



## Concrete Creep Coefficients in Codes and References

<sup>1</sup>Vojislav Mihailovic

<sup>1</sup>(University of Novi Sad, Faculty of Civil Engineering, Subotica, Serbia)

Corresponding Author: Vojislav Mihailovic

**ABSTRACT :** *The largest part of this paper will deal with the procedures for obtaining data for the creep and shrinkage coefficients of concrete, using the current recommendations of Eurocodes, regional regulations (of Serbia, etc.) and selected proposals of three authors. Possible difficulties in correctly defining the creep coefficient will be pointed out.*

*The work is intended for researchers, designers and engineers when creating structures of larger spans, or special requirements, in order to facilitate the application of codes (regulations) for the calculation of concrete structures.*

**KEYWORDS:** *Concrete, rheological data, creep (viscosity), codes*

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### I. INTRODUCTION

The paper is a continuation of the problematic matter from the previous two papers of the author [20][21]. In them, it is shown how to easily determine the experimental, numerical characteristics of the creep coefficient functions and creep measures function and their application in theories of concrete. These will be the tasks of determining the data using the recommendations of Eurocodes, regional regulations for selected climatic regions and according to three selected description in references.

The application of experimental values found for creep coefficients, and then, the theoretical finding of creep measures requires more experience in this special field of concrete. The success of the measurement depends primarily on the conditions of the laboratory for this special type of examination, as well as on the commitment and qualifications of the group that is able to guarantee and monitor the obtained measurement results and perform the necessary analysis. The accuracy of the test results depends a lot on the type of concrete theory that will be adopted when calculating the parameters in the expressions for the creep coefficients. A special task is the preliminary adoption of the dimensions and shape of the concrete samples, which should correspond to the tested material. In known papers, relatively small samples of prisms 7.5 x 7.5 x 21cm were used, which overestimate the role of concrete creep and shrinkage. Small cross-sections of concrete samples correspond to materials with fine-grained fractions, such as fine-grained concrete, polymers, epoxies, etc. [8].

Creep coefficients serve as the basis for the formation of theoretical stress-strain relationships for concrete. That is why one should be careful in applying the values found in the calculation and analysis of <sup>2</sup>RC, <sup>3</sup>PC and composite structures. The degree of agreement between experimental data and theoretical values can sometimes be unacceptable for practical application. How much that difference will be depends a lot on the type of concrete theory, which will be adopted during calculation and design. Until now, the theory of aging of concrete, which is called, due to its simple application, the Technical theory of calculation of RC and PC structures[5] has been mostly applied.

### II. ANALYSIS OF CONCRETE BEHAVIOR

The behavior of hardened concrete depends a lot on the duration of the load. Therefore, it differs: extremely short-term, short-term and long-term loading of concrete structures. The presentation in this paper refers to a constant or monotonic variable load with respect to the rate of load changes.

Based on measurements of longitudinal strains of concrete samples, most often on 12 x 12x 36 cm, and now 20 x 20 x 60 cm, creep curves  $\varphi(t)$  are formed. For each initial loading time, a curve was obtained as a function of the observation time, which is chosen at the time  $t = 3, 7, 14, 21, 28, 90, 180$  and 365 days for young concrete and middle-aged concrete. In the case of old concrete, the same time intervals are chosen, but for the sake of easier work, it is assumed that then  $\tau_{0,tot} = (365) + \tau_0$  days. The value in parentheses is rarely used and only in heritage theory.

In the analysis of the influence of long-term load on the deformation of creep and shrinkage of concrete, only the macroreological approach will be applied. It is considered that the composition and many other characteristics of concrete, which are detailed in the part of this paper under point 3, affect only the concrete hardening phase. The data from that concrete phase are combined in accordance with Table 1. The work procedure in that phase is completely independent of the procedure of measuring the parameters of the second concrete phase, in which they are determined, and which quite well define the behavior of concrete during long-term actions.

The following current approaches have been applied to these analyzes:

a). Direct application of concrete creep coefficients is common in the application for rapid analysis of experimental results and long-term deformations in structures. Analyzes are performed to compare them with data in regulations and literature. In addition, these values can be used in the design and monitoring of deformations of concrete structures. A special area of application is in interventions on the elements by prestressing, moving supports and the use of hydraulic jacks to obtain the desired shape of deformations of structures.

b). The microreological approach to the formation of rheological models has attracted a great deal of attention from many researchers of concrete behavior. It contains an attempt to use many factors used in concrete technology to obtain concrete of a certain strength or special properties of concrete to assess the shrinkage and creep of concrete. Hansen and Jordan were the most famous proponents of these procedures, who did not find, it seems, greater applications in practice [3] [10].

c). The macro-rheological approach to rheological models is based on attempts to form functions of concrete creep measures that can successfully show the results of measuring strains of concrete samples, in certain climatic or thermohygro-metric conditions. There are a large number of proposals for creep measures, which, most often, do not match the experiments well enough.

The most significant contributions, only from this area of concrete (RC and PC) before 1970., in the opinion of this author, were by: Boltzmann (1877), Volterr (1931), Dischinger (1937), Ulickij, Djuric and Sattler. New attempts began with the proposals of Arutjunyan (1967) [5], Jodaan (1980) [13] and many other researchers (see:[4] [15][17]). The theory of aging has been criticized for 'not showing viscoelastic strains during unloading'. It seems that this kind of thinking is present even now. In the papers of the this author, it is shown that the theory of aging and the theory of heritage represent borderline theories and the best show the behavior of young and old concrete [1] [5][9] [11]. The author's new proposals with criteria for their application are given in point 5.3 .

d). The thermodynamic approach to  $\sigma$ - $\epsilon$  relations for concrete arose as an attempt to bind the viscous and mechanical properties of concrete with the intense effects of temperature and the occurrence of dynamic stresses in structures. The need for special analyzes appears during the design, construction and operation AB or PB strutures of nuclear reactors during chemical reactions. [18]. Significant works in this area have Argiris J. and Willam K. Model of nonlinear creep of concrete suggested in [15].

### III. INFLUENCES ON CREEP AND SHRINKAGE OF CONCRETE

Very often in works dealing with the behavior of hardened concrete, equal importance is given to the data for hardened and concrete in the phase of hardening in the estimating of stresses and deformations of structures. This does not mean that data on the composition and its care are not needed in order to correctly obtain the material with the desired characteristics in the following tasks.

That is why, in this paper, two phases of concrete are strictly separated in its applications. Therefore, in this paper, the two phases of concrete in its applications are now strictly separated.

#### a) In the fase of hardening concrete

Concrete hardening tasks belong to the highest areas of concrete technology. However, in this paper, the minimum data from this area should be stated in order to establish an appropriate connection with the occurrence of creep and shrinkage of concrete. If we split the consideration of the phases of concrete hardening from the phase of hardened concrete, it is much easier to conclude what are the proper influences on the characteristics of concrete in both phases.

The limit of  $\tau_0 = 28$  days defines with sufficient precision the concrete zones in which the parameter of relations for  $\sigma$ - $\epsilon$  are considered more reliably. Special concretes, instead of the named specified limit, are used for  $\tau_0 > 3$  and 7 days.

When forming the report on the creep and shrinkage of concrete it is good that it contains basic technological data on the tested concrete, although it will not be used to directly determine the behavioral parameters of hardened concrete. Broader research programs would be significantly larger than the content in Table 7.

**Table1. An example of a possible composition of a concrete mix**

Descriptions	Data	Notes
Concrete fractions(mm)	33%	Aggregate humidity HP90%
0-4	19%	
4-8	48%	
8-16		
PC	450kg 12% slag	Fineness of grinds, etc.
$m_w / m_c$	0.38	Water from the water supply network-in the settlement of NN
Additive, porosity, etc.	Plasticizer-2% Porosity-6%	For concreting, etc
Samples	15x 15 x5 Class: C35.5 (MPa)	Dimensions ( cm) Concrete strength(exper.)

**Table.2 Data for the duration of concrete care**

Nurturing time (days)	HP( %)	T( $^{\circ}$ C )	A form of nurturing	Note
1 dan	90	20	in molds	
2 dana	90	20	in water or damp cloths	
od 3 do 28 dana	40	20	out of the mold	

The example shown in Tables 1 and 2 contains only one possible form of concrete composition and care, which could possibly find its place in a Database for Concrete Technology in the climatic areas of the regions, as a faster insight into the data of the concrete hardening phase.

It is considered that most of the hardening process takes up to 28 days (alt. 3. or 7 dana). It can be argued that only after the hardening time can concrete be considered as a known material for calculating the influence of creep and shrinkage of concrete.

However, it has already been stated that most of the hardening of concrete is always done in  $\tau_0 > 7$  days, so for special structure technologies, it can be successfully adopted in structure control program is provided. There are examples of the application of special concretes that are loaded after age of 3 days.

#### b) In the stage of hardened concrete

The main influences on the intensity of creep and shrinkage of concrete are: initial loading time for series of concrete samples or of the structure ( $\tau_0$ ), observation time of strains ( $t_1$ ), duration of loading ( $t - \tau_0$ ). The influence of humidity and temperature in the local environment is especially introduced for samples of series of concrete or structures on the change of the measurement results. The influence of the dimensions of the cross-section of the samples (or structures) can be significant on the obtained measurement results.

For the sake of easy comparison with the results of other research sources, the characteristics of concrete in the hardening phase should also be useful.

Macroreological models show well how creep parameters should be defined to fulfill experimental conditions [17][19]. The principle is applied that only parameters can be used in the relations( $\sigma$ -  $\epsilon$ ), which can be measured directly or indirectly and have physical significance.

## IV. EUROCODES AND REGIONAL CODES

The information in this section aims to fulfill the requirements of designers, researchers, builders and facilitate the work on the adoption of appropriate data. For this purpose, a concise form of an overview of

selected known proposals (according to the author of the paper) for the functions of the creep coefficient and, sometimes, the function of the creep measure of concrete will be presented.

#### 4.1 <sup>1</sup>EC '70(end <sup>2</sup>CEB'70)[2]

In the proposal, the creep coefficient function is represented by the expression:

$$\varphi(t) = \beta_1 \cdot \beta_2(\tau) \cdot \beta_3 \cdot \beta_4 \cdot \beta_5(t_1 - \tau_0) \quad (1)$$

where  $\beta_1$ - factor for climatic conditions of the environment,  $\beta_2$ - for the initial loading,  $\beta_3$ - composition of concrete,  $\beta_4$  -size of elements and  $\beta_5$  ( $t_1 - \tau_0$ ) duration of concrete loading.

This proposal should be appreciated as one of the first attempts to take into account more realistic properties of materials. A very similar procedure is shown in Ulicki's book [5].

#### 4.2 <sup>3</sup>EC2 '93[2]

The recommendations for the adoption of data on the effects of shrinkage and creep of concrete are contained in EC2-'93. The model of codes should connect the codes of most <sup>4</sup>European countries for easier application in practice and for easier communication in solving the problems of concrete structures at the time of initial loading ( $\tau_0=3, 7, 28, 90, 180$  and  $365$  days).

##### a) Total values of creep coefficient for $t=t_n$

For practical purposes, it is most important to know the values of the creep coefficients ( $\varphi_{tot}$ ) in the time that corresponds to its ultimate value  $t=t_n$  (365 days). For each individual value of the initial loading time ( $\tau_0$ ), the corresponding total values of the creep coefficient can be obtained, which are shown in Table 3. Most often, in practice, only the values given in Table 3 are required.

<sup>1</sup>EC '70 - European model code 1970.

<sup>2</sup>CEB-FIP'78 -Committee European model code 1978.

<sup>3</sup>EC 2 '90 - European model code 1990.

<sup>4</sup>FIP - Federation International for Prestressing

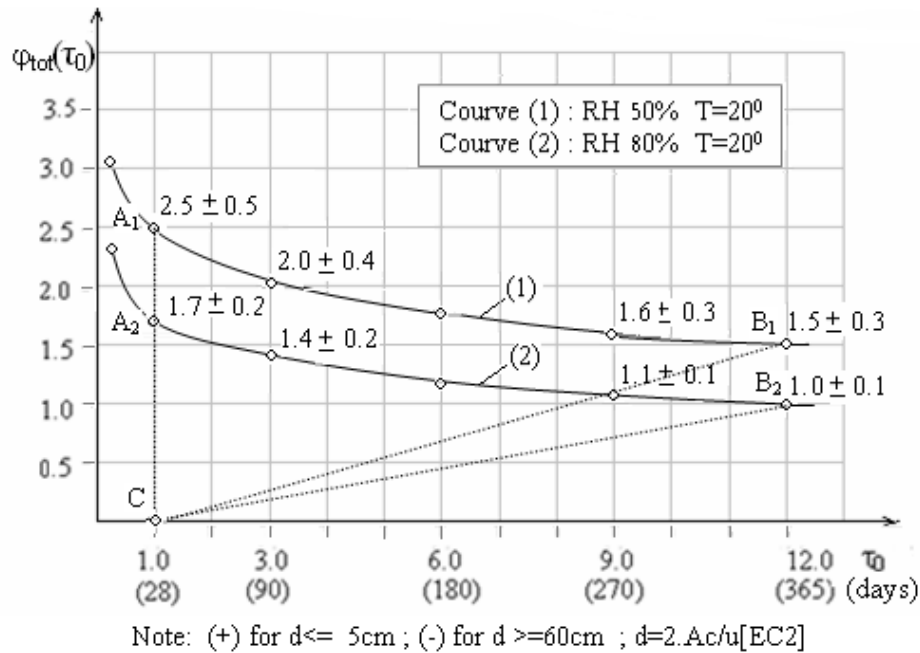
**Table3. Total values ( $\varphi_{tot}$ ) of the creep coefficients ( $\varphi$ ) for concrete (EC2)**

Initial loading time $t_0$ (days)	Mean size section $d_m = 2A_c/u$ (cm)					
	5.0	15.0	60.0	5.0	15.0	60.0
	RH=50%			RH=80%		
1	5.5	4.6	3.7	3.6	3.2	2.9
7	3.9	3.1	2.6	2.6	2.3	2.0
28	3.0	2.5	2.0	1.9	1.7	1.5
90	2.4	2.0	1.6	1.5	1.4	1.2
365	1.8	1.5	1.2	1.1	1.0	1.0

$d_m$  – replacement thickness of the circular section ( $d_m = 2r_m$ )

Total values ( $\varphi_{tot}$ ).are given in Table 3. The values ( $\varphi_{tot}$ )in the table mostly depend on the initial time ( $\tau_0$ ), then relative humidity (RH%), and mean section thickness ( $d_m$ ). In EC2, it is considered that the dry environment at relative humidity RH = 50%, and if the structure is in the open space, then RH = 80%.

Instead of Table 3, the following diagram can be used to estimate the creep of concrete:



**Fig.1 Total values of the concrete creep coefficient ( $\phi_{tot}$ ) as a function of initial time ( $\tau_0$ )**

Curves  $A_1B_1$  and  $A_2B_2$  were drawn taking values from Table 3. The division of the zone under the drawn curve (1) into two parts was adopted, the part under the curve (1) up to the dotted line  $CB_1$  and the part between that line and the axis  $\tau_0$ . The first part refers to the coefficient of creep depending on the theory of aging of concrete, and the second in relation to the theory of heritage of concrete.

For the curve (2), which refers to the humidity of the environment HP 80%, the partition of its zone under the curve (2) in relation to the dotted line  $CB_2$  is valid. (Alt.: For environmental conditions in the region of Serbia, the position of point C on the abscissa  $\tau_0 = 90$  days can be chosen).

The description of the validity of concrete theories is the same as for the description of the curve (1), but in relation to the line  $CB_2$ .

The overall effect of concrete creep is in full agreement with EC2 recommendations [2]. The method of using a similar diagram is shown in more detail in the author's published work (see: [18])

**b) Creep coefficients in time  $t < t_n$**

For the purposes of practice, it is necessary to know the values of creep coefficients ( $\phi$ ) during the observation time, which is less than its final value  $t_n$  (365 days). Such a need appears in the case of phased construction of RC, PC and CS structures, control of force, loss of force in cables due to prestressing, when caused by displacement of structural supports, etc.

In cases where greater accuracy of the creep and shrinkage effects of concrete is required in practice, EC2 recommended a 'creep function':

$$J(t, \tau_0) = 1 / E_c(t_0) + \Phi(t, \tau_0) / E_{28} \tag{2}$$

, and creep coefficient :

$$\Phi(t, \tau_0) = \Phi_0 \beta_c(t - t_0) \tag{3}$$

,wherein:

- $\Phi_0$  – the final value of the creep coefficient
- $\beta_c(t_1 - t_0)$  – coefficient showing the increase in creep strains in depending on the duration of the constant load.

The coefficient  $\beta_c$  is represented in EC2 by the expression:

$$\beta_c(t_1 - t_0) = [(t - t_0) / (\beta_H + (t - t_0))]^{0.3} . \tag{4}$$

The structure of expressions (3) and (4) and the given constants show that they have an apparently only empirical character. It is known that the empirical form is adopted for expressions, most often, if these terms do not have a physical meaning. The expression  $J(t, \tau_0)$  was previously called a specific function of the total strains of the samples (denoted in my paper by  $\delta_v(t_1 - \tau_0)$ ) [20].

**c) Final strains values of the concrete shrinkage  $\epsilon_{sn}$**

Concrete shrinkage is an appearance that takes place in parallel with the development of concrete creep. The total (or final) values ( $\epsilon_{sn}$ ) of shrinkage strains are given in Table 4. depending on the relative humidity of the environment and the mean thickness of the cross section.

**Table4. Final strains values of the concrete shrinkage  $\epsilon_{sn}$  ( EC2 )**

RH(%)	Nominal size for section $d_m = 2A_c/u$ (cm)	
	$\leq 15.0$	$\leq 60.0$
50	0.60	0.50
80	0.33	0.28

$\epsilon_{sn} = \text{abs}(\epsilon_{sn})$  - the absolute value of the constant valid for  $t = t_n$  (365 days)

If  $t < t_n$  an appropriate reduction should be made in proportion to  $\varphi(t)/\varphi_n$ .

CS – Composite structure

The following expression applies to strains due to shrinkage at any time of observation (t):

$$\epsilon_s(t) = -\varphi(t)\epsilon_n/\varphi_n \tag{5}$$

The

sign (-) indicates that the value corresponds to negative strains in concrete (shrinkage).

**4.3 PBAB '87 [2]**

If experimental (measured) data for creep coefficients are not available, the provisions entered in PBAB'87 (Yu-code) can be of great help to designers and researchers in the region of Serbia.

**a) Total values of creep coefficient for  $t=t_n$**

Table 5 shows the final values of creep coefficients ( $\varphi_{tot}$ ). The values are given depending on the initial loading time of the structure ( $\tau_0 = 7, 14, 28, 90, 365, 3$  years), relative humidity of the environment (HP%) and the medium thickness of the replacement cross section of concrete element ( $d_m = 2 r_m$ ).

**Table5. Total values of the creep coefficients  $\varphi_{tot}$  for concrete (PBAB'87)**

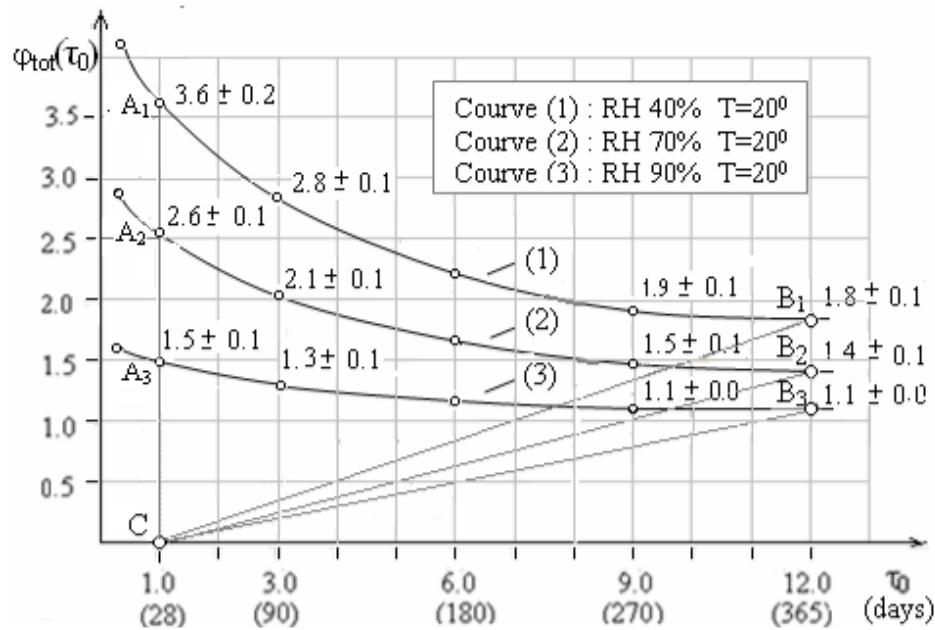
$\tau_0$ (days)	$r_m$ (cm)	Relative humidity (RH)			
		40%	70%	90%	In the water
7	$\leq 10$	4.3	3.1	1.7	1.4
	20	4.1	2.9	1.6	
	$\geq 40$	3.8	2.7	1.6	
14	$\leq 10$	4.0	2.9	1.6	1.3
	20	3.8	2.7	1.5	
	$\geq 40$	3.6	2.5	1.5	
28	$\leq 10$	3.7	2.6	1.6	1.3
	20	3.6	2.6	1.5	
	$\geq 40$	3.4	2.5	1.4	
90	$\leq 10$	2.9	2.1	1.3	1.2
	20	2.8	2.1	1.3	
	$\geq 40$	2.7	2.0	1.3	
365	$\leq 10$	2.0	1.5	1.1	1.0
	20	1.8	1.4	1.1	
	$\geq 40$	1.7	1.3	1.0	
3 years	$\leq 10$	1.2	1.0	0.8	0.8

	20	1.1	0.9	0.8	
	≥ 40	0.9	0.8	0.7	

Note:  $d_m$  – replacement thickness of the circular section ( $d_m=2r_m$ )

Total values ( $\varphi_{tot}$ ) are given in Table 5. The values ( $\varphi_{tot}$ ) in the table mostly depend on the initial time ( $\tau_0$ ), then relative humidity (HP%), and mean section thickness ( $d_m$ ). In PBAB'87, it is considered that the dry environment at relative humidity is PH = 40%, if the structure is in the open space PH = 70%. If the structure is directly above the water then it is H=90%.

Instead of a table, this data can also be seen in Fig. 2 in the form of a diagrams.



Note: (+) for  $d \leq 10\text{cm}$  ; (-) for  $d \geq 40\text{cm}$  ;  $d = 2.A_c/u$  [P'87].

**Fig.2 Total values of concrete creep coefficient( $\varphi_{tot}$ ) as a function of initial time ( $\tau_0$ )**

The total values of the creep coefficient of concrete ( $\varphi_{tot}$ ) and the proposal of its distribution by means of straight line CB1, CB2 and CB3 are drawn in the shown Fig.2. The detailed description of obtaining the diagram is exactly the same as for the previous Fig.1.

It can be pointed out that the regulations (PBAB'87) provide a parallel possibility to calculate the value for ( $\varphi_{tot}$ ) if the observation time is  $t_1 < t_n(365)$ , i.e. according to the theory of aging. The reduced value of the coefficient ( $\varphi_{tot}$ ) according to the Table5 for relations  $\varphi_t / \varphi_n$  , as a function of observation time, can be seen in the regional code (PBAB'87). The table should be given only as a function of creep coefficients, because the influence of the thickness of the cross-section has already been introduced in Table 5 and in Fig. 2.

**b) Final strains values of concrete shrinkage  $\epsilon_{sn}$**

Shrinkage of concrete often occurs in parallel with the development of creep strains. The final values of concrete shrinkage expansions ( $\epsilon_{sn}$ ) are given in Table.6 depending on the relative humidity and the replacement thickness of the elements. It is assumed that the average temperature of the environment is 20°C.

**Table6. Final values of strains ( $\epsilon_{sn}$ ) due to concrete shrinkage (P'87)**

$r_m$ (cm)	Relative humidity (RH)			
	40%	70%	90%	In water
≤ 10	0.56	0.40	0.15	0.00
≥ 40	0.48 0.42	0.34	0.12	
		0.30	0.10	



Concrete shrinkage often takes place in parallel with the development of creep strains. The final values of strains of concrete shrinkage ( $\epsilon_{sn}$ ) are given in Table.6 depending on the relative humidity and thickness of the elements. The mean medium temperature is assumed to be 20°C.

The same effects of concrete are observed as in the case of concrete creep. If the concrete structure is outdoors, it is considered to be in relative humidity HP = 70%. If the structure is directly above water, the relative humidity is higher HP = 90%. Internal structures (buildings, halls, etc.) are located in a dry environment HP = 40% [PAB'87].

## V. CREEP COEFFICIENTS IN THE REFERENCES

The information in this part is intended to facilitate the requirements of designers, researchers, builders of structures and to better understand the work on the adoption of appropriate data. For this purpose, a concise form of selected well-known propositions, according to the opinion of the author of the paper, for the creep coefficient functions will be presented. The symbols in the formulas are the

### 5.1 Proposal of Rüsç [2]

The proposal belongs to a group of expressions that are formed as a possible form of adjusting mathematical expressions to be as close as possible to the real elastic and viscous behavior of concrete.

Based on tests of creep and shrinkage of concrete, Rüsç proposed the following formula for the coefficient of creep:

$$\varphi(t) = \beta_0(t_0) + \varphi_d \beta_d(t-t_0) + \varphi_f [\beta_f(t) - \beta_f(t_0)] \quad (6)$$

The proposal contains in expression (6) three members, quite similar to the proposed rheological model, for the modified Burger's rheological model ( ) for the theor  $\bar{E}_u$  hereditary age of concrete [19]. The first term of the expression, which does not contain Burger's model, depends on the strength of the concrete at time  $t_0$ , the second term on the properties of the inheritance and the third term on the aging properties of the concrete. Individual articles can be seen in more detail in the paper mentioned in the footnote. They also depend on a larger number of numerical coefficients. The proposal is significant mostly because of the transfer of experience on how to accomplish a large-scale experimentally set task.

The proposal of Rüsç and a group of his collaborators have been accepted by the committee \*CEB-FIP, as a future basis for Eurocode recommendations for regional regulations.

### 5.2 Proposal of Bazant [8]

The proposal belongs to the same group of researchs as described for Rüsç. This proposal caused significant interest in the professional and scientific public. It is based on the search for the concrete creep coefficient function during the observation time (t) so that it can define the conditions of real behavior of concrete during experimental testing of concrete samples and structures.

In the EC2 model, a general algebraic relation  $\sigma$ - $\epsilon$  for concrete is recommended:

$$\epsilon(t, t_0) = \epsilon_n(t) + \sigma(t_0) J(t_0, t_0) + [\sigma(t) - \sigma(t_0)] [1/E(t_0) + \chi \phi(t, t_0) / E28] \quad (7)$$

Instead of this relationship, Bazant defines an incremental expression for stresses in concrete:

$$\Delta\sigma = E''(t, t_0) [\Delta E(t) - \Delta E''(t)] \quad (8)$$

\* CEB-FIP International Recommendations, JGC, Belgrade, 1978.

, where is the 'fictitious' modulus of elasticity

$$E''(t, t_0) = E(t_0) / [1 + \chi(t, t_0) \cdot \Phi(t, t_0)] \quad (9)$$

The well-known researcher Trost proposed an expression for the measure of creep of concrete, which he called the 'creep function', in the form adopted in EC2'90 and PBAB'87 regulations according to expression (2). The discussion on the correctness of the name is given in the paper of the author [20]. Expression (9) depends on the concrete aging coefficient  $\chi(t, t_0)$ . In the book of the author [14], it was shown that the recommended values for  $\delta_v(t, \tau_0)$ , of several authors can give wrong results for the displacements and stresses in AB and PB structures.

An expression for the creep coefficient was recommended by Bazant:

$$\Phi(t, t') = \Phi_u(t') (t - t')^{0.6} / [10 + (t - t')^{0.6}] \quad (10)$$



,where is

$$\Phi_u(t') = \Phi(\infty, 7) 1.25 t'^{-0.118} \quad (11)$$

Formulas (10) and (11) have an empirical character because they are not based on rheological models. Proposal contains constants that have, meseem, no physical meaning. Expression (10) emphasizes the hereditary properties of concrete because it contains only the difference  $(t - t')$ . A similar solution was proposed by Trost based on the consideration of his experimental researchs and rheological models [7]

In the work of the author, the influence of the creep coefficient on the behavior of concrete samples was analyzed in detail and it was determined that the results for stresses in concrete elements for higher values of creep coefficients  $\phi(t)$  are uncertain, as can be seen in the book [14].

### 5.3 Proposal of the author in this paper [19]

There are many proposals for a total creep coefficient, which are more or less successful, to present the solution with functions that can be easily accepted. The main reason(probably) is because the source of the solution is not visible, or the derivation way of expressions is unclear. Therefore, someone should strive for the simplest possible forms of expression with a clear physical meaning, easy to understand and work in practice.

In dissertation, the author gave preference to boundary theories: Theory of Aging (1974) and Theory of Heritage (1978). Applying these theories, explicit solutions for deformations and stresses in RC, PC and Composite structures were obtained. Several forms of monotonically variable loads and proposed relations (Rusch, Ivkovic, Jordan) were observed. Solving statically determinate and indeterminate composite structures has exactly, the same or similar workflow, as in the theory of elastic structures.

Following the above explanation, proposals were formed for the concrete creep coefficient functions  $\phi(t)$  for three concrete theories. It should be noted that from Dishiger (1937) until the adoption of the CEB-FIP recommendations (1978), the designation  $\phi(t)$  was by far the most widely used in works in the field of civil engineering, so now, for the sake of easier understanding, it has been given priority over  $\phi(t)$  in the previous description ( under 5.2.).

#### a) Young concrete (According to the theory of aging-Viscoplastic material)

The theory of aging is the most widely used in practice so far. The following form is proposed for the measure creep function (specific value of strain  $\delta_v(t, \tau_0)$ ) and the creep coefficient function  $\phi(t)$ :

$$\begin{aligned} \delta_v(t, \tau_0) &= [\phi(t) - \phi(\tau_0)] / E_0 \\ \phi(t) &= \phi_n (1 - e^{-\beta_n t}) ; \quad \phi_n = \phi_{tot} \text{ (From codes see: Fig.1 or Fig 2)} \\ \text{Valid for : } &7 \leq \tau_0 \leq 90 \text{ days} \quad (\text{see: [19] [20]}) \end{aligned} \quad (12)$$

The \*parameter  $\beta_n = \beta_n(\tau_0)$  depends on the selected initial loading time  $(\tau_0)$ . It can be taken from Table 7. depending on its value  $(\tau_0)$ :

**Table 7. Data for initial time  $(\tau_0)$  and its parametar  $\beta_n$  to find ratios  $\phi(t) / \phi_n$**

$\tau_0$ [days]	$\beta_n(\tau_0)$ [1/days]	Time of Load duration (t) [days]					
		7	14	28	90	180	365
7	0.04	0.242	0.429	0.674	0.973	0.999	1.000
14	0.03	0.190	0.343	0.568	0.939	0.996	0.999
28	0.02	0.131	0.214	0.429	0.850	0.973	0.999
90	0.015	0.086	0.165	0.343	0.741	0.933	0.996
180	0.0125	0.084	0.160	0.295	0.675	0.895	0.989

Work procedure: You need to select the value  $(\tau_0)$ , then in the same row  $(\beta_n)$  [1/day], and at the time of observation time  $(t_i)$ , you can find  $\phi(t_i) / \phi_n$ . See expression (12) to find  $\phi(t_i)$ .

The table was formed on the basis of test data for concrete samples, in a paper in which the characteristic values of  $(\beta_n)$  were only for three values  $\tau_0$  [19].

How they can be obtained by interpolating new values of  $(\beta_n)$  for new  $(\tau_0)$  will be given in the procedure in the example at the end of this paper.

The final value of  $\phi_n$  should be taken from the tables (or diagrams) in the regulations under point 4. 2 or point 4. 3 if we do not have sample test data.

When the selected observation value ( $t$ ) is not in the table, the appropriate interpolation between adjacent values can be performed.

Formula for Modified Maxwell's model corresponds to young concrete is ([9] [19]):

The coefficients  $\beta_n$  and  $E_0$  according to these recommendations and earlier [19] [21].

In short, it can be said: In the theory of aging, the behavior of concrete in area I ( $\tau_0 \leq 90$  days) is idealized as a viscoplastic material, which corresponds to the modified Maxwell model with changing body characteristics over time.

**b) Old concrete** (According to the heritage theory - viscoelastic material)

The expressions for the for the measure creep function (specific value of strain  $\delta_v(t, \tau_0)$ ) and the creep coefficient function  $\varphi(t)$ , which are quite simple, were proposed in a new form and with different application criteria:

$$\begin{aligned} \delta_v(t - \tau_0) &= \varphi_\infty [1 - e^{-\beta_\infty(t - \tau_0)}] / E_\infty ; \\ \varphi(t - \tau_0) &= \varphi_\infty (1 - e^{-\beta_\infty(t - \tau_0)}); \varphi_\infty = \varphi_{tot} \text{ (From codes see: Fig.1 or Fig 2)} \\ \text{Valid for: } &\tau_0 \geq 365 \text{ days and } t > \tau_0 \quad (\text{see: [19] [20]}) \end{aligned} \quad (13)$$

\*alt. name for the coefficient ( $\beta$ )

Modified viscoelastic model corresponds to old concrete [19] :

Constants  $\varphi_\infty$ ,  $\beta_\infty$  and  $E_\infty$  should be taken according to an earlier presentation [19].

In short, it can be said: In the heritage theory, the behavior of concrete in area III is idealized (for  $\tau_0 > 365$  days) as a viscoelastic material, which corresponds to a modified elastically extended model ( ) with characteristics of the body over time.

**c) medium-aged concretes** (Heritage theory of aging - 'bimaterial')

The heritage of aging theory was considered the most complete because it takes into account the aging properties and the inherited properties of concrete. In addition to several proposals made (see: [5] [1]), there are still large deviations of experimental and theoretical values for stresses and strains of concrete in laboratories, calculations of displacement of structures and displacements registered on structures. The name 'bimaterial' refers to its dual properties according to the theory of aging and according to the theory of heritage.

One of the successful proposals is Burger's rheological model in A. Naaman's book [16]. In this model, the characteristics of concrete are given as constants. However, there are more attempts to show them as functions of time [13].

In this paper, the opinion that the Burger's model is the best, but with variable characteristics for the modulus of elasticity and the coefficient of viscosity in one Newtonian body, is maintained for medium-aged concrete [19] [11].

Therefore, the Modified Burger's model is suitable for medium-aged concretes whose structural formula is [19]:

$$\tilde{E}u = \tilde{M} + \tilde{K}$$

Then the sum of the creep measure function is  $\delta v(t, \tau_0)$ , and in particular, the partial functions of the creep coefficients according to the theory of aging  $\varphi(t)$  and the theory of heritage  $\varphi(t - \tau_0)$  were found in the observation time ( $t$ ) (see: [18]):

$$\begin{aligned} \delta_v(t, \tau_0) &= [\varphi(t) - \varphi(\tau_0)] / E_0 + \varphi_\infty [(1 - e^{-\beta_\infty(t - \tau_0)}) / E_\infty] \\ \text{, where is } &\varphi(t) = \varphi_n (1 - e^{-\beta_n t}); \quad \varphi(t - \tau_0) = \varphi_\infty (1 - e^{-\beta_\infty(t - \tau_0)}) \\ \text{, end } &\varphi_n = k_n \cdot \varphi_{tot}; \quad \varphi_\infty = k_\infty \cdot \varphi_{tot} \text{ (From codes see: Fig.1 or Fig 2)} \\ \text{Valid for: } &90 \text{ days} < \tau_0 \leq 365 \text{ days} \\ \text{Check: } &\varphi_{tot} = \varphi_n + \varphi_\infty \quad (\text{see: [19] [20]}) \end{aligned} \quad (14)$$

The final values of the creep coefficient  $\varphi_n$  and  $\varphi_\infty$  are obtained from (14) by applying the partial factors  $k_n$  and  $k_\infty$  for the values found under (12) and (13) [11].

In summary, it can be said: In the theory of hereditary age, the behavior of concrete is idealized, the behavior of concrete in area II ( $90 < \tau_0 \leq 365$  days) is parallel idealized as 'bimaterial' (a viscoplastic and

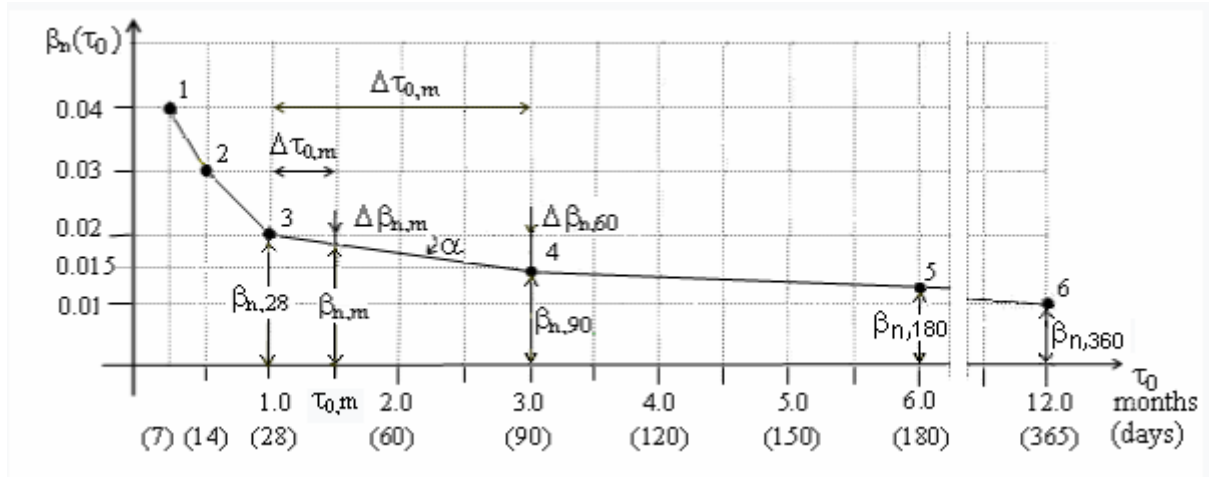
viscoelastic material), which corresponds to a modified Burger’s model ( ) with changing characteristics over time in its ( ) model.

The application of concrete theories corresponding to the proposed modified rheological models is a reliable way to more accurately estimate deformations and stresses under long-term loads[14].

**d) New values for 'fluidity' coefficients and creep coefficients**

In the author's work, which dealt with the determination of the rheological characteristics of concrete, the task was, for three series of samples with different ages of concrete, i.e. with different initial times ( $\tau_0$ ), to find the appropriate value for  $\beta_n(\tau_0)$  and the limit value of the creep coefficient  $\varphi_n(t)$  when  $t \rightarrow t_n$  [19].

How it can be obtained by interpolation new values of  $\beta_n(\tau_0)$  for any new ( $\tau_0$ ) will be given within the procedure in the example at the end of this text.



**Fig.3 Diagram of the 'fluidity' coefficient  $\beta_n(\tau_{0m})$  depending on the initial loading time of the concrete samples ( $\tau_0$ )**

Now only for characteristic days of age of concrete (node points)  $\tau_0=7, 14, 28, 90, 180$  and 365 days in Table 7 was formed, similar to the one in the paper [19]. It was used to create Fig. 3, which is adapted to the search for the 'fluidity' coefficient between the nodal points.

The procedure of linear interpolation between nodal points 3 and 4 is applied, which relates to the behavior of concrete according to the theory of aging (area I). The same can be done for the part between the points from 1 to 4 for theory of aging, and from point 4 to 6 for 'bimaterial' behavior of concrete. By applying the well-known linear interpolation for any value  $\tau_0=\tau_{0m}$ , the 'fluidity' coefficient  $\beta_n(\tau_{0m})$  is easily obtained, which is shown in this picture and in the numerical example.

**Example:** Coefficient of 'fluidity' and coefficient of creep

Find:  $\beta_n(\tau_{0m})=?$ ,  $\varphi_n(t_1)=?$

Date:  $\tau_{0m}=45$  dana;  $\beta_n(\tau_{0,28})=0.020$ ;  $\beta_n(\tau_{0,90})=0.015$ ;  $\varphi_n=1.94$  (for  $t \rightarrow t_n$  experimental result)

Results (see: Fig. 3):

$$\Delta\tau_{0m} = \tau_{0m} - \tau_{028} = 45 - 28 \approx 15 \text{ (days)}$$

$$\Delta\tau_{090} = \tau_{090} - \tau_{028} = 90 - 28 \approx 60 \text{ (days)}$$

$$tg\alpha_m = \Delta\beta_n(\tau_{0,90}) / \Delta\tau_{0,90} = \text{abs}(-0.015+0.020) / 60 = 8.333 \times 10^{-5}$$

Coefficient of 'fluidity':

$$\beta_{n,45} = \beta_{n,28} - \Delta\tau_{0m} tg\alpha_m = 0.02000 - (0.02000 - 15 \times 8.333 \times 10^{-5}) = 0.01875 \text{ (1/days)}$$

$$(\Delta\tau_{0m} tg\alpha_m = 0.00125)$$

Coefficient of creep in time  $t_1=28$  (days):

$$\varphi(t_1) = \varphi_n (1 - e^{-\beta_n t_1}) = 1.94 (1 - \exp(-0.0200 \times 28)) = 1.94 \times 0.429 = 1.65$$

Instead of the expression for  $\varphi(t)$  the same is obtained using the factor  $\varphi(t) / \varphi_n = 0.429$  from Tabele 7.

**VI. CONCLUSIONS**

Based on the above, the following conclusions are formed:

1. The final values of the creep coefficient function ( $\varphi_{tot}$ ), are given in the tables of recommendations EC2 ('90) and regional code PBAB ('87). When experimental results for ( $\varphi_{tot}$ ), are not available, their

appearance provides a valuable opportunity to be used for various calculation tasks in concrete laboratories, design, calculation and research of concrete structures.

2. Diagrams of creep coefficients ( $\phi_{tot}$ ), are drawn according to the data of the mentioned recommendations, depending on the initial loading time ( $\tau_0$ ). They facilitate the selection of areas in which it is important that the concrete is young, old and middle-aged.

3. Direct application of creep coefficients ( $\phi_{tot}$ ), on the basis of the presented tables and diagrams, practical work in assessing the deformations and stresses of concrete structures due to creep and shrinkage of concrete can be significantly facilitated.

4. The application of regional regulations, if they rely on empirical methods of determining the value of the creep coefficient, sometimes have large errors in the calculation of stresses and deformations of concrete [ 1] [ 8 ].

5. The presentation of three references on the adopted conception of the creep coefficient function, which should be in agreement with the rheological models, facilitates the acceptance of concrete theories.

6. It is necessary to continue researching the application criteria of concrete theories for different regions, then for climatic and other conditions for different types of RC, PC and CS structures.

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