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Research Paper

Reliability- Based Failure Verification of Angala Timber Column

Obianime, T. S.; Sule, S. and Nwofor T.C.
Department of Civil and Environmental Engineering,
University of Port Harcourt,
P.M.B 5323, Port Harcourt, Rivers State, Nigeria

Abstract
This paper carried out reliability-based failure verification of a solid square simply supported Angala timber column in axial compression, bending and combined axial compression and bending respectively to Eurocode 5 (EC5). The limit state function in axial compression, bending and combined axial compression and bending were developed in accordance with the design provisions of EC5. The limit state function in axial compression was developed considering buckling failure of the column in axial compression. The parameters of the limit state functions were considered as random variables with their type of probability distribution obtained from literature. The First Order Reliability Method coded in MATLAB language was invoked in the reliability estimate. It was found that reliability of the column decreased with increase in load ratio, decreased with increase in column slenderness ratio and increased with increase in column dimension considering failure in axial compression, bending and combined axial compression and bending respectively. The design was found to be conservative in bending but satisfactory in axial compression and combined axial compression and bending respectively when compared with the target safety index value of 3.8 for 50-year reference period at ultimate limit state. The computer program developed is easy to run and interactive and it therefore suitable for structural safety estimation.

Keywords: *Failure verification, Angala timber column, Limit state function, First Order Reliability Method, Slenderness Ratio, EC5, Buckling*

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I. Introduction

A column is a fundamental structural element designed to carry loads primarily in compression. While columns mainly resist axial loads, they can also experience bending moments due to eccentric loading, lateral forces, or imperfections in construction and material properties. The structural design of columns must ensure they satisfy both ultimate limit states (ULS), which prevent catastrophic failure, and serviceability limit states (SLS), which ensure acceptable levels of deflection, durability, and long-term functionality (Ogork and Nakore, 2017; Abubakar, 2006; Ranganathan, 1990).

Structural engineering problems are inherently stochastic rather than purely deterministic due to the randomness in material properties, loading conditions, and environmental factors (El-Reedy, 2013; Abejide, 2012). Traditional design approaches rely on partial safety factors, which are specified in design codes such as Eurocode 5 (EN 1995-1-1:2004) to account for these uncertainties. However, such factor-based designs cannot guarantee absolute structural safety due to their empirical nature (Afolayan and Abubakar, 2003). When a column exceeds its ultimate strength or fails to meet serviceability requirements, the consequences can be severe, including structural collapse, property damage, and potential fatalities (Sule and Benu, 2012; Sule and Benu, 2019). This highlights the critical need for reliability-based approaches that quantitatively assess the probability of failure at the design stage rather than relying solely on deterministic safety factors.

In addition to axial compression and bending, slender columns, such as those considered in this study, are susceptible to buckling failure. Buckling occurs when a column experiences lateral instability due to excessive axial compressive loading, leading to a sudden sideways deflection and potential collapse. The inclusion of wind loads and lateral forces further complicates the column's behavior, introducing additional bending stresses that increase the likelihood of failure due to combined bending and compression.

Due to the inherent uncertainties in material properties, loading conditions, and design assumptions, conventional deterministic approaches may underestimate or overestimate the column's reliability. Variability in timber strength, moisture content, creep effects, and long-term degradation further contributes to potential structural failures. According to Melchers (1999), failures in structural elements are primarily caused by human error, negligence, poor workmanship, or unforeseen loading conditions during the design and construction phases. These uncertainties highlight the need for probabilistic design frameworks, such as structural reliability analysis, to improve the predictability and safety of engineered structures.

Afolayan and Abubakar (2003) and Afolayan (2005) advocate for probabilistic methods as the most rational approach to account for these uncertainties. Reliability-based analysis provides a more robust means of verifying structural performance by evaluating failure probabilities rather than relying on fixed safety factors. This ensures that load-resisting capacities are assessed within a realistic range of variability, enhancing structural resilience and efficiency.

This study presents a reliability assessment of a simply supported solid Angala timber column with a square cross-section subjected to:

1. Axial Compression: Evaluating the probability of failure due to pure compressive loading along the column's length.
2. Bending: Assessing structural performance under lateral forces that induce bending stresses.
3. Combined Compression and Bending: Considering the interactive effects of axial and lateral loads, particularly under wind-induced forces and eccentric loading.

The limit state functions for these failure modes are developed based on Eurocode 5 (EN 1995-1-1:2004) design provisions. The analysis further incorporates First Order Reliability Method (FORM) to evaluate reliability indices (***β***) and failure probabilities (P_f) for varying load ratios and column cross-sectional dimensions.

By adopting a probabilistic reliability approach, this study aims to provide quantitative failure verification of Angala timber columns, ensuring optimal design, material efficiency, and improved safety margins for timber structures.

II. Development of the Limit State Function

The limit state function in bending and compression are derived in accordance with the EC5 design rules for timber structures. A pin-ended solid square Angala timber column under axial compression and bending (**Figure 1**) is considered in this study.

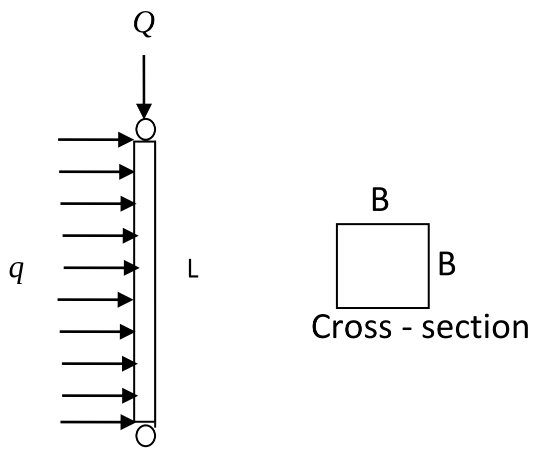

Figure 1. A simply supported solid square Angala timber column under axial compression and bending

2.1 Compression Limit State

The design compressive stress in column parallel to grain is given by:

$$\sigma_c = \frac{Q}{A} = \frac{Q}{B^2} \tag{1}$$

Where σ_c = design stress, A = cross- sectional area, B = cross-section dimension of the column, Q = compressive load on column

The factored compressive load on column is given by:

$$Q = 1.35G_k + 1.5Q_k \tag{2}$$

Let:

$$\alpha_1 = \frac{G_k}{Q_k} \tag{3}$$

Application of Equation (3) changes Equation (2) to:

$$Q = Q_k\left(1.35\alpha + 1.5\right) \tag{4}$$

Therefore, the compressive strength parallel to the grain is given by:

$$f_c = \frac{K_{\text{mod}}\, f_{c,k}}{\gamma_m} \tag{5}$$

Where K_{mod} = modification factor for duration of loading and moisture content, Q = short term axial load, γ_m = partial safety factor for the material property based on EC5, $f_{c,k}$ = characteristic value of the compressive strength based on timber strength class, α_1 = load ratio under axial compression

The buckling criterion of a column with cross-sectional area loaded in axial compression

$$\frac{\sigma_c}{k_c f_c} \leq 1 \tag{6}$$

The Euler's buckling load of a pin-ended column is given by:

$$P_E = \frac{\pi^2 EI}{L^2} \tag{7}$$

The second moment of area I of a rectangular section is given by:

$$I = Ar^2 \tag{8}$$

Where A, r represents the cross-sectional area and radius of gyration of the section respectively

Substituting for I in equation (7) and dividing both sides of Equation (7) by A gives:

$$\frac{P_E}{A} = \frac{\pi^2 EAr^2}{L^2 A} \tag{9}$$

$$\sigma_E = \frac{P_E}{A} \tag{10}$$

And

$$\lambda = \frac{l}{r} \tag{11}$$

Where l = free column length

The Application of Equation (10) and Equation (11) changes Equation (9) to:

$$\sigma_E = \frac{\pi^2 E}{\lambda^2} \tag{12}$$

The second moment of area of a square section is given by:

$$I = \frac{B^4}{12} \tag{13}$$

Applying equation (8), the radius of gyration of the column section is given by:

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{B^4}{12B^2}} = \frac{B}{\sqrt{12}} \qquad (14)$$

According to EC5, the slenderness of a timber column is described by the so-called relative slenderness ratio given by:

$$\lambda_{rel} = \sqrt{\frac{f_c}{\sigma_E}} \qquad (15)$$

Substituting for σ_E using Equation (12) changes Equation (15) to:

$$\lambda_{rel} = \lambda\sqrt{\frac{f_c}{\pi^2 E_{0,05}}} \qquad (16)$$

According to EC5, the column instability factor is given by:

$$k_c = \frac{1}{k + \sqrt{k^2 + \lambda^2{}_{rel}}} \qquad (17)$$

Where k_c, k = column instability factor and axial instability factor respectively, λ_{rel} represents relative slenderness ratio corresponding to bending, $E_{0,05}$ = characteristic value of modulus of elasticity, λ = slenderness ratio corresponding to bending

According to EC5, the column buckling factor k is given by:

$$k = 0.5\left(1 + \beta\left(\lambda_{rel} - 0.3\right) + \lambda^2{}_{rel}\right) \qquad (18)$$

Where β represents the column straightness factor = 0.2 for structural timber

Application of equation (6) gives the limit state function of the column failure due to buckling in axial compression as:

$$g(x) = k_c f_c - \sigma_c \qquad (19)$$

Where f_c, σ_c represents the design strength in axial compression and stress in axial compression respectively

Applying equations (1), (3), (4) and (5), the limit state function in axial compression is given by:

$$g(x) = k_c \frac{K_{\text{mod}} f_{c,k}}{\gamma_m} - \frac{Q_k\left(1.35\alpha_1 + 1.5\right)}{B^2} \qquad (20)$$

Substituting for r in Equation (11) using Equation (14) gives the equation for slenderness ratio as:

$$\lambda = \frac{l\sqrt{12}}{B} \qquad (21)$$

Multiplying the second part of Equation (21) by $l\sqrt{12}$ and dividing by same changes Equation (20) to:

$$g(x) = k_c \frac{K_{\text{mod}} f_{c,k}}{\gamma_m} - \frac{\lambda Q_k\left(1.35\alpha_1 + 1.5\right)}{Bl\sqrt{12}} \qquad (22)$$

2.2 Bending Limit State

The applied bending stress parallel to grain is given by:

$$\sigma_m = \frac{M}{Z} \qquad (23)$$

The induced bending moment on beam under uniform loading is given by:

$$M = 0.125ql^2 \qquad (24)$$

From strength of materials, the section modulus of a solid square section is given by:

$$Z = \frac{B^3}{6} \tag{25}$$

Applying Equations (23), (24) and (25), the load induced bending stress parallel to grain is given by:

$$\sigma_m = \frac{0.75ql^2}{B^3} \tag{26}$$

Where q = lateral load on column

The factored applied lateral load is given by:

$$q = q_k\left(1.35\alpha_2 + 1.5\right) \tag{27}$$

According to EC5, the design bending strength parallel to grain is given by:

$$f_m = \frac{K_{\text{mod}}\, f_{m,k}}{\gamma_m} \tag{28}$$

Where $f_{m,k}$ = characteristic value of the bending strength

Applying Equations (26), (27) and (28), the limit state function in bending is given by:

$$g(x) = \frac{K_{\text{mod}}\, f_{m,k}}{\gamma_m} - \frac{0.75q_k\left(1.35\alpha_2 + 1.5\right)l^2}{B^3} \tag{29}$$

Similarly, multiplying the second part of Equation (29) by $l\sqrt{12}$ and dividing by same changes Equation (30) to:

$$g(x) = \frac{K_{\text{mod}}\, f_{m,k}}{\gamma_m} - \frac{0.75q_k\left(1.35\alpha_2 + 1.5\right) l\lambda}{B^2\sqrt{12}} \tag{31}$$

According to EC5, for column under combined axial compression and bending,

$$\frac{\sigma_c}{k_c f_c} + \frac{\sigma_m}{f_m} \leq 1 \tag{32}$$

Therefore, the limit state function under combined axial compression and bending is given by:

$$g(x) = 1 - \left(\frac{\sigma_c}{k_c f_c} + \frac{\sigma_m}{f_m}\right) \tag{33}$$

Substituting for σ_c, f_c, σ_m and f_m using Equation (1), Equation (5), Equation (27) and Equation (27) changes Equation (30) to:

$$g(x) = 1 - \left(\frac{Q\gamma_m}{k_c B^2 k_{\text{mod}} f_{c,k}} + \frac{0.75q\gamma_m\, l^2}{B^3 k_{\text{mod}} f_{m,k}}\right) \tag{34}$$

Substituting for Q and q using Equation (4) and Equation (26) changes Equation (31) to:

$$g(x) = 1 - \left(\frac{Q_k\left(1.35\alpha + 1.5\right)\gamma_m}{k_c B^2 k_{\text{mod}} f_{c,k}} + \frac{0.75q_k\left(1.35\alpha_2 + 1.5\right)\gamma_m\, l^2}{B^3 k_{\text{mod}} f_{m,k}}\right) \tag{35}$$

Multiplying the second and third parts of Equation (35) by $l\sqrt{12}$ and dividing by same changes Equation (35) to:

$$g(x) = 1 - \left(\frac{Q_k\left(1.35\alpha + 1.5\right)\gamma_m * \lambda}{k_c B k_{\text{mod}} f_{c,k} * l\sqrt{12}} + \frac{0.75q_k\left(1.35\alpha_2 + 1.5\right)\gamma_m\, l * \lambda}{B^2 k_{\text{mod}} f_{m,k} * \sqrt{12}}\right) \tag{36}$$

III. Materials and Methods

Let the failure surface in x-space be given by:

$$g(x) = g(x_1, x_2, ..., x_n) = 0 \tag{37}$$

The vector of the random variables in x-space is given by:

$$x = [x_1, x_2, ..., x_n]^T \tag{38}$$

The vector of the basic random variables $x = (x_1, x_2, ..., x_n)$ are the basic variables with joint probability function given by

$$F_X(x) = P\left(\bigcap_{i=1}^{n}\{X_i \leq x_i\}\right) \tag{39}$$

For First Order Reliability Method, $F_X(x)$ is continuous and differentiable with respect to the basic random variables. This means that the probability density of $F_X(x)$ exists. The limit state function, $g(x)$ of a structure at a particular limit state is usually a function of the basic variables that affect structural performance. The performance function $g(x) > 0$ represents safe domain, $g(x) = 0$ represents limit state surface and $g(x) < 0$ represents failure domain.

First order approximation to probability of failure is given by:

$$P_f = P(X \in F) = P(g(X) \leq 0 = \int_{g(X) \leq 0} dF_X(X) = \phi(-\beta) \tag{40}$$

Where β = reliability index which represents the minimum distance between the origin and the failure surface and it is given by:

All non-normal variables in the limit state functions are first transformed into equivalent normally distributed variables. According to Ranganthan (1999), the equivalent normal variables are made equal at the design point, $x = \left(x_1^*, x_{2,}^* \, x_3^*, ..., x_n^*\right)$ on the failure surface.

Each statistically independent non-normal variable is equated with an equivalent normal variable at the design point to yield:

$$\Phi\left(\frac{x_i^* - \mu_{xi}^N}{\sigma_{xi}^N}\right) = Fx_i(x_i^*) \tag{41}$$

where $\Phi(\cdot)$ = cumulative distribution function of the standard normal variable at the design point; μ_{xi}^N, σ_{xi}^N represents the mean and standard deviation of the equivalent normal variable at the design point respectively, $Fx_i(x_i^*)$ = cumulative distribution function of the original non-normal variables.

The mean of the equivalent normal variable at the design point is given by:

$$\mu_{xi}^N = x_i^* - \Phi^{-1}\left[Fx_i\left(x_i^*\right)\right]\sigma_{xi}^N \tag{42}$$

Equating the probability distribution functions of the original variable and the equivalent normal variable at the design point yields:

$$\frac{\phi}{\sigma_{xi}^N}\left(\frac{x_i^* - \mu_{xi}^N}{\sigma_{xi}^N}\right) = fx_i(x_i^*) \tag{43}$$

Where $\phi(\cdot)$ and $fx_i(x_i^*)$ = probability distribution function of the equivalent standard normal and the original non-normal random variable respectively.

$$\Phi^{-1}\left[Fx_i\left(x_i^*\right)\right]\sigma_{xi}^N = x_i^* - \mu_{xi}^N \tag{44}$$

Applying Equation (41), (42), (43) and (44), the standard deviation of the equivalent normal variables is given by:

$$\sigma_{xi}^{N} = \phi \frac{\left(\Phi^{-1}\left[F_{xi}\left(x_i^{*}\right)\right]\right)}{fx_i\left(x_i^{*}\right)} \tag{45}$$

The strength classification of Angala timber specie has been carried out and found to belong to structural timber specie of strength Class D70 by the author. The results of some of the mechanical properties are presented in Table 1.

Table 1: Statistics of the Basic Random Variables

Random Variable	Mean	Standard Deviation	Coefficient of Variation	Type of Probability Distribution
Q	65,000N	1950N	0.30	Gumbel
q	3.25N/mm	0.975N/mm	0.30	Gumbel
K_{mod}	0.90	0.135	0.15	Lognormal
L	3000mm	30mm	0.01	Normal
B	300mm	3mm	0.01	Normal
$f_{m,k}$	70N/mm^2	10.5N/mm^2	0.15	Lognormal
$f_{c,k}$	34N/mm^3	5.1N/mm^3	0.15	Lognormal
γ_m	1.30	0.195	0.15	Lognormal
α_1	Varying	Varying	Fixed	Fixed
α_2	Varying	Varying	Fixed	Fixed
$E_{0,05}$	16800N/mm^2	2520	0.15	Lognormal
k_c	0.294	0.0441	0.15	Lognormal

Results of the Reliability Based Analysis

The reliability-based analysis of the Angala solid square timber column under axial compression, bending, and combined axial compression and bending was conducted using the First Order Reliability Method (FORM). MATLAB-based computational models were employed to determine the reliability indices and perform sensitivity analyses for different random variables. The findings from the analysis are presented in Figures 2 to 7.

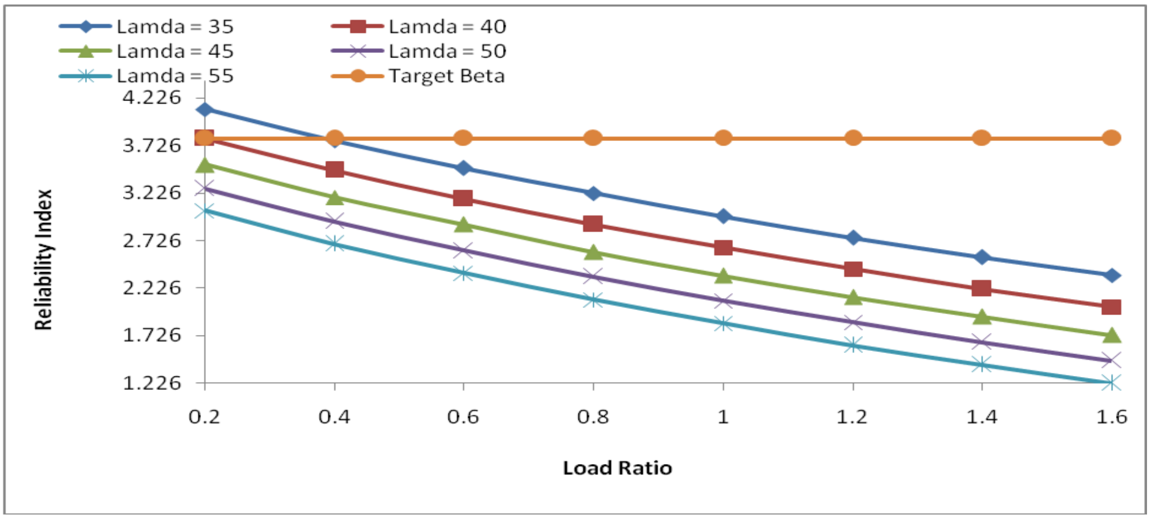

Figure 2: Relationship between Reliability Index and Load Ratio at varying Slenderness Ratios (Buckling Criterion)

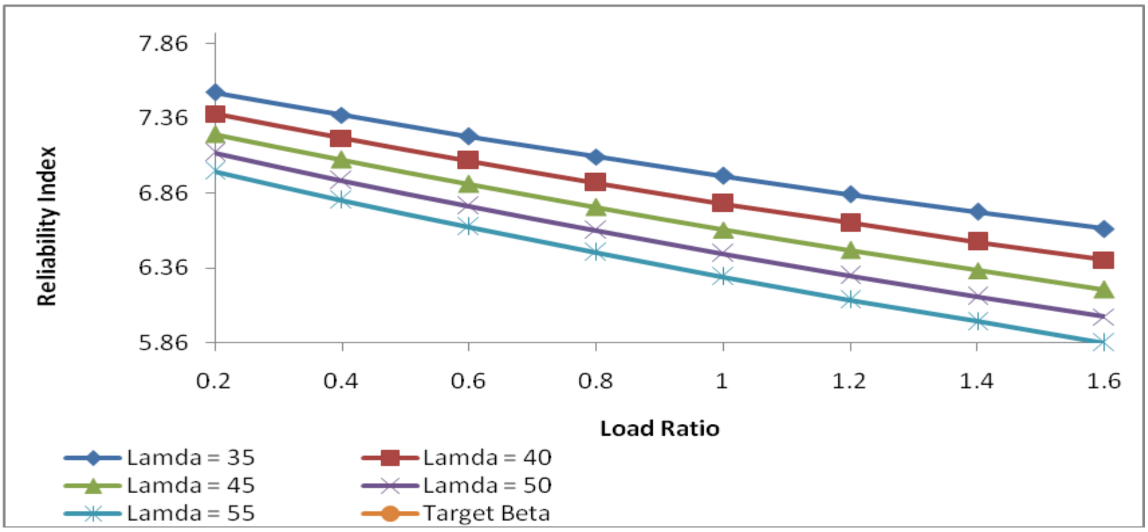

Figure 3: Relationship between Reliability Index and Load Ratio at varying Slenderness Ratios (Bending Criterion)

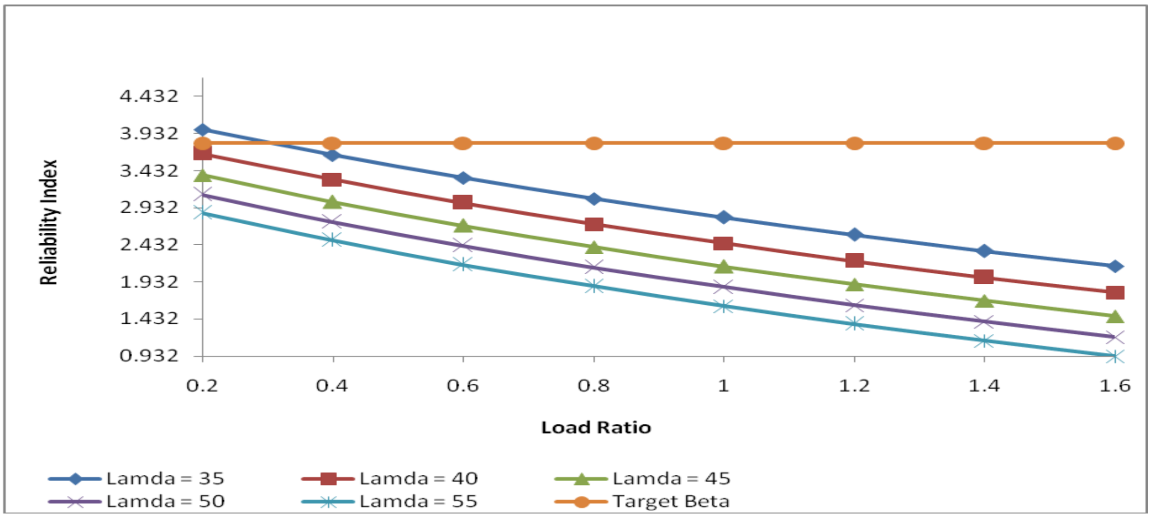

Figure 4: Relationship between Reliability Index and Load Ratio at varying Slenderness Ratios (Combined Axial Compression and Bending Criterion)

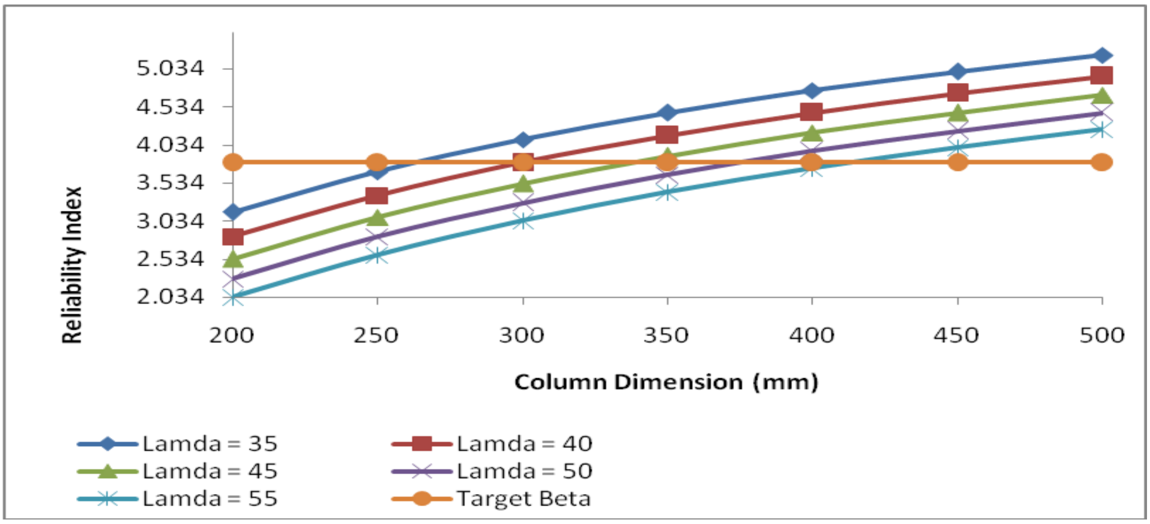

Figure 5: Relationship between Reliability Index and Column Dimension at varying Slenderness Ratios (Buckling Criterion)

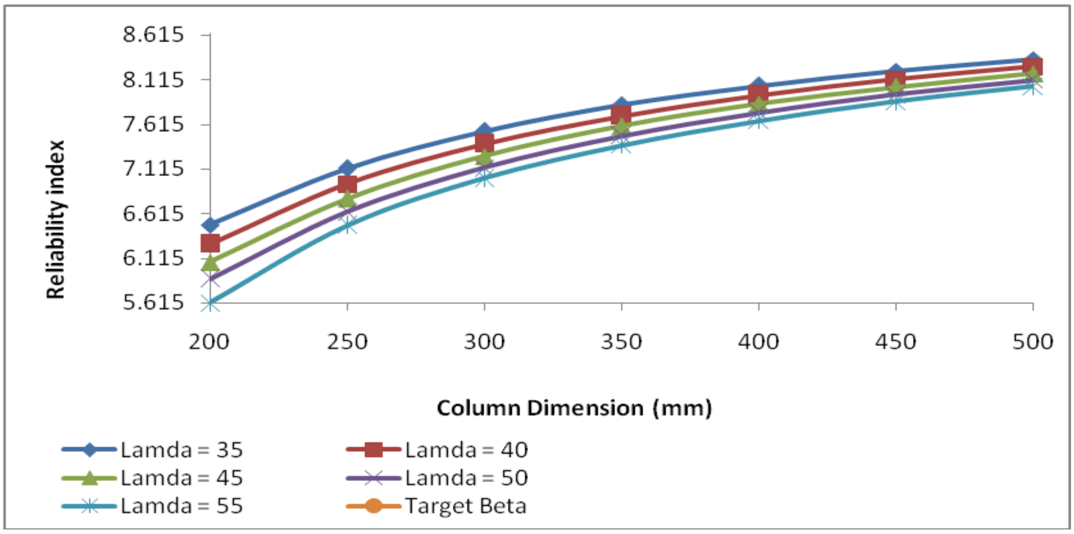

Figure 6: Relationship between Reliability Index and Column Dimension at varying Slenderness Ratios (Bending Criterion)

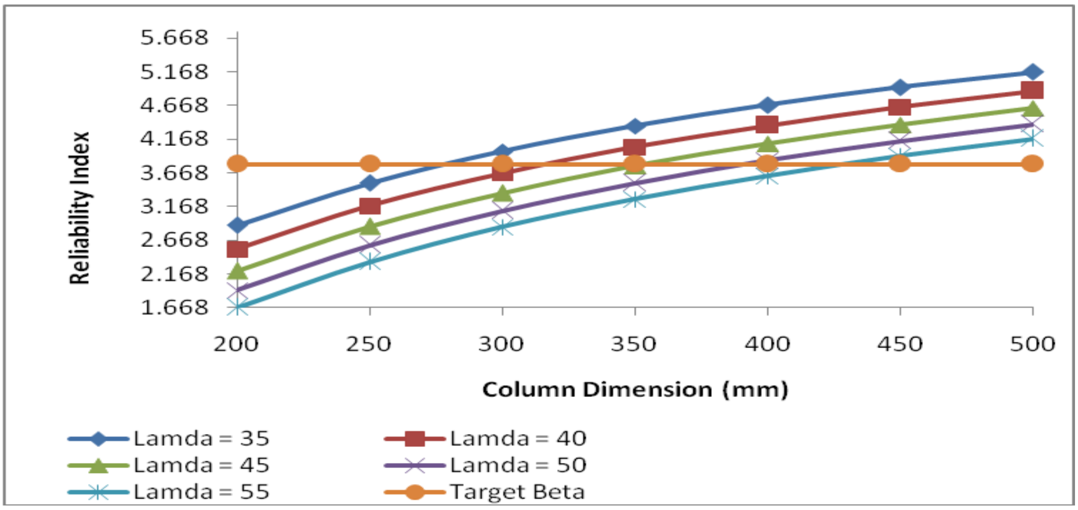

Figure 7: Relationship between Reliability Index and Column Dimension at varying Slenderness Ratios (Combined Axial Compression and Bending Criterion)

IV. Discussion of Results

Effect of Load Ratio and Slenderness Ratio on Reliability Index

Figures 2 to 4 illustrate the relationship between the reliability index and the load ratio at different slenderness ratios. The results indicate that an increase in load ratio leads to a decrease in the reliability index across all failure modes—axial compression, bending, and combined axial compression and bending. Additionally, higher slenderness ratios resulted in lower reliability indices. This reduction is attributed to the increase in axial compressive forces and transverse loads, which intensify column instability due to buckling and cause a rise in bending moments. These observations align with previous studies by Abubakar and Edache (2002), which reported that increasing the load ratio negatively impacts the reliability of structural members.

Influence of Column Dimensions and Slenderness Ratio on Reliability Index

Figures 5 to 7 present the relationship between reliability index and column dimensions for various slenderness ratios. The results demonstrate that increasing column dimensions enhances the reliability index across all failure modes. This is because larger column dimensions improve the load-bearing capacity, making the column more resistant to failure. Conversely, higher slenderness ratios result in lower reliability indices due to a reduction in the column's load-carrying capacity. These findings corroborate the work of Sule and Benu (2019), who emphasized the importance of selecting appropriate column dimensions to enhance structural reliability.

Comparison with Target Reliability Index

The computed reliability indices were compared with the target reliability index of 3.8 for a 50-year reference period at the ultimate limit state, as recommended by Eurocode EN 1990 (2002). The results indicate that:

- Bending failure results were conservative, implying that the design of Angala timber columns under bending may be overly safe, potentially leading to uneconomical designs.
- Buckling and combined axial compression and bending failure results were satisfactory, meaning that the columns performed well within acceptable safety margins.

V. Conclusion

The reliability analysis of a simply supported Angala timber column was conducted across different slenderness ratios, evaluating its structural performance under axial compression, bending, and combined axial compression and bending using the First Order Reliability Method (FORM). The results demonstrated that the reliability index decreases as both the load ratio and slenderness ratio increase, indicating that higher loads and increased column slenderness reduce structural stability and increase the probability of failure. Conversely, increasing the cross-sectional dimensions of the column significantly improves its reliability, highlighting the importance of optimizing column geometry to enhance structural safety.

A key observation from the analysis is that bending failure yielded highly conservative reliability indices, suggesting that current design provisions may lead to overestimated safety margins. This finding implies that there is potential for design optimizations to ensure a more balanced, cost-effective structural solution without compromising safety. The study's results align with existing literature, reaffirming the crucial role of selecting appropriate column dimensions and slenderness ratios in ensuring the safety, durability, and overall performance of timber structures. These findings contribute to the broader field of timber engineering, providing essential insights for improving design methodologies and structural reliability assessments for Angala timber columns.

References

[1]. Abejide, O. S. (2012). Incorporation of advanced second moment reliability assessment method into assessment and design. World Journal of Engineering and Pure and Applied Science, 2 (4), pp. 112-117.

[2]. Abubakar, I. (2006). Reliability analysis of structural design parameters of reinforced concrete strip footings. Journal of Applied Science and Research, 2 (7), pp. 397-401.

[3]. Abubakar, I., and Edache (2007). Reliability analysis of simply supported steel beams. Australian Journal of basic and Applied Sciences, 1 (1), 20-21.

[4]. Afolayan, J.O. (2002). Parameter selection for safe design under uncertain conditions: A truss example. Journal of Engineering and Applied Science, 21, pp. 77-82, 2002.

[5]. Afolayan, J.O. (2005). Probability based design of glued thin-webbed timber beams. Asian Journal of Civil Engineering (Building and Housing), 6(1-2), pp. 75-84, 2005

[6]. Afolayan, J.O and Abubakar, I. (2003). Reliability analysis of reinforced concrete one-way slabs: The ultimate conditions. Nigerian Journal of Engineering, 11(2), pp. 28-31.

[7]. Benu, M.J. and Sule, S. (2012). Probabilistic failure analysis of a solid timber column. Nigerian Journal of Tech. (NIJOTECH), 31(3), 405-408.

[8]. Eurocode O. (2002). Eurocode-basis of structural design, British Standards, CEN, Brussels.

[9]. Eurocode S. (1995). Design of timber structure's part 1.1. General rules and rules for buildings. Publication of European Committee for Standardization.

[10]. El-Reedy, M. (2013). Reinforced concrete structural reliability. Taylor and Francis, Boca Raton, FL.

[11]. Obianime, T.S. (2025). Strength Classification and Reliability assessment of Three selected Timber species as bridge Beams in the Niger Delta. Unpublished PhD Thesis for the award of PhD in Civil Engineering Department, University of Port Harcourt, Rivers State, Nigeria.

[12]. Ogork, E.N. and Nakore, A.K. (2017). Reliability analysis of a two span floor designed according to Eurocode 2. Nigerian Journal of Technology (NIJOTECH), 36 (1), pp. 18-25.

[13]. Melchers, R. E. (1999). Structural Reliability Analysis and Prediction. Second Edition, John Wiley and Sons, Chichester, England.

[14]. Ranganathan, R. (1990). Structural Reliability, Analysis and Design, Jaico Publishing House, Mumbai.

[15]. Salisu, D., Abubakar, I., Ejeh, S.P. and Afolayan, J.O. (2009). Probabilistic Design of mechanically-jointed timber beams using Eurocode 5. Indian Journal of Science and Technology, 2 (10), pp. 62-65, 2009.

[16]. Sule, S. and Benu, M.J. (2019). Reliability Analysis of a Solid Timber Column Subjected to Axial And Lateral loading. FUOYE Journal of Engineering and Technology, 4(2), pp. 1-5.