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Research Paper

Existence of periodic solutions to impulsive Cohen-Grossberg shutting inhibitory cellular neural networks on time Scales

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ABSTRACT: *In this paper, by using the continuation theorem of coincidence degree theory, some criteria are established for the existence of periodic solution for a Cohen-Grossberg shutting inhibitory cellular neural networks (CGSICNNs) with variable coefficients and impulses on time scales.*
KEYWORDS: *Cohen-Grossberg shutting inhibitory cellular neural networks; Periodic solution; Impulse; Time scale*

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I. INTRODUCTION

Consider the following Cohen-Grossberg shunting inhibitory cellular neural networks (CGSICNNs) with delays:

$$\begin{cases} \dot{x}_{ij}(t) = -a_{ij}(x_{ij}(t))\Big\{b_{ij}(x_{ij}(t)) + \sum\limits_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t) f_{ij}(t, x_{kl}(t-\tau_{kl}(t)))x_{ij}(t) - L_{ij}(t)\Big\}, t\geqslant 0, \\ x_{ij}(t) = \varphi_{ij}(t), t\in[-\tau,0], i=1,2\ldots,n, j=1,2\ldots,m, \end{cases}$$

where $\tau_{ij}(t)$ represents axonal signal transmission delays and is continuous with $0\leqslant\tau_{ij}(t)\leqslant\tau$; $C_{ij}(t)$ denotes the cell at the (i,j) position of the lattice at the t ; the r -neighborhood $N_r(i,j)$ of $C_{ij}(t)$ is

$$N_r(i,j) = \{C_{kl}(t) : \max(|k-i|,|l-j|)\leqslant r, 1\leqslant k\leqslant n, 1\leqslant l\leqslant m\},$$

$x_{ij}(t)$ is the activity of the cell $C_{ij}(t)$; $L_{ij}(t)$ is the external inputs to $C_{ij}(t)$; $a_{ij}(x_{ij}(t))$ and $b_{ij}(x_{ij}(t))$ represent an amplification function at time t and an appropriately behaved function at time t, respectively; $C_{ij}^{kl}(t)\geqslant 0$ is the connection or coupling strength of postsynaptic activity of the cell transmitted to the cell C_{ij} ; the activity functions $f_{ij}(t,\cdot)$ are continuous functions representing the output or firing rate of the cell $C_{kl}(t)$, $\varphi_{ij}(t)$ are the initial functions.

Since Bouzerdout and Pinter in [4-6] described SICNNs as a new cellular neural networks (CNNs), SICNNs have been extensively applied in psychophysics, speech, perception, robotics, adaptive pattern recognition, vision, and image processing. Hence, they have been the object of intensive analysis by numerous authors in recent years. In particular, there have been extensive results on the problem of the existence and stability of periodic and almost periodic solutions of SICNNs with constant time delays and time-varying delays in the literature. We refer the reader to [7-10,13-15] and the references cited therein. Moreover, it is well known that the discrete systems are more important than their continuous counterparts in implementing and application. In addition, it is essential to formulate discrete-time counterparts of the continuous-time functional differential systems when one wants to simulate or compute the continuous-time systems after we obtained its dynamical characteristics, but it is troublesome to study the existence and stability of periodic solutions for continuous and discrete systems respectively. Therefore, it is meaningful to study that on time scale which can unify the continuous and discrete situations.

However, to the best of our knowledge, few authors have considered CGSICNNs with delays on time scales. The main purpose of this paper is to study the existence of periodic solution of the following Cohen-Grossberg shutting inhibitory cellular neural networks with variable coefficients and impulses on time scales:

$$\begin{cases} x_{ij}^{\Delta}(t) = -a_{ij}(x_{ij}(t))\left\{b_{ij}(x_{ij}(t)) + \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t) f_{ij}(t, x_{kl}(t-\tau_{kl}(t)))x_{ij}(t) - L_{ij}(t)\right\}, \\ \qquad t, t_k \in \mathbb{T}, t \geq 0, t \neq t_k, i = 1,2,\ldots,n, j = 1,2,\ldots,m, \\ \triangle x_{ij}(t_k) = x_{ij}(t_k^+) - x_{ij}(t_k^-) = e_k(x_{ij}(t_k)), i = 1,2,\ldots,n, j = 1,2,\ldots,m, k = 1,2,\ldots, \end{cases} \tag{1.1}$$

where $\mathbb{T}$ is an ω-periodic time scale, $e_k : \mathbb{R} \to \mathbb{R}$ is continuous and there exists a positive integer q such that $t_{k+q} = t_k + \omega, e_{k+q}(\cdot) = e_k(\cdot), t_{k+1} > t_k, \lim_{k\to\infty} t_k = \infty$, $\triangle x_{ij}(t_k) = x_{ij}(t_k^+) - x_{ij}(t_k^-)$ are the impulses at moments t_k, for $t_k \neq 0 (k = 1,2,\ldots)$, $[0,\omega]_{\mathbb{T}} \cap \{t_k\} = \{t_1, t_2, \ldots, t_q\}$.

The initial conditions of system (1.1) are of the form

$$x_{ij}(s) = \phi_{ij}(s) \neq 0, s \in [-\tau, 0], i = 1,2,\ldots,n, j = 1,2,\ldots,m,$$

where $\phi_{ij} \in C([-\tau,0], \mathbb{R}^{n\times m}), \tau = \max_{1\leqslant k\leqslant m, 1\leqslant l\leqslant n}\{\max_{t\in[0,\omega]_{\mathbb{T}}} \tau_{kl}(t)\}$.

For the sake of convenience, we denote

$$\overline{g} = \max_{t\in[0,\omega]_{\mathbb{T}}} |g(t)|, \underline{g} = \min_{t\in[0,\omega]_{\mathbb{T}}} |g(t)|, \|g\|_2 = \left(\int_0^{\omega} |g(t)|^2 \Delta t\right)^{\frac{1}{2}}, \tilde{g} = \frac{1}{\omega}\int_0^{\omega} g(t)\Delta t,$$

where g is an ω-periodic function.

Throughout this paper, we assume that:

$(\mathrm{H}_1) a_{ij}(\cdot) \in C(\mathbb{R}, \mathbb{R}^+), L_{ij}, C_{ij}^{kl} \in C(\mathbb{T}, \mathbb{R}), (i = 1,2,\ldots,n, j = 1,2,\ldots,m)$ are ω-periodic functions, $0 < \underline{a}_{ij} \leqslant a_{ij}(\cdot) \leqslant \overline{a}_{ij} < \frac{1}{\omega}; b_{ij}(\cdot) \in C(\mathbb{R}, \mathbb{R})$ are delta differential and $0 < \rho_{ij} \leq b_{ij}^{\Delta}(\cdot) \leq \delta_{ij}$, $b_{ij}(0) = 0$.

$(\mathrm{H}_2) f_{ij}(t,\cdot), i = 1,2,\ldots,n, j = 1,2,\ldots,m$ are continuous ω-periodic functions respect to t and there exist constants $f_{ij}^M > 0$ such that $|f_{ij}| < f_{ij}^M$.

$(\mathrm{H}_3) e_k \in C(\mathbb{R}, \mathbb{R})$ are bounded functions, $k = 1,2,\ldots$.

(H_4) There exists positive constants K_{ij} such that $|f_{ij}(t,x)-f_{ij}(t,y)|\leqslant K_{ij}|x-y|$, for all $t\in\mathbb{T}, i=1,2,\ldots,n, j=1,2,\ldots,m.$

II. PRELIMINARIES

According The theory of time scales, one may see [12].

Definition 2.1 ([12]) For each $t\in\mathbb{T}$, let N be a neighborhood of t. Then, for $V\in C_{rd}[\mathbb{T}\times\mathbb{R}^n,\mathbb{R}^+]$, define $D^+V^\Delta(t,x(t))$ to mean that, given $\varepsilon>0$, there exists a right neighborhood $N_\varepsilon\subset N$ of t such that

$$\frac{1}{\mu(t,s)}[V(\sigma(t),x(\sigma(t)))-V(s,x(\sigma(t)))-\mu(t,s)f(t,x(t))]<D^+V^\Delta(t,x(t))+\varepsilon$$

for each $s\in N_\varepsilon, s>t$, where $\mu(t,s)\equiv\sigma(t)-s.$ If t is right-scattered and $V(t,x(t))$ is continuous at t, this reduces to

$$D^+V^\Delta(t,x(t))=\frac{V(\sigma(t),x(\sigma(t)))-V(t,x(\sigma(t)))}{\sigma(t)-t}.$$

Definition 2.2 ([11]) We say that a time scale $\mathbb{T}$ is periodic if there exists $p>0$ such that if $t\in\mathbb{T}$, then $t\pm p\in\mathbb{T}$. For $\mathbb{T}\neq\mathbb{R}$, the smallest positive p is called the period of the time scale.

Definition 2.3 ([11]) Let $\mathbb{T}\neq\mathbb{R}$ be a periodic time scale with period p. We say that the function $f:\mathbb{T}\to\mathbb{R}$ is periodic with period ω if there exists a natural number n such that $\omega=np,\ f(t+\omega)=f(t)$ for all $t\in\mathbb{T}$ and ω is the smallest number such that $f(t+\omega)=f(t)$.

If $\mathbb{T}=\mathbb{R}$, we say that f is periodic with period $\omega>0$ if ω is the smallest positive number such that $f(t+\omega)=f(t)$ for all $t\in\mathbb{T}$.

Lemma 2.1 ([2]) Assume that $p,q:\mathbb{T}\to\mathbb{R}$ are two regressive functions, then

$$(i) e_p(\sigma(t),s)=(1+\mu(t)p(t))e_p(t,s);$$
$$(ii) \frac{1}{e_p(t,s)}=e_{\ominus p}(t,s);$$
$$(iii) e_p(t,s)=\frac{1}{e_p(s,t)}=e_{\ominus p}(s,t);$$
$$(iv) e_p(t,s)e_p(s,r)=e_p(t,r).$$

Lemma 2.2 ([2]) Assume that $f,g:\mathbb{T}\to\mathbb{R}$ are delta differentiable at $t\in\mathbb{T}^k$. Then

$$(fg)^\Delta(t)=f^\Delta(t)g(t)+f(\sigma(t))g^\Delta(t)=f(t)g^\Delta(t)+f^\Delta(t)g(\sigma(t)).$$

Lemma 2.3 If $a,b\in\mathbb{T},\alpha,\beta\in\mathbb{R}$ and $f,g\in C(\mathbb{T},\mathbb{R})$, then

(i) $\int_a^b [\alpha f(t)+\beta g(t)]\Delta t = \alpha\int_a^b f(t)\Delta t + \alpha\int_a^b g(t)\Delta t;$

(ii) If $f(t)\geq 0$ for all $a\leq t<b$, then $\int_a^b f(t)\Delta t\geq 0$;

(iii) If $|f(t)|\leq g(t)$ on $[a,b):=\{t\in\mathbb{T}: a\leq t<b\}$, then $|\int_a^b f(t)\Delta t|\leq\int_a^b g(t)\Delta t$.

The proofs of the following lemmas can be found in [1,3,17], respectively.

Lemma 2.4 Let $t_1,t_2\in[0,\omega]_{\mathbb{T}}$. If $x:\mathbb{T}\to\mathbb{R}$ is ω -periodic, then

$$x(t)\leqslant x(t_1)+\int_0^{\omega}|x^{\Delta}(s)|\,\Delta s,\ x(t)\geqslant x(t_2)-\int_0^{\omega}|x^{\Delta}(s)|\,\Delta s.$$

Lemma 2.5 (Cauchy-Schwarz inequality) Let $a,b\in\mathbb{T}$. For rd-continuous functions $f,g:[a,b]\to\mathbb{R}$ we have

$$\int_a^b|f(t)g(t)|\,\Delta t\leqslant\left(\int_a^b|f(t)|^2\,\Delta t\right)^{\frac{1}{2}}\left(\int_a^b|g(t)|^2\,\Delta t\right)^{\frac{1}{2}}.$$

Lemma 2.6 (compact result [18]) Assume that $\{f_n\}_{n\in N}$ is a function sequence on J such that

(i) $\{f_n\}_{n\in N}$ is uniformly bounded on J;

(ii) $\{f_n^{\Delta}\}_{n\in N}$ is uniformly bounded on J.

Then there is a subsequence of $\{f_n\}_{n\in N}$ converges uniformly on J.

Lemma 2.7 (Mean value theorem) Let function f be continuous on $[a,b]_{\mathbb{T}}$ and delta differentiable on $[a,b)_{\mathbb{T}}$, then there exist ξ, $\varsigma\in[a,b)_{\mathbb{T}}$ such that

$$f^{\Delta}(\xi)(b-a)\leq f(b)-f(a)\leq f^{\Delta}(\varsigma)(b-a).$$

III. MAIN RESULTS

In this section, by using the Mawhin's continuation theorem, we shall study the existence of at least one periodic solutions of system (1.1).

Let X,Y be normed Banach spaces, $L:\mathrm{Dom}L\subset X\to\dim Y$ be a linear mapping, and $N:X\to Y$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim\mathrm{Ker}L=\mathrm{codim}\mathrm{Im}L<+\infty$ and $\mathrm{Im}L$ is closed in Y. If L is a Fredholm mapping of index zero and there exist continuous projector $P:X\to X$ and $Q:Y\to Y$ such that $\mathrm{Im}P=\mathrm{Ker}L,\mathrm{Ker}Q=\mathrm{Im}(I-Q)$, it follows that mapping $L|_{\mathrm{Dom}L\cap\mathrm{Ker}P}:(I-P)X\to\mathrm{Im}L$ is invertible. We denote the inverse of that mapping by K_P. If Ω is an open bounded subset of X, the mapping N will be called L -compact on $\bar{\Omega}$, if

$QN(\bar{\Omega})$ is bounded and $K_P(I-Q)N:\bar{\Omega}\to X$ is compact. Since $\mathrm{Im}Q$ is isomorphic to $\mathrm{Ker}L$, there exists an isomorphism $J:\mathrm{Im}Q\to \mathrm{Ker}L$.

In order to obtain the main results, we introduce the Mawhin's continuation theorem as follows.

Lemma 3.1 ([16]) Let $\Omega\subset X$ be an open bounded set and let $N:X\to Y$ be a continuous operator which is L-compact on $\bar{\Omega}$. Assume

(a) for each $\lambda\in(0,1), x\in\Omega\cap \mathrm{Dom}L, Lx\neq\lambda Nx,$
(b) for each $x\in\Omega\cap\mathrm{Ker}L, QNx\neq 0$, and $\deg(JQN,\Omega\cap\mathrm{Ker}L,0)\neq 0$.

Then $Lx=Nx$ has at least one solution in $\bar{\Omega}\cap\mathrm{Dom}L$.

Theorem 3.1 Assume that $(H_1)-(H_3)$ hold, $D=\mathrm{diag}(d_{11},\ldots,d_{1m},d_{21},\ldots,d_{2m},\ldots,d_{n1},\ \ldots,d_{nm})$ is a diagonal matrix, and the following condition (H_5) hold

$$(H_5)\ \ d_{ij}=\frac{\underline{a}_{ij}(1-\omega\bar{a}_{ij}\delta_{ij})}{\omega\underline{a}_{ij}\bar{a}_{ij}+\dfrac{\bar{a}_{ij}}{\rho_{ij}}}-f_{ij}^{M}\sum_{C^{kl}\in N_r(i,j)}\bar{C}_{ij}^{kl}>0, i=1,2,\ldots,n,\ j=1,2,\ldots,m,$$

then system (1.1) has at least one ω-periodic solutions.

Proof. Let $C[0,\omega;t_1,t_2,\ldots,t_q]_{\mathbb{T}}=\{x:[0,\omega]_{\mathbb{T}}\to\mathbb{R}^{nm}$ is a piecewise continuous map with first-class discontinuous points in $[0,\omega]_{\mathbb{T}}\cap\{t_k\}$ and at each discontinuous point it is continuous on the left }. Take

$$X=\{x\in C[0,\omega;t_1,\ldots,t_q]_{\mathbb{T}}:x(t+\omega)=x(t)\}, Z=X\times\mathbb{R}^{nm\times(q+1)}$$

and $\|x\|=\sum_{(i,j)}\max_{t\in[0,\omega]_{\mathbb{T}}}|x_{ij}(t)|$, then X is a Banach space. Set

$$L:\mathrm{Dom}L\cap X\to Z, x\to(x^{\Delta},\vartriangle x(t_1),\ldots,\vartriangle x(t_q),0),$$

where $\mathrm{Dom}L=\{x\in C^1[0,\omega;t_1,\ldots,t_q]:x(0)=x(\omega)\}$, and $N:X\to Z$,

$$Nx=\left(\begin{pmatrix}A_{11}(t)\\ \vdots\\ A_{1m}(t)\\ \vdots\\ A_{n1}(t)\\ \vdots\\ A_{nm}(t)\end{pmatrix},\begin{pmatrix}e_1(x_{11}(t_1))\\ \vdots\\ e_1(x_{1m}(t_1))\\ \vdots\\ e_1(x_{n1}(t_1))\\ \vdots\\ e_1(x_{nm}(t_1))\end{pmatrix},\begin{pmatrix}e_2(x_{11}(t_2))\\ \vdots\\ e_2(x_{1m}(t_2))\\ \vdots\\ e_2(x_{n1}(t_2))\\ \vdots\\ e_2(x_{nm}(t_2))\end{pmatrix},\ldots,\begin{pmatrix}e_q(x_{11}(t_q))\\ \vdots\\ e_q(x_{1m}(t_q))\\ \vdots\\ e_q(x_{n1}(t_q))\\ \vdots\\ e_q(x_{nm}(t_q))\end{pmatrix},0\right),$$

where

$$A_{ij}(t) = -a_{ij}(x_{ij}(t))\Big\{b_{ij}(x_{ij}(t)) + \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t) f_{ij}(t, x_{kl}(t-\tau_{kl}(t)))x_{ij}(t) - L_{ij}(t)\Big\}, i = 1,2,\ldots,n,\ j = 1,2,\ldots,m.$$

Take $z = (f, C_1, \ldots, C_q, d) \in \mathrm{Im}L \subset Z$, then

$$\mathrm{Ker}L = \{x \in X \mid x = h \in R^{nm}\},\ \ \mathrm{Im}L = \Big\{(f, C_1, \ldots, C_q, d) \in Z : \int_0^{\omega} f(s)\Delta s + \sum_{k=1}^{q} C_k + d = 0\Big\}$$

and $\dim\mathrm{Ker}L = nm = \mathrm{codim}\,\mathrm{Im}L.$

So, $\mathrm{Im}L$ is closed in Z, L is a Fredholm mapping of index zero. Define two projectors

$$Px = \frac{1}{\omega}\int_0^{\omega} x(t)\Delta t,$$

$$Qz = Q(f, C_1, C_2, \ldots, C_n, d) = \Big(\frac{1}{\omega}\Big[\int_0^{\omega} f(s)\Delta s + \sum_{k=1}^{q} C_k + d\Big], 0, \ldots, 0, 0\Big).$$

It is easy to show that P and Q are continuous and satisfy

$$\mathrm{Im}P = \mathrm{Ker}L, \mathrm{Im}L = \mathrm{Ker}Q = \mathrm{Im}(I-Q).$$

Further, let $L_P^{-1} = L|_{\mathrm{Dom}L\cap\mathrm{Ker}P}$ and the generalized inverse $K_P = L_P^{-1}$ is given by

$$K_P z = \int_0^{t} f(s)\Delta s + \sum_{t>t_k} C_k - \frac{1}{\omega}\int_0^{\omega}\int_0^{t} f(s)\Delta s\Delta t - \sum_{k=1}^{q} C_k.$$

Thus, the expression of QNx is

$$\left(\begin{pmatrix} \frac{1}{\omega}\int_0^{\omega} A_{11}(t)\Delta t + \frac{1}{\omega}\sum_{k=1}^{q} e_k(x_{11}(t_k)) \\ \vdots \\ \frac{1}{\omega}\int_0^{\omega} A_{1m}(t)\Delta t + \frac{1}{\omega}\sum_{k=1}^{q} e_k(x_{1m}(t_k)) \\ \vdots \\ \frac{1}{\omega}\int_0^{\omega} A_{n1}(t)\Delta t + \frac{1}{\omega}\sum_{k=1}^{q} e_k(x_{n1}(t_k)) \\ \vdots \\ \frac{1}{\omega}\int_0^{\omega} A_{nm}(t)\Delta t + \frac{1}{\omega}\sum_{k=1}^{q} e_k(x_{nm}(t_k)) \end{pmatrix}, 0, \ldots, 0, 0\right),$$

and then

$$K_P(I-Q)Nu=\begin{pmatrix}\int_0^{\omega}A_{11}(s)\Delta s+\sum_{t>t_k}e_k(x_{11}(t_k))\\ \vdots\\ \int_0^{\omega}A_{1m}(s)\Delta s+\sum_{t>t_k}e_k(x_{1m}(t_k))\\ \vdots\\ \int_0^{\omega}A_{n1}(s)\Delta s+\sum_{t>t_k}e_k(x_{n1}(t_k))\\ \vdots\\ \int_0^{\omega}A_{nm}(s)\Delta s+\sum_{t>t_k}e_k(x_{nm}(t_k))\end{pmatrix}-\begin{pmatrix}\frac{1}{\omega}\int_0^{\omega}\int_0^{t}A_{11}(s)\Delta s\Delta t\\ \vdots\\ \frac{1}{\omega}\int_0^{\omega}\int_0^{t}A_{1m}(s)\Delta s\Delta t\\ \vdots\\ \frac{1}{\omega}\int_0^{\omega}\int_0^{t}A_{n1}(s)\Delta s\Delta t\\ \vdots\\ \frac{1}{\omega}\int_0^{\omega}\int_0^{t}A_{nm}(s)\Delta s\Delta t\end{pmatrix}$$

$$-\begin{pmatrix}(\frac{t}{\omega}-\frac{1}{2})\int_0^{\omega}A_{11}(s)\Delta s\\ \vdots\\ (\frac{t}{\omega}-\frac{1}{2})\int_0^{\omega}A_{1m}(s)\Delta s\\ \vdots\\ (\frac{t}{\omega}-\frac{1}{2})\int_0^{\omega}A_{n1}(s)\Delta s\\ \vdots\\ (\frac{t}{\omega}-\frac{1}{2})\int_0^{\omega}A_{nm}(s)\Delta s\end{pmatrix}-\begin{pmatrix}\sum_{k=1}^{q}e_k(x_{11}(t_k))\\ \vdots\\ \sum_{k=1}^{q}e_k(x_{1m}(t_k))\\ \vdots\\ \sum_{k=1}^{q}e_k(x_{n1}(t_k))\\ \vdots\\ \sum_{k=1}^{q}e_k(x_{nm}(t_k))\end{pmatrix}.$$

Thus, QN and $K_P(I-Q)N$ are both continuous. Using the Lemma 2.6, it is easy to show that $K_P(I-Q)N(\bar{\Omega})$ is compact for any open bounded set $\Omega\subset X$. Moreover $QN(\bar{\Omega})$ is bounded. Thus, N is L -compact on $\bar{\Omega}$ for any open bounded set $\Omega\subset X$.

Now, it needs to show that there exists a domain Ω, that satisfies all the requirements given in lemma 3.1. Corresponding to operator equation $Lx=\lambda Nx$, $\lambda\in(0,1)$ we have

$$\begin{cases}x_{ij}^{\Delta}(t)=\lambda\Big[-a_{ij}(x_{ij}(t))\Big\{b_{ij}(x_{ij}(t))+\sum_{C^{kl}\in N_r(i,j)}C_{ij}^{kl}(t)f_{ij}(t,x_{kl}(t-\tau_{kl}(t)))x_{ij}(t)-L_{ij}(t)\Big\}\Big]\\ \qquad t\neq t_k, t\in[0,\omega]_{\mathbb{T}}, i=1,2,\dots,n,\ j=1,2,\dots,m,\\ \triangle x_{ij}(t_k)=x_{ij}(t_k^+)-x_{ij}(t_k^-)=\lambda e_k(x_{ij}(t_k)), k=1,2,\dots,q, i=1,2,\dots,n,\ j=1,2,\dots,m.\end{cases}\tag{3.1}$$

Suppose that $x(t)=(x_{11}(t),\dots,x_{1m}(t),x_{21}(t),\dots,x_{2m}(t),\dots,x_{n1}(t),\dots,x_{nm}(t))^T\in X$ is a solution of system (3.1) for a certain $\lambda\in(0,1)$. Set $t_0=t_0^+=0, t_{q+1}=\omega$, from (3.1), Lemma 2.5 and Lemma 2.7, we have

$$\int_0^{\omega}|x_{ij}^{\Delta}(t)|\Delta t=\sum_{k=1}^{q+1}\int_{t_{k-1}^{+}}^{t_k}|x_{ij}^{\Delta}(t)|\Delta t+\sum_{k=1}^{q}|x_{ij}(t_k^{+})-x_{ij}(t_k)|$$
$$\leqslant\bar{a}_{ij}\Big[\int_0^{\omega}b_{ij}(x_{ij}(t))\Delta t+\int_0^{\omega}|L_{ij}(t)|\Delta t+\int_0^{\omega}\sum_{C^{kl}\in N_r(i,j)}|C_{ij}^{kl}f_{ij}(t,x_{kl}(t-\tau_{kl}(t)))||x_{ij}(t)|\Delta t\Big]+\sum_{k=1}^{q}|e_k(x_{ij}(t_k))| \quad (3.2)$$
$$\leqslant\bar{a}_{ij}\delta_{ij}\sqrt{\omega}\,\|x_{ij}\|_2+\sum_{C^{kl}\in N_r(i,j)}\bar{a}_{ij}\bar{C}_{ij}^{kl}f_{ij}^{M}\sqrt{\omega}\,\|x_{ij}\|_2+\bar{a}_{ij}\sqrt{\omega}\,\|L_{ij}\|_2+\sum_{k=1}^{q}|e_k(x_{ij}(t_k))|,$$
$$i=1,2,\dots,n,\ j=1,2,\dots,m.$$

Integrating both sides of (3.1), over the interval $[0,\omega]_{\mathbb{T}}$, we obtain

$$\int_0^{\omega}A_{ij}(t)\Delta t+\sum_{k=1}^{q}e_k(x_{ij}(t_k))=0, i=1,2,\dots,n,\ j=1,2,\dots,m,$$

namely,

$$\int_0^{\omega}a_{ij}(x_{ij}(t))b_{ij}(x_{ij}(t))\Delta t=$$
$$-\int_0^{\omega}\sum_{C^{kl}\in N_r(i,j)}a_{ij}(x_{ij}(t))C_{ij}^{kl}(t)f_{ij}(t,x_{kl}(t-\tau_{kl}(t)))x_{ij}(t)\Delta t+\int_0^{\omega}a_{ij}(x_{ij}(t))L_{ij}(t)\Delta t+\sum_{k=1}^{q}e_k(x_{ij}(t_k)),$$

where $i=1,2,\dots,n,\ j=1,2,\dots,m$. By Lemma 2.5 we get

$$\Big|\int_0^{\omega}a_{ij}(x_{ij}(t))b_{ij}(x_{ij}(t))\Delta t\Big|$$
$$\leqslant\int_0^{\omega}\sum_{C^{kl}\in N_r(i,j)}\bar{a}_{ij}\bar{C}_{ij}^{kl}f_{ij}^{M}|x_{ij}(t)|\Delta t+\bar{a}_{ij}\sqrt{\omega}\,\|L_{ij}\|_2+\sum_{k=1}^{q}|e_k(x_{ij}(t_k))|$$
$$\leqslant\bar{a}_{ij}\sqrt{\omega}\sum_{C^{kl}\in N_r(i,j)}\bar{C}_{ij}^{kl}f_{ij}^{M}\|x_{ij}\|_2+\bar{a}_{ij}\sqrt{\omega}\,\|L_{ij}\|_2+\sum_{k=1}^{q}|e_k(x_{ij}(t_k))|,$$

where $i=1,2,\dots,n,\ j=1,2,\dots,m.$ Then by (H_1), we obtain

$$\Big|\int_0^{\omega}a_{ij}(x_{ij}(t))x_{ij}(t)\Delta t\Big|\leqslant\frac{\bar{a}_{ij}}{\rho_{ij}}\sqrt{\omega}\sum_{C^{kl}\in N_r(i,j)}\bar{C}_{ij}^{kl}f_{ij}^{M}\|x_{ij}\|_2$$
$$+\frac{\bar{a}_{ij}}{\rho_{ij}}\sqrt{\omega}\,\|L_{ij}\|_2+\frac{1}{\rho_{ij}}\sum_{k=1}^{q}|e_k(x_{ij}(t_k))|, \quad (3.3)$$

From Lemma 2.4, for any $t_1^{ij},t_2^{ij}\in[0,\omega]_{\mathbb{T}}, i=1,2,\dots,n,\ j=1,2,\dots,m$, we have

$$\int_0^{\omega}a_{ij}(x_{ij}(t))x_{ij}(t)\Delta t\leqslant\int_0^{\omega}a_{ij}(x_{ij}(t))x_{ij}(t_1^{ij})\Delta t+\int_0^{\omega}a_{ij}(x_{ij}(t))\Big(\int_0^{\omega}|x_{ij}^{\Delta}(t)|\Delta t\Big)\Delta t$$

and

$$\int_0^{\omega}a_{ij}(x_{ij}(t))x_{ij}(t)\Delta t\geqslant\int_0^{\omega}a_{ij}(x_{ij}(t))x_{ij}(t_2^{ij})\Delta t-\int_0^{\omega}a_{ij}(x_{ij}(t))\Big(\int_0^{\omega}|x_{ij}^{\Delta}(t)|\Delta t\Big)\Delta t.$$

Dividing by $\int_0^\omega a_{ij}(x_{ij}(t))\Delta t$ on the two sides of the inequalities above, we obtain that

$$\begin{aligned} & x_{ij}(t_1^{ij}) \\ & \geqslant \frac{1}{\int_0^\omega a_{ij}(x_{ij}(t))\Delta t}\int_0^\omega a_{ij}(x_{ij}(t))x_{ij}(t)\Delta t - \int_0^\omega |x_{ij}^\Delta(t)|\Delta t \\ & \geqslant -\Big[\frac{1}{\omega \underline{a}_{ij}}\Big|\int_0^\omega a_{ij}(x_{ij}(t))x_{ij}(t)\Delta t\Big| + \int_0^\omega |x_{ij}^\Delta(t)|\Delta t\Big] \end{aligned}$$

and

$$\begin{aligned} x_{ij}(t_2^{ij}) & \leqslant \frac{1}{\int_0^\omega a_{ij}(x_{ij}(t))\Delta t}\int_0^\omega a_{ij}(x_{ij}(t))x_{ij}(t)\Delta t + \int_0^\omega |x_{ij}^\Delta(t)|\Delta t \\ & \leqslant \frac{1}{\omega \underline{a}_{ij}}\Big|\int_0^\omega a_{ij}(x_{ij}(t))x_{ij}(t)\Delta t\Big| + \int_0^\omega |x_{ij}^\Delta(t)|\Delta t, \end{aligned}$$

by the arbitrariness of t_1^{ij}, t_2^{ij}, let $\underline{t}_{ij}, \overline{t}_{ij} \in [0,\omega]_{\mathbb{T}}$ such that $x_{ij}(\underline{t}_{ij}) = \min_{t\in[0,\omega]_{\mathbb{T}}} x_{ij}(t)$,

$$x_{ij}(\overline{t}_{ij}) = \max_{t\in[0,\omega]_{\mathbb{T}}} x_{ij}(t), i=1,2,\ldots,n,\ j=1,2,\ldots,m,$$

so

$$x_{ij}(\underline{t}_{ij}) \geqslant -\Big[\frac{1}{\omega \underline{a}_{ij}}\Big|\int_0^\omega a_{ij}(x_{ij}(t))x_{ij}(t)\Delta t\Big| + \int_0^\omega |x_{ij}^\Delta(t)|\Delta t\Big]$$

and

$$x_{ij}(\overline{t}_{ij}) \leqslant \frac{1}{\omega \underline{a}_{ij}}\Big|\int_0^\omega a_{ij}(x_{ij}(t))x_{ij}(t)\Delta t\Big| + \int_0^\omega |x_{ij}^\Delta(t)|\Delta t.$$

Hence,

$$\max_{t\in[0,\omega]_{\mathbb{T}}} |x_{ij}(t)| \leqslant \frac{1}{\omega \underline{a}_{ij}}\Big|\int_0^\omega a_{ij}(x_{ij}(t))x_{ij}(t)\Delta t\Big| + \int_0^\omega |x_{ij}^\Delta(t)|\Delta t, \tag{3.4}$$

where $i=1,2,\ldots,n,\ j=1,2,\ldots,m$. In addition, from (3.4) we have that

$$\begin{aligned} & \underline{a}_{ij}\|x_{ij}\|_2 = \underline{a}_{ij}\Big(\int_0^\omega |x_{ij}(t)|^2\ \Delta t\Big)^{\frac{1}{2}} \leqslant \underline{a}_{ij}\sqrt{\omega}\max_{t\in[0,\omega]_{\mathbb{T}}}|x_{ij}(t)| \\ & \leqslant \frac{1}{\sqrt{\omega}}\Big|\int_0^\omega a_{ij}(x_{ij}(t))x_{ij}(t)\Delta t\Big| + \underline{a}_{ij}\sqrt{\omega}\int_0^\omega |x_{ij}^\Delta(t)|\Delta t, i=1,2,\ldots,n,\ j=1,2,\ldots,m. \end{aligned} \tag{3.5}$$

Combining (3.2), (3.3) and (3.5), we obtain

$$
\begin{aligned}
&\underline{a}_{ij}\|x_{ij}\|_2 \leqslant \\
&\frac{1}{\sqrt{\omega}}\Big(\frac{\overline{a}_{ij}}{\rho_{ij}}\sqrt{\omega}\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl} f_{ij}^M \|x_{ij}\|_2+\frac{\overline{a}_{ij}}{\rho_{ij}}\sqrt{\omega}\|L_{ij}\|_2+\frac{1}{\rho_{ij}}\sum_{k=1}^{q}|e_k(x_{ij}(t_k))|\Big) \\
&+\underline{a}_{ij}\sqrt{\omega}\Big(\overline{a}_{ij}\delta_{ij}\sqrt{\omega}\|x_{ij}\|_2+\sum_{C^{kl}\in N_r(i,j)}\overline{a}_{ij}\overline{C}_{ij}^{kl} f_{ij}^M\sqrt{\omega}\|x_{ij}\|_2+\overline{a}_{ij}\sqrt{\omega}\|L_{ij}\|_2+\sum_{k=1}^{q}|e_k(x_{ij}(t_k))|\Big) \\
&\leqslant(\omega\underline{a}_{ij}\overline{a}_{ij}+\frac{\overline{a}_{ij}}{\rho_{ij}}) f_{ij}^M\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl}\|x_{ij}\|_2+\omega\underline{a}_{ij}\overline{a}_{ij}\delta_{ij}\|x_{ij}\|_2+(\omega\underline{a}_{ij}\overline{a}_{ij}+\frac{\overline{a}_{ij}}{\rho_{ij}})\|L_{ij}\|_2 \\
&+\Big(\sqrt{\omega}\underline{a}_{ij}+\frac{1}{\sqrt{\omega}}\frac{1}{\rho_{ij}}\Big)\sum_{k=1}^{q}|e_k(x_{ij}(t_k))| \\
&=(\omega\underline{a}_{ij}\overline{a}_{ij}+\frac{\overline{a}_{ij}}{\rho_{ij}}) f_{ij}^M\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl}\|x_{ij}\|_2+\omega\underline{a}_{ij}\overline{a}_{ij}\delta_{ij}\|x_{ij}\|_2+(\omega\underline{a}_{ij}\overline{a}_{ij}+\frac{\overline{a}_{ij}}{\rho_{ij}})\alpha_{ij},
\end{aligned}
$$

where $\alpha_{ij}=\|L_{ij}\|_2+\frac{1}{\sqrt{\omega}\overline{a}_{ij}}\sum_{k=1}^{q}|e_k(x_{ij}(t_k))|, i=1,2,\ldots,n,\ j=1,2,\ldots,m$.

Hence,

$$
\frac{\underline{a}_{ij}(1-\omega\overline{a}_{ij}\delta_{ij})}{\omega\underline{a}_{ij}\overline{a}_{ij}+\dfrac{\overline{a}_{ij}}{\rho_{ij}}}\|x_{ij}\|_2\leqslant f_{ij}^M\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl}\|x_{ij}\|_2+\alpha_{ij}, i=1,2,\ldots,n,\ j=1,2,\ldots,m. \qquad (3.6)
$$

Denote

$$
\begin{aligned}
&\|x\|_2=(\|x_{11}\|_2,\ldots,\|x_{1m}\|_2,\|x_{21}\|_2,\ldots,\|x_{2m}\|_2,\ldots,\|x_{n1}\|_2,\ldots,\|x_{nm}\|_2)^T, \\
&C=(\alpha_{11},\ldots,\alpha_{1m},\alpha_{21},\ldots,\alpha_{2m},\ldots,\alpha_{n1},\ldots,\alpha_{nm})^T.
\end{aligned}
$$

Thus, (3.6) can be rewritten in the matrix form

$$D\|x\|_2\leqslant C.$$

From the assumptions of Theorem 3.1, we obtain

$$\|x\|_2\leqslant D^{-1}C\triangleq(B_{11},\ldots,B_{1m},B_{21},\ldots,B_{2m},\ldots,B_{n1},\ldots,B_{nm})^T,$$

that is $\|x_{ij}\|_2\leqslant B_{ij}, i=1,2,\ldots,n,\ j=1,2,\ldots,m$.

By (3.2), (3.3) and (3.4) we obtain

$$\max_{t\in[0,\omega]_{\mathbb{T}}} |x_{ij}(t)|$$
$$\leqslant \frac{1}{\omega \underline{a}_{ij}}\Big(\frac{\overline{a}_{ij}}{\rho_{ij}}\sqrt{\omega}\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl} f_{ij}^{M}\,\|x_{ij}\|_2+\frac{\overline{a}_{ij}}{\rho_{ij}}\sqrt{\omega}\,\|L_{ij}\|_2+\frac{1}{\rho_{ij}}\sum_{k=1}^{q}|e_k(x_{ij}(t_k))|\Big)$$
$$+\overline{a}_{ij}\delta_{ij}\sqrt{\omega}\,\|x_{ij}\|_2+\sum_{C^{kl}\in N_r(i,j)}\overline{a}_{ij}\overline{C}_{ij}^{kl} f_{ij}^{M}\sqrt{\omega}\,\|x_{ij}\|_2+\overline{a}_{ij}\sqrt{\omega}\,\|L_{ij}\|_2+\sum_{k=1}^{q}|e_k(x_{ij}(t_k))|$$
$$\leqslant \frac{1}{\sqrt{\omega}\underline{a}_{ij}}\Big[\Big(\frac{\overline{a}_{ij}}{\rho_{ij}}+\omega\overline{a}_{ij}\underline{a}_{ij}\Big) f_{ij}^{M}\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl}\,\|x_{ij}\|_2+\omega\delta_{ij}\underline{a}_{ij}\overline{a}_{ij}\,\|x_{ij}\|_2+\Big(\frac{\overline{a}_{ij}}{\rho_{ij}}+\omega\overline{a}_{ij}\underline{a}_{ij}\Big)\|L_{ij}\|_2$$
$$+\Big(\sqrt{\omega}\underline{a}_{ij}+\frac{1}{\sqrt{\omega}\rho_{ij}}\Big)\sum_{k=1}^{q}|e_k(x_{ij}(t_k))|\Big]$$
$$\leqslant \frac{1}{\sqrt{\omega}\underline{a}_{ij}}\Big[\Big(\frac{\overline{a}_{ij}}{\rho_{ij}}+\omega\overline{a}_{ij}\underline{a}_{ij}\Big) f_{ij}^{M}\sum_{C^{kl}\in N_r(i,j)}\overline{C}_{ij}^{kl}B_{ij}+\omega\delta_{ij}\underline{a}_{ij}\overline{a}_{ij}B_{ij}+\Big(\frac{\overline{a}_{ij}}{\rho_{ij}}+\omega\overline{a}_{ij}\underline{a}_{ij}\Big)\alpha_{ij}\Big]$$
$$=: A_{ij},\ i=1,2,\cdots,n,\ j=1,2,\cdots,m.$$

Denote $A=\sum_{(i,j)}A_{ij}+A^*$, where A^* is a sufficiently large positive constant. Now, we take

$$\Omega=\{x(t)\in X: \|x\|< A\}.$$

Obviously, Ω satisfies the condition (a) of Lemma 3.1.

When $x\in\partial\Omega\cap \mathrm{Ker}L=\partial\Omega\cap\mathbb{R}^{nm}$, $x=(x_{11},\dots,x_{1m},x_{21},\dots,x_{2m},x_{n1},\dots,x_{nm})^T$ is a constant vector with $\|x\|=A$. Furthermore, take $J:\mathrm{Im}Q\to\mathrm{Ker}L,(r,0,\dots,0,0)\to r$, then

$$JQN\begin{pmatrix}x_{11}\\ \vdots\\ x_{1m}\\ \vdots\\ x_{n1}\\ \vdots\\ x_{nm}\end{pmatrix}=\begin{pmatrix}-a_{11}(x_{11})\Big[b_{11}(x_{11})+\sum_{C^{kl}\in N_r(i,j)}\widetilde{G_{11}^{kl}}x_{11}-\widetilde{L_{11}}\Big]+\frac{1}{\omega}\sum_{k=1}^{q}e_k(x_{11}(t_k))\\ \vdots\\ -a_{1m}(x_{1m})\Big[b_{1m}(x_{1m})+\sum_{C^{kl}\in N_r(i,j)}\widetilde{G_{1m}^{kl}}x_{1m}-\widetilde{L_{1m}}\Big]+\frac{1}{\omega}\sum_{k=1}^{q}e_k(x_{1m}(t_k))\\ \vdots\\ -a_{n1}(x_{n1})\Big[b_{n1}(x_{n1})+\sum_{C^{kl}\in N_r(i,j)}\widetilde{G_{n1}^{kl}}x_{n1}-\widetilde{L_{n1}}\Big]+\frac{1}{\omega}\sum_{k=1}^{q}e_k(x_{n1}(t_k))\\ \vdots\\ -a_{nm}(x_{nm})\Big[b_{nm}(x_{nm})+\sum_{C^{kl}\in N_r(i,j)}\widetilde{G_{nm}^{kl}}x_{nm}-\widetilde{L_{nm}}\Big]+\frac{1}{\omega}\sum_{k=1}^{q}e_k(x_{nm}(t_k))\end{pmatrix},$$

where $\widetilde{G_{ij}^{kl}}=\frac{1}{\omega}\int_0^{\omega}C_{ij}^{kl}(t)\,f_{ij}(t,x_{ij})\Delta t$.

If necessary, we can take A sufficiently large such that

$$x^T JQNx = \sum_{i=1}^{n}\sum_{j=1}^{m}\Big[-a_{ij}(x_{ij})x_{ij}\Big[b_{ij}(x_{ij}) + \sum_{C^{kl}\in N_r(i,j)} \widetilde{G_{ij}^{kl}}x_{ij} - \widetilde{L_{ij}}\Big] + \frac{1}{\omega}x_{ij}\sum_{k=1}^{q} e_k(x_{ij})\Big] < 0,$$

so for any $x \in \partial\Omega \cap \mathrm{Ker}L, QNx \neq 0$. Moreover, let $\psi(r;x) = -rx + (1-r)JQNx$, then for any $x \in \partial\Omega \cap \mathrm{Ker}L, x^T\psi(r;x) < 0$, we get

$$\deg\{JQN, \Omega \cap \mathrm{Ker}L, 0\} = \deg\{-x, \omega \cap \mathrm{Ker}L, 0\} \neq 0.$$

So, condition (b) in Lemma 3.1 is also satisfied, we now know that Ω satisfies all the requirements in Lemma 3.1. Therefore, (1.1) has at least one ω -periodic solution. The proof is complete.

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