = 0) to the observed cost. This ratio gives after simplification:

$$EE_i = \mathrm{Exp}(-U_i)$$

The value of the efficiency level is between 0 and 1. Economic efficiency ($EE_i$) can be broken down into technical and allocative efficiency when the production function is explicitly derived from the estimated cost function. This decomposition is often possible when the Cobb-Douglas function is used because it is dual. The allocative efficiency ($AE_i$) is therefore estimated by the equation:

$AE_i = EE_i / TE_i$

With: $EE_{i,}$ economic efficiency and technical efficiency. However, Coelliand al. (1998) indicate that estimating the function of costs and factor demand equations by the maximum likelihood method provides a more appropriate estimate of the parameters of cost function than a simple estimating equation. The maximum likelihood method also has the advantage of directly calculating allocative inefficiency.

**2-5-The estimate of the production boundary**

With regard to estimating the production boundary, it is a central step in any analysis of technical efficiency and different methodologies have been developed (Fried and al, 1993). The literature mentions two main approaches: the non-parametric approach and the parametric approach.

Whether or not a functional form of the production boundary is the distinguishing feature of these approaches. Regardless of the approach chosen, there are several methods for estimating the border. These methods can be categorized according to the presumed form of the production boundary or cost, according to the estimation technique used to obtain it, and the nature and assumed properties of the difference between observed production and optimal production. The parametric approach is one with a function with explicit parameters. For example, a functional form of the CES, Cobb-Douglas or trans-logarithmic type can be used.

In the case of a parametric function, several econometric and non-econometric techniques can be used to estimate the parameters of the production or cost boundary: the smallest square method or the maximum likelihood method, linear programming and quadratic programming. Aigner and Chu (1968) were the initiators of parametric production boundaries, notably the non-econometric deterministic approach that the gap between frontier production and observed production is due to the inefficiency of the operator. The disadvantage of this method is that it does not take into account random phenomena that can influence the level of effectiveness.

The nature of the differences between observed production and maximum production differentiates stochastic boundaries from deterministic boundaries. The choice between the stochastic border and the deterministic border is purely optional. It depends on the researcher. Some researchers assume that any observed discrepancy is solely due to the inefficiency of the producer, and they describe the boundary as deterministic. While the researchers believe that the discrepancies are explained both by the inefficiency of the producer and by random elements beyond the control of the operator, the production or cost boundary is said to be stochastic in nature (N'Gbo, 1994). The random effect was introduced by Aignerand al. (1977) and Meeusen and Van den Broek (1977) to account for factors beyond the operator's control. The estimation of the stochastic boundary can be done using econometric methods (the method of the smallest squares corrected and the method of maximum likelihood).

The non-parametric approach has the particularity of not imposing any pre-established form at the production or cost boundary since the convex and efficient isoquant associated with a given level of production is estimated from a cloud of points all above and to the right of this isoquant, Farrell (1957) was the first to propose the convex non-parametric boundary as part of a study to envelop the observed production activities so that all production possibilities thus formed would be convex. Farrell's idea was developed by Charnes and al. (1978) and Banker and al. (1984) through the DEA method, which takes into account scale yields. Farrell's method imposed consistent yields of scale, but this hypothesis was released by Charnes et al. (op. cit). The evaluation of efficiency indices is done by linear programming or quadratic programming that minimizes the gap between observed production and optimal production.

The non-convex non-parametric approach was proposed by Deprins et al. (1984) with the only hypothesis being the free disposal of productions and inputs hence its name Free Disposal Hull (FDH). Under this method, for single-product and multi-input operations, one operation is classified as inefficient if it is possible to find another one belonging to the sample with a higher quantity produced and the volume of inputs used is lower.

and a particular probability law; this is not the case in the non-Parametric approach. It is then interesting to ask what is the effect of using a functional form?

**2.6 Non-parametric versus parametric approach**

The methods of boundaries of productive efficiency can be distinguished according to whether they are parametric Aigner D.J. & Chu S.F. (1968)[3] ;Aigner, D.J., Lovell, L.M. & Schmidt, P. (1977)[4] or non-parametric Charnes, A. Cooper, W.W. & Rhodes, E. (1978); Banker, R.D., Charnes A. & Cooper, W.W. (1984). The objective is to present a comparative analysis of these two approaches in the boundary methodology, trying to highlight the benefits and weaknesses of each approach.

The fundamental difference between the parametric and non-parametric approach is that the former is based on an explicit statistical model that is established by the use of a functional form

Using less information than in the parametric approach, the results in the non-parametric approach should be less accurate but there is a risk of influencing the results by imposing a functional form that is not the most appropriate. The trade-off between imposing more structures and more flexibility is a permanent problem as more constraints in a model lead to better estimates; indeed strong assumptions generate strong results as long as the constraints (for example the chosen functional form) are true.

Another difference between these two approaches is that in the non-parametric approach, everything happens as if, only companies operating near the border have very important informational content in determining the non-parametric boundary. In the parametric approach, however, all observations are relevant to the determination of the border. This aspect is worth noting, because depending on the circumstances, and depending on the information available, one approach may be preferred to the other and vice versa.

The advantage of the non-parametric approach is that it makes it easier to take into account multi-production technology, on the contrary, it does not directly assess the elements of the technology since it only provides measures of productive efficiency. As before, the advantage of an approach may be a weakness depending on the objectives pursued. In general, it is difficult to say definitively, which of the two approaches is more relevant, because it is an appropriate choice depending on the problem posed and also depends on the researcher's own sensitivity. The choice between parametric and non-parametric approaches can be made on the basis of available information and objectives. For example, if we are only interested in measuring the effectiveness of firms in a sector or economy, the non-parametric approach can be used. On the contrary, if in addition to efficiency, production technology is of interest, then a parametric approach should be adopted.

## III. METHODOLOGY

This section includes the processes or procedures, methods or approaches, instruments and techniques used in the analysis to assess the main factors contributing to the economic performance of businesses in Côte d'Ivoire. We will first present the efficiency estimation plan, the description of the data of Ivorian companies and the variables, the model used to evaluate the level of technical inefficiency of the 12 sectors of activity of the Ivorian economy, the model for estimating the determinants of efficiency and finally the model for estimating scores of economic, allocative and technical efficiency.

### 3-1 Efficiency Estimation Plan

Given the random nature of production, linked to the nature of the climate and the price fluctuations of input products, the choice of the stochastic parametric method to estimate economic efficiency seems justified to us. The choice of the Cobb-Douglas functional form to represent the production function is explained by the fact that this function admits its own dual, property which allows us, subsequently, to obtain the minimum cost function necessary to determine the economic and allocative efficiency scores.

The stochastic border method developed by Aignerand al. (1977) and meeusen and van den Broek (1977) breaks down the error of the function studied in two independent terms. The first represents random effects and measurement errors, it is distributed on both sides of the production border. The second term, which represents the degree of productive inefficiency, is distributed on one side of the production border. Assuming that the term random error follows a symmetrical normal distribution, while the term efficiency follows an asymmetrical distribution defined positively for a cost function and negatively for a production and profit function.

Efficiency studies have the merit of indicating the level of performance of the sectors and determining the optimal production plans. It is also possible to determine the factors that explain the level of efficiency of producers. Knowledge of these factors improves producer performance through public policy recommendations.

### 3.2 Description of Data from Ivorian Firms and Variables

We will therefore estimate the boundaries of stochastic production in (12) twelve sectors of activity of the Ivorian economy. We have a database provided by the Financial Data Bank (BDF) of the National Statistical Institute (INS). The study sample covers a total of 4443 companies. This database is observed over the period 2008 to 2012. The variables are defined as:

- $Y_{it}$ is the output representing the added value.
- $PTr_{it}$ is labour productivity, measured by the ratio of value added to the number of workers
- $PCa_{it}$ is the productivity of capital measured by the ratio of value added to own fixed capital
- $IEp_{it}$ are investments in equipment and other infrastructure.
- CP the ownfixed capital

In addition, analysis of the determinants of effectiveness shows that the choice of determinants of efficacy depends on the scale of analysis and the objective of the study. Factors that explain the level of efficiency of individual farms may be the age of the operator, the size of the operation, the level of education of the operator, the distance from the market, the method of claiming, the work experience.

However, at the panel data level, we assume in this research that the factors that affect the level of technical, allocative and economic efficiency of the twelve business lines are: the size of the company (TE) as measured by staff, the institutional environment (EVI), value added (VA), financial debt (DF) and national savings. The effects of these factors on efficiency levels are estimated by the Tobit regression method, which takes into account the truncated nature of efficiency scores that take values between 0 and 1.

**3.3 Efficiency Determinants**

Measuring economic efficiency can identify potential profit gains in the sector studied. The resulting inefficiency can be explained by factors such as the size of the operation, the age and level of education of the chief operating officer, etc. From a political point of view, it is interesting to look for sources of inefficiency and identify the determinants. Governments can act on the determinants identified to improve overall efficiency. The first usual question asked is how to explain the perceived efficiency differential between farms and/or between sectors. Several methods are used for this purpose. There is the one-step method called the compound error and incorporated error-effect production frontier proposed by Battese and Coelli (1992). Another method also used to explain inefficiencies proceeds in two stages: first inefficiencies are estimated from a parametric or non-parametric boundary, and then a regression of efficiency scores is carried out on the determining variables.

In general, this assumes that the variables explaining inefficiency are those relating to the characteristics of operators and farms, they are different from the factors of production. This hypothesis is introduced to avoid the bias included in the first step, whereby the level of effectiveness is independent of these variables while in the second stage they are considered dependent. According to Murillo Zamorano (2004), the methods yield equivalent results. The advantage of this method is that in the event of a specification error in the second stage, the bias only affects the estimated coefficients of the determinants and not the boundary coefficients. As Lovell (2000) argues, this method can be used for both the non-parametric and parametric approach. Regression, carried out during the second stage, is possible thanks to the MCO method or a Tobit model to account for the truncated character (between 0 and 1) of the dependent variable (efficiency).

**3.4 The border model**

The model used to assess the level of technical inefficiency of the 12 sectors of activity in the Ivorian economy is the stochastic production boundary, still known as the compound error and incorporated inefficiency production frontier proposed by Battese and Coelli (op.cit). This model was introduced into the literature by Aigner and al. (op.cit) and Meusen and Van den Broeck (op.cit). The advantage of this type of production boundary is that it helps to explain the deviations observed between the maximum production and the production actually obtained by the operator as well as the effects of random factors beyond the control of the operator. We assume that the discrepancies are not explained solely by the inefficiency of the producer. They are the result of both the inefficiency of the producer and the random elements that do not depend on the producer.

Following Meeusen and Van den Broeck (1977) and Lovell and Schmidt (1977) who proposed independently of stochastic production border models, and N'Gbo (1994) who worked from non-cylinder panel data, we consider the following production boundary:

$$\ln Y_{it} = \ln[f(X_{it,}\beta)] + V_{it} - U_i \qquad (7)$$

with, t = 1,2,……,T andi = 1,2,……..,N

Where $Y_i$: is the production of the**i$^{ème}$** firm at the **t$^{ème}$**period,$X_{it,\,;}$ is a vector (lxk) of the inputs of the**i$^{ème}$** firm at the **t$^{ème}$**period$\beta$: is a vector (lxk) of technological parameters unknown to the border; $T_i$ : represents the number of observations of the i$^{ème}$firm, N represents the number of firms; $V_{it}$ : is the usual symmetrical error term. . It represents the discrepancy due to the vagaries that influence production and which are not directly under the control of the manager. $U_i$is a non-negative error term representing inefficiency and assumed to be invariant over time. The border defined above is stochastic in the sense that it combines both error and error terms$V_{it}$and $U_i$. Within the deterministic boundary, the term $V_{it}$does not appear explicitly. This type of border does not take into account the term classic error, and any deviation from the production boundary is considered to be due to inefficiency.

The relationship (7) can still be written in the form:

$$\ln Y_{it} = \beta_0 + \sum \beta_j \ln X_{it} + \varphi_{it} \qquad (8)$$

With,$\varphi_{it} = V_{it} - U_i$

The input vector$X_{it}$ can be disintegrated into several explanatory variables such as capital productivity (PCa), own fixed capital (CFp), labour productivity (PTr), investments in equipment (IEp) Thus, the equation (8) becomes:

$$\ln Y_{it} = \beta_0 + \beta_1 \ln PTr_{it} + \beta_2 \ln PCa_{it} + \beta_3 \ln CFp_{it} + \beta_4 \ln IEp_{it} + \varphi_{it} \qquad (9)$$

The production boundary can be estimated by least-squares or by maximum likelihood if the distributions of the terms of error $V_{it}$ and $U_i$ are specified.

- *Maximum likelihood*

Following Aigner, Lovell and Schmidt (1977), we take a normal distribution for V which means that V $\rightarrow$N $(0, \sigma_v^2)$and a normal distribution centered, truncated left to zero for U, which means that U $\rightarrow |N(0, \sigma_u^2)|$. The attached density for V and U knowing that the two distributions are independent is written:

$f\,(V, U) = \frac{1}{\pi\sigma_v\sigma_u}\exp\left(\left(-\frac{U^2}{2\sigma_u^2}\right) - \left(\frac{V^2}{2\sigma_v^2}\right)\right)$ (10)

If you replace V based on U, you get:

$f\,(U,\varphi) = \frac{1}{\pi\sigma_v\sigma_u}\exp\left(-\frac{U^2}{2\sigma_u^2} - \frac{(\varphi^2+U^2+2U\varphi)}{2\sigma_v^2}\right)$ (11)

Now let's calculate the density of integrating the relationship (11) versus U. We've got

$f(\varphi) = \left(\frac{2}{\sigma}\right) f^*\left(\frac{\varphi}{\sigma}\right)\left[1 - F^*\left(\frac{\lambda\varphi}{\sigma}\right)\right]$ (12)

$-\infty < \varphi < +\infty$ avec $\sigma^2 = \sigma_u^2 + \sigma_u^2$ et $\lambda = \frac{\sigma_u}{\sigma_v}$

And F*(.) defines the distribution function of a normal, reduced centered distribution and $f$*(.) its density is defined as the distribution function of a normal centered distribution. The timing of order one and the variance are given by:

$$E(\varphi) = \frac{\sqrt{2}}{\sqrt{\pi}}\sigma_u$$

$$V(\varphi) = V(u) + V(v)$$

$V(\varphi) = \left[\frac{\pi-2}{\pi}\right]\sigma_u^2 + \sigma_v^2$ (13)

It should be noted that the parameterization $\lambda = \frac{\sigma_u}{\sigma_v}$is interesting;$\lambda$ is considered a measure of the relative variability of two sources of inefficiency. $\lambda^2 \longrightarrow 0$ Implies that ${\sigma_v}^2 \longrightarrow +\infty$and/or that${\sigma_u}^2 \longrightarrow 0$, which means that random shocks dominate in explaining inefficiency. Similarly, when${\sigma_v}^2 \longrightarrow 0$ the gaps at the border are mainly due to technical inefficiency.

- *The method of the smallest squares*

If we consider the method of the smallest squares, the model (8) can be written

$Lny_{it} = \beta_0 + \sum \beta_j LnX_{it} + (V_{it} - (U_i - \mu))$ (14)

It can be re-set as follows:

$Lny_{it} = \beta_0' + \sum \beta_j LnX_{it} + \varphi_i'$ (15)

With $\beta_0' = \beta_0 - \mu$ and $\varphi_i' = V - [U_i - E(U)] = V_{it} - (U_i - \mu) = \varphi + \mu$

The process of estimating (14) can be done in two stages:

At first, since the distribution of$\varphi_{it}'$ is symmetrical, one can estimate (15) by the method of the smallest ordinary squares; all$\beta_j$ willbe unbiased. Secondly, we completely identify the border by estimating$\beta_0$ and therefore$\mu$. To do this, you have to specify a specific distribution for each of the error terms. One can then estimate$\mu$ by the method of moments and, subsequently $\beta_0$ (Aigner et ali, 1977).

We determine in the following pages the economic efficiency from the estimation of an equation system composed of a production function and the first-rate conditions of minimization of the cost of production. This method developed by Schmidt and Lovell (1979) is implemented by Ferriera and Steel (2007).

### 3.5 Economic Efficiency Scores

Economic efficiency scores are obtained by the product between technical efficiency scores and those of allocative efficiency. The profit efficiency of each operation can be estimated from the border models used to measure the technical efficiency of companies (Ali and Flinn, 1989). It takes a contained value between 0 and 1. And is defined as:

$ET_i = \left[\prod X_{ij}^{\beta_{ij}} \exp(U_{it})\right] / \prod X_{ij}^{\beta_{ij}} = Exp\left(-\widehat{U}_{it}^*\right)$ (16)

$$\text{with } \widehat{U}_i = \max_j(\widehat{\sigma}_{it}^*) - \widehat{\sigma}_i^*$$

Following Schmidt and Sickles (1984) and Goaïed and Ben Ayed-Mouelhi (2000), we consider that the estimation of technical effectiveness is based on the use of the predictor which is BLUP (Best linear unbiaised predictor). After the border estimate (9), one obtains:

$\widehat{U}_i^* = \frac{-\sigma_u^2 \sum_{t=1}^{T} Y_{it} - Ln\,(X_{it})\widehat{\beta} - \widehat{\beta}_0^*}{T_i\widehat{\sigma}_u^2 + \widehat{\sigma}_v^2}$(17)

Using the parametric approach to determining technical efficiency scores, we will estimate a Cobb-Douglas-type stochastic production boundary using the Frontier 4.1 program (Coelli, 1996). Frontier 4.1 software provides iteration of production boundary elasticities, technical efficiency scores and determinant coefficients. The parameters of the stochastic production boundary will be estimated by the maximum likelihood[5] method. After starting the function by the method of the smallest ordinary squares, the program carries out iterations.

### 3.6 Estimates of economic and allocative efficiency scores

This measure determines the ratio between the levels of output produced by a firm i and the level of output produced by a fully efficient firm using the same level of output. Then we determine the economic and allocative efficiency from the estimation of an equation system composed of a production function and the first-rate conditions of the minimization of the cost of production, a method first developed by Schmidt and Lovell (1979) starting from the Cobb-Douglas type production function:

$$Y = f(X_1 \ldots\ldots X_4) \quad = AX_1^{\beta_1} X_2^{\beta_2} X_3^{\beta_3} X_4^{\beta 4} \qquad (18)$$

Therefore, in choosing its production program, the firm must in addition to the technical parameters, take into account their relative prices in the market. Allocative efficiency is therefore to choose the best productive combination of inputs based on their prices in order to optimize profit or minimize costs at a given level of production. Thus, for a factor price vector, the cost of production is given by the following equation

$$C = \sum_{i=1}^{4} P_i X_i \qquad (19)$$

$X_i$ Quantities of the factors of production and i ranging from 1 to 4 and corresponding to the different variables of the C border. Assuming that all factors are variable and have market prices, we associate prices ($P_i$) with inputs. Companies therefore seek to minimize their cost of production. Cost minimization in the aforementioned industries will be considered a stressed optimization problem:

$$\text{Min } C = \sum_{i=1}^{4} P_i X_i \quad \text{S/C } Y = f(X_i, \ldots\ldots, X_4, \beta) \text{with } X_i > 0 \qquad (20)$$

Such as $Y = f(X_i, \ldots\ldots, X_4, \beta)$

The Lagrangian of this optimization problem is as follows:

$$L\,(.) \quad = \sum_{i=1}^{4} P_i X_i + \lambda f\left[Y - \left(X_{1,} \ldots . X_4, \beta\right)\right] \qquad (21)$$

These techniques are described in BatieseCoelli and Colby 1989 Fixed or random effect models can also be used The choice of an estimation method depends essentially on the assumptions. Under the previously made assumptions maximum likelihood estimators are efficient Schmidt and Sickles 1984])

Withλ it is the multiplier of lagrangian. λis the marginal cost of producing output Y,$\left(\lambda = \frac{\partial L}{\partial Y}\right)$. Hence, the conditions of the first order are:

$$\begin{cases} \frac{\partial L}{\partial X_1} = P_1 - \lambda f_1\left[X_{1,} \ldots\ldots X_4, \beta\right] = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \qquad (22) \\ \frac{\partial L}{\partial X_4} = P_4 - \lambda f_4\left[X_{1,} \ldots\ldots X_4, \beta\right] = 0 \\ \frac{\partial L}{\partial \lambda} = Y - f\left[X_{1,} \ldots\ldots X_4, \beta\right] = 0 \qquad (23) \end{cases}$$

Where $f_i$is the first partial derivative of the production function in relation to the input. This is the marginal production of the postman $i^{ème}$ with i= 1, ... 4: Assuming second-order conditions are met, we can resolve the dual cost function from the production function (18) sequentially. To derive the dual cost function of this Cobb-Douglas-type production function, simply substitute the equation (18) in the equation (21) and write the corresponding first-order conditions in the equation system (18) and (21). After calculation, we get the following input demand equation:

$$X_i = \frac{\lambda Y \beta_i}{P_i} \quad (24)$$

Consider the ratios of the required amounts of inputs, either$\frac{X_j}{X_1}$, $(j = 2, 3, 4)$ to eliminate the variables λ and Y. After solving these ratios, we can deduce the equations $X_2$,$X_3$, $X_4$and substitute them in the equation (16). We get so $X_1$as a production function Y, Parameters of the Cobb-Douglas production function and the prices of production factors. Now just repeat the same procedure to get the derivative applications$X_2$,$X_3$, $X_4$. By replacing the variables $X_i$in the cost function (20), we get the dual cost function. This gives, after algebraic calculations, the following expression:

$C = KY_{zt}^{*1/r} P_1^{\beta_1/r} P_2^{\beta_2/r} P_3^{\beta_3/r} P_4^{\beta_4/r}$ (25)Or r =$\sum_{i=1}^{4} \beta_i$and K =r*$\left[A^* \beta_1^{\beta_1 *} \beta_2^{\beta_2 *} \beta_3^{\beta_3 *} \beta_4^{\beta_4}\right]^{-1/r}$ then you get the new value of the dual cost C defined as: $C = \left(\sum_{i=1}^{4} \beta_i\right)^* \left[A^* \beta_1^{\beta_1 *} \beta_2^{\beta_2 *} \beta_3^{\beta_3 *} \beta_4^{\beta_4}\right]^{-\frac{1}{\sum_{i=1}^{4} \beta_i}} x$

$$\left(Y_{zt}^{*1/\sum_{i=1}^{4} \beta_i} P_1^{\beta_1/\sum_{i=1}^{4} \beta_i} P_2^{\beta_2/\sum_{i=1}^{4} \beta_i} P_3^{\beta_3/\sum_{i=1}^{4} \beta_i} P_4^{\beta_4/\sum_{i=1}^{4} \beta_i}\right) \qquad (26)$$

With r: scale performance$Y_{zt}^* = Y_{zt} - V_{zt}$: which is defined as the observed output of sector z (with z ranging from 1 to 12 to account for the 12 sectors of activity[6]) during the t year, adjusted by the term random error. In its linear form the dual cost border becomes:

$$LnC_{zt} \quad = K + 1/rLnY_{zt}^* + \alpha_1 Ln\left(P_{1_{zt}}\right) + \alpha_2 Ln\left(P_{2_{zt}}\right)$$

$$+\alpha_3 \mathrm{Ln}(\mathrm{P}_{3_{zt}}) + \alpha_4 \mathrm{Ln}(\mathrm{P}_{4_{zt}}): \quad (27)$$

With $\alpha = \frac{\beta_i}{r}$. the different $P_i$ are characteristics of the prices of the factors of production. The K, $\alpha_1 \ldots \ldots \alpha_4$ coefficients, are parameters obtained analytically and minimizing the cost function under the constraint of the level of production reached. For a given level of production, economic efficiency is by definition the ratio between the minimum cost and the observed cost. The application of Shephard'sLemme to the equation (18) allows us to derive input demand equations at the minimum cost level $(\mathrm{X}_{dizt})$ from which we obtain the quantities of economically efficient inputs. According to Shephard's Lemme, with the behavioral assumption of cost minimization, input demand functions are simply derivatives of the dual cost function versus the price of the input in question:

$$\mathrm{Ln}(\mathrm{X}_{dizt}) = \mathrm{LnK}_i + 1/r\, \mathrm{LnY}^*_{zt} + \mathrm{Ln}\left[\prod \mathrm{P}_{izt}^{\alpha_j/\mathrm{P}_{izt}}\right] \quad (28)$$

With i =1... 4.

The equation (27) represents the input demand function constrained by the output level. This is, in consumer theory, the offset demand equation, or Hicksian. This equation is used to derive the cost boundary.

The cost line provides the minimum cost demand equations; second, these equations are used to calculate the amount of economically effective factors$(\mathrm{X}_{iezt})$. By substituting expressions $\mathrm{X}_i$ for the objective function (cost minimization), the dual cost function is achieved, which expresses the total cost based on input prices and output level. For a given level of production, economic efficiency is by definition the ratio between the minimum cost and the observed cost. Following Albouchi, Bachta and Jacquet (2005), we note that economic efficiency is written as follows:

$$\mathrm{EE}_{izt} = \sum_{i=1}^{4} \mathrm{X}_{iezt}{}^* \mathrm{P}_{izt} \Big/ \sum_{i=1}^{4} \mathrm{X}_{izt}{}^* \mathrm{P}_{izt} \quad (29)$$

The acronym EE denoting economic efficiency and $(\mathrm{X}_{iezt})$ being the amount of factor economically effective.
Finally, as according to Farrell, allocative efficiency will be deducted from the following equation:

$$\mathrm{EA}_{izt} = \mathrm{EE}_{zt}/\mathrm{ET}_{zt} = \frac{\sum_{i=1}^{4} \mathrm{X}_{iezt}{}^* \mathrm{P}_{izt} \Big/ \sum_{i=1}^{4} \mathrm{X}_{izt}{}^* \mathrm{P}_{izt}}{\left[\prod \mathrm{X}_{ij}^{\beta_{ij}} \exp(\mathrm{U}_{it})\right] \Big/ \prod \mathrm{X}_{ij}^{\beta_{ij}}} \quad (30)$$

## IV-Presentation of results

The results will be presented in four phases. The first step will be to indicate the representativeness of the sectoral coverage of the panel of a few variables within the border. Value added is implicitly taken into account in the calculation of capital productivity and labour productivity, after the results of the estimate of the production border are presented, then the efficiency, technical, allocative and economic scores are presented and finally the determinants of the efficiency of Ivorian enterprises are presented.

**Table 1: The average representativeness of the panel's sector coverage**

| Business Sectors | Firm number | Average value Added value |
|---|---|---|
| Agriculture and food processing industries | 108 | 3554 |
| Manufacture of materials | 83 | 756 |
| Textile and leather industries | 56 | 2545 |
| Other industries | 103 | 978 |
| Transport and communication | 299 | 4756 |
| Trade and distribution | 2797 | 3212 |
| Financial institutions and insurance | 68 | 1234 |
| Construction | 123 | 453 |
| Agriculture and mining | 101 | 2124 |
| Wood industry and printing | 244 | 2455 |
| Hotel and restaurant | 144 | 2234 |
| Other services | 317 | 3435 |
| **Total** | **4443** | |

*Source: Data from the National Statistical Institute of Côte d'Ivoire, calculations by the author*

The total size of the panel used is 22,215 observations as 4,443 companies are observed over five years from 2008 to 2012. With regard to the table, a good representativeness of the *"trade and distribution"* sector is remarkable. It is by far the sector of activity that includes the largest number of companies. Next, we have, in order of representativeness, the sectors *"other services"*, *"transport and switching"* and *"wood industry and printing"*. The sectors of activity with the fewest number of firms are the *"textile and leather industry"* sector, the *"financial institution and insurance"* sector and the *"manufacture of materials"* sector. These sectors remain under-represented due to the unavailability of data. However, this sampling bias does not affect the empirical results, as estimates of the stochastic production frontier are made by sector. Looking at the averages of the contributions in terms of value added, it can be observed that the most represented sectors are *"transport and communication"*, *"agriculture and agro-food industry"* and *"other services"*, "trade and distribution" and "textile and leather industries". There seems to be a very strong correlation between the number of firms in a given sector and its contribution in terms of added value. Four (4) variables have been retained in the production boundary. The availability of the variables relating to capital productivity and labour productivity was not evident. The different capital productivities were calculated from the gross value added by own fixed capital by sector of activity. Labour productivity takes into account the number of workers: It is the ratio of value added by the number of workers. In other words, it is assumed that the value of available capital is better approximated by taking into account the legal depreciation arrangements than by ignoring its wear and tear.
The table below gives an overview of the statistics for the different variables used in the production boundary.

**Table 2: Descriptive statistics of production boundary variables**

| Boundary variables | Minimum | Maximum | Average | Std. Dev |
|---|---|---|---|---|
| Own fixed capital | 7,19E+10 | 1,02E+11 | 8,56E+10 | 1,26E+10 |
| Investment and equipment | 5,38E+10 | 8,34E+10 | 6,44E+10 | 1,25E+10 |
| Labour productivity | 7,13E+06 | 2,04E+07 | 1,08E+07 | 5,42E+06 |
| Capital productivity | 1,88 | 3,82 | 2,77 | 0,72 |

*Source: Data from the National Institute of Statistics of Côte d'Ivoire, author's calculations*.

Analysis of the last column of the above table shows that the differences between industries for the same variable are very large except for capital productivity. These high values can potentially introduce biases into the estimates. To correct for this bias, we will use the logarithm. This mathematical procedure for transforming variables into time series has the advantage of normalizing and stabilizing them (Greene, 2005).

### 4.1 Production boundary

To obtain the results of the production boundary estimation, we used the maximum likelihood method implemented by the Frontier 4.1 programme.

$$\ln Y_{it} = 5{,}3995 - 0.11.10^{-8}\ln CFp_{it} + 0{,}4952\ln IEp_{it} + 0.68.10^{-9}\ln PTr_{it} + 0{,}0426\ln PCa_{it} + \varphi_{it}$$

The results of estimating the production frontier specified in equation (3) are presented in the table below.

**Table 3: The estimation of the parameters of the stochastic production frontier**

| Explanatory variables | Coefficients | values | t-stat |
|---|---|---|---|
| Constant | $\beta_0$ | 5,3995* | 3,0517 |
| Own fixed capital | $\beta_1$ | -0.11E-08* | -6,4358 |
| Investment and equipment | $\beta_2$ | 0,4952** | 3,2422 |
| Labour productivity | $\beta_3$ | 0.68E-09*** | 1,9073 |
| Capital productivity | $\beta_4$ | 0,0426 | 0,4408 |
| Square Sigma | $\sigma^2$ | 0,3506** | 2,1206 |
| Gamma | $\gamma$ | 0,8206* | 8,4130 |
| Eta | $\eta$ | 0,0619 | 1,0992 |
| Log Likelihood | LL | -15,8085 | - |

(*) Significant at1%, (**) Significant at5%, (***) Significant at10%

*Source: Data from the National Institute of Statistics of Côte d'Ivoire, author's calculations.*

These results concern all 12 sectors of activity observed over the period from 2008 to 2012. We note that the gamma value ($\gamma$) (0.82) is greater than 0. This result allows us to reject the hypothesis that the variance of efficiency is zero. This justifies the existence of a stochastic frontier. Therefore the integration of the

inefficiency term in the random term is justified. This gamma value ($\gamma$) also teaches us that the deviation from the frontier is explained by the inefficiency of the sectors at 82%. The evaluation of ($\gamma$), significantly different from zero, indicates the existence of productive inefficiencies. This result means that the gap between observed production and potential production of the sectors studied is partly due to their inefficiency. Indeed, in this study, only 18% of the gaps between observed output and potential output of the 12 sectors of activity are related to random effects including measurement errors, even if this value may seem a little high, which may be due
to the nature of the data. The closer the value of ($\gamma$) is to 1, the smaller the difference between the results of a stochastic estimate and those of a deterministic estimate (Briec, Comes and Kerstens, 2006). On the other hand, the value of ($\eta$) is not significantly different from zero, which shows that the level of technical inefficiency has not changed much over the observation period.

**4.2 Costboundary**

For further analysis, we now consider the cost frontier as identified in equation (19). The dual cost frontier is analytically derived from the stochastic production frontier. It has been used to estimate and decompose economic efficiency into two components: technical efficiency and allocative efficiency. The results are presented in the table below by sector of activity.

**Table 4: Comparison of efficiency scores by sector**

| activity sector | technical efficiency | allocative efficiency | Economic efficiency |
|---|---|---|---|
| Agriculture and food processing industries | 0,7563 | 0,6840 | 0,5173 |
| Manufacture of materials | 0,9169 | 0,8345 | 0,7652 |
| Textile and leather industries | 0,5081 | 0,7073 | 0,3594 |
| Other industries | 0,4299 | 0,8261 | 0,3551 |
| Transport and communication | 0,7854 | 0,6928 | 0,5441 |
| Trade and distribution | 0,7809 | 0,6821 | 0,5326 |
| Financial institutions and insurance | 0,3010 | 0,8267 | 0,2489 |
| Construction | 0,5520 | 0,7181 | 0,3964 |
| Agriculture and mining | 0,7555 | 0,8045 | 0,6078 |
| Wood industry and printing | 0,4957 | 0,7155 | 0,3547 |
| Hotel and restaurant | 0,4996 | 0,7129 | 0,3562 |
| Other services | 0,9098 | 0,8246 | 0,7502 |
| Maximum | 0,9169 | 0,8345 | 0,7652 |
| Minimum | 0,3010 | 0,6821 | 0,2489 |
| Average | 0,6409 | 0,7524 | 0,4823 |

*Source: Data from the National Institute of Statistics of Côte d'Ivoire, author's calculations*

The results of the cost boundary estimate inspire the following comments: none of the sectors of activity is 100% efficient. From the efficiency scores obtained, it can be concluded that the Materials *"Manufacturing sector"* is the most efficient both technically and economically. This sector therefore makes better use of its productive resources than the others. In order of economic efficiency, the *"Other services","Agriculture* and *mining"* and *"Trade and distribution"* sectors come next. However, the sectors that are technically less efficient are *"Financial institutions and insurance","Wood and printing industry", "Textile* and *leather industries"* and *"Other industries"*. Overall, over the period 2008-2012 with a panel of 4443 companies, the Ivorian economy can be considered economically inefficient. The average score for economic efficiency is 0.4823. These results indicate that locally produced scientific knowledge and innovation are used to produce goods with less efficient employment and combinations of available inputs. The corruption index, which was used as a proxy for the institutional environment (IE), shows that a very high level of corruption prevented people from benefiting from the revenues of certain areas that were not under the control of the public authorities. This situation disturbed the productive apparatus and had negative effects on economic efficiency in Côte d'Ivoire.

**4-3 Estimating the determinants of effectiveness**

The results from the Tobit model for explaining the technical, economic and allocative efficiency levels of the 12 industries shown in the table below. These results relate to the explanatory effect of the factors retained from the efficiency levels and their significance. The Tobit model was used to account for the truncated character (between 0 and 1) of the efficiency scores. In this study, we assume that the factors that affect the level of technical, allocative and economic efficiency of the 12 business lines are: the size of the company (TE) as measured by staff, the institutional environment (EVI), value added (VA), national savings (EN), and financial debts (DF). The corruption index has been used as a proxy for the institutional environment (EVI). It comes from the TRADING ECONOMICS database. The effects of these factors on efficiency levels are estimated by the Tobit regression method using Stata 14.1 software. The model is as follows: $EFF_{it} = f(EVI_{it}, TE_{it}, VA_{it}, DF_{it}, EN_{it,})$ With t = 2008 to 2012, i = ET, EA, EE and ET representing technical efficiency, EA designating allocative efficiency and EE being economic efficiency. The results are as follows:

**Table 5: Estimating the Determinants of Efficiency Scores**

| Determinants | Technical efficiency | | Allocative efficiency | | Economic efficiency | |
|---|---|---|---|---|---|---|
| | coefficients | t-test | coefficients | t-test | coefficients | t-test |
| Constant | -0,322 | -1,37 | 3,148* | 4,660 | 0,588** | 2,440 |
| VA | -0,011** | -2,220 | -0,0023** | -2,060 | 0,013*** | 1,850 |
| TE | 0,014** | 2,150 | -0,003 | -0,170 | 0,0828** | 2,060 |
| EVI | 0,096* | 7,030 | -0,184** | -3,170 | -0,038*** | -1,910 |
| DF | -0,001* | -8,120 | 0,003** | 3,070 | 4,31E-04** | 2,070 |
| EN | 0,099* | 7,940 | -0,223** | -3,700 | -0,019 | -1,320 |
| Log likelihood | 145.50 | - | 78.249 | - | 130.187 | - |

(*) Significant at 1%, Significant at 5%, significant at 10%

*Source: Data from the National Statistical Institute of Côte d'Ivoire, calculations by the author*

The results show that there is a correlation between efficiency levels and certain factors among the factors selected, whereas for the rest of the factors or their effect on efficiency levels to be negligible or non-significant.

In view of these results, we can say that the effects of the size of the company, the institutional environment, the financial debts are statistically significant. A larger firm improves its economic, allocative and technical efficiency. On the other hand, national savings are not statistically significant.

**4.3.1 Effect of added value and size of the business**

The results of the estimates presented in the table above show a statistically significant effect on the size of the business and the productivity measured by the value-added of the firms considered over the period 2008-2012 on economic efficiency. This is the example of large companies based in Côte d'Ivoire such as Orange, Mtn, Total or Nestlé which are very well listed on the Regional Securities Exchange (BRVM).
Similarly, an institutional environment that is less and less altered and credible promotes economic and allocative efficiency.

**4.3.2 Effect of the institutional environment**

With regard to the institutional environment, the table above shows that at the 5% threshold it has a negative impact on allocative and economic efficiency, while its impact on technical efficiency is significantly positive. Thus, an increasingly corrupt environment would help to make Ivorian companies economically inefficient. On the other side, an increasingly corrupt and credible environment promotes economic and allocative efficiency. This assertion is consistent with the conclusions of Girod's (2006) analysis.

**4.3.3 Effect of financialdebt**

The relationship between efficiency and financial debt shows that the most indebted companies are technically inefficient, while from an economic point of view they are more efficient. This is because the debt burden on suppliers is destroying strategic organization and productive investment efforts. Explains that the debt burden on suppliers is destroying strategic organization and productive investment efforts. A company with financing capacity and a company in need of financing do not have the same general strategic policies. These external debt efficiency issues were recently addressed by Loxley and Sackey (2008). The results they achieve are identical to ours.

### 4.3.4 Effect of national savings

Analysis of the impact of national savings on the efficiency of Ivorian companies shows that on the range of companies considered, increasing national savings are helping to reduce their economic and allocative efficiency. This result leads us to understand the negative effects of the crisis on the granting of bank loans. Especially between the period at the end of 2009 and the end of 2011 only 8% of the bank credit rate was granted to Ivorian firms. This low credit rate has had a significant impact on the economic efficiency of Ivorian businesses. In total, we can remember that the Ivorian productive system has been disorganized since the post-election crisis in the country. It partly justifies the empirically observed economic inefficiency. This inefficiency is mainly determined by the size of companies, the institutional environment, national savings, financial debts and value added.

## V. CONCLUSION

Our analysis sought to understand and explain the determinants of economic efficiency or inefficiency in Côte d'Ivoire. This study draws its justification for the more favour of the post-election crisis that has created socio-political instability and disrupted the production apparatus. The additional effect of the unrugly use of production factors and skilled labour that has migrated to other countries has probably had an influence on economic efficiency. Also: is it possible to argue that Ivorian companies are economically efficient? So what are the determinants of this effectiveness or inefficiency?

As a result, we used data from the INS Financial Data Bank for 4,443 companies observed over the 2008 to 2012 period. Estimating a stochastic production boundary and analyzing efficiency scores have shown that the production system is not economically efficient. The results indicate that the gap between observed production and potential output is 18%. This significant gap between the efficiency of the 12 industries shows that there are huge opportunities to increase their efficiency. Estimating a Tobit model then identified the explanatory factors for economic inefficiency. These include the size of companies, the institutional environment, national savings, financial debts and value added.

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