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Research Paper

Geometric characteristics of Doppler shift in two dimensional plane

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ABSTRACT: *By approximating the rate of change of the radial distance, the transformation relation between Doppler frequency shift and path difference can be obtained. Thus, it is revealed that the Doppler shift represents the path difference formed by a virtual single-base array on a two-dimensional plane. The analysis shows that the virtual path differential is composed of two parts in two - dimensional plane. One is caused by the initial Doppler shift, which represents the effect of the initial velocity. The other part is generated by the current Doppler rate of change and represents the effect of the acceleration. The new concept proposed in this paper will provide a new solution to the existing passive detection..*
KEYWORDS: *Doppler shift, Path difference equation, Virtual array, Doppler changing rate, Single station location, Airborne passive positioning*

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I. INTRODUCTION

The author's existing research shows that the Doppler frequency shift corresponding to the motion radius vector can be decomposed into the sum of binomial terms under the assumption that the measured target is moving at a constant speed, or the detection platform is moving at a uniform speed. One term relates only to the initial radial velocity, and the other to the current radial acceleration[1]. The research results presented by the author at that time mainly emphasize that Doppler frequency shift is a two-dimensional distribution function.

Recently, by using the approximate difference method, the author has derived the calculation formula of virtual path difference based on the Doppler frequency shift measurement from the Doppler frequency shift equation based on the rate of change of radial distance[2]. But at that time, the author only used this result for the location calculation of passive detection, and did not realize the geometric meaning of virtual path difference based on Doppler frequency shift measurement.

As a complement, this paper focuses on the geometrical properties of the Doppler shift, which represent the path difference formed by the detection of a virtual single-base array on a two-dimensional plane, on the basis of the existing research. It also explains the two-dimensional distribution characteristics of Doppler frequency shift..

II. GEOMETRIC MODEL

2.1 Motion platform

As shown in Fig. 1, a detection platform is assumed to move uniformly along a straight line in a two-dimensional plane. Doppler receiver R is arranged on the moving platform to detect stationary or slow-moving target T on the ground. The Doppler frequency shift received is

$$\lambda f_d = v\cos\beta \qquad (1)$$

Where: f_d is Doppler frequency shift; λ the wavelength; v the moving speed of the moving platform; β advance angle.

Regardless of how the Doppler shift is detected and computed. Geometrically, there is only one receiving unit at point R, which represents the position of the receiver, and it may also be a receiving array. The black dot to the right of point R only represents the geometric support point of a virtual single-base array.

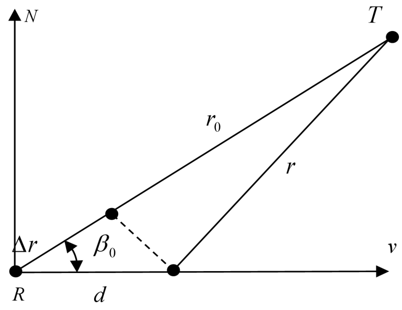

Figure1: Doppler shift detection of a moving single station

2.2 Moving target

Fig. 2 shows the plane geometry of the ground fixed radar in the process of detecting a moving target, and it is assumed that the measured target moves in a straight line at an even speed.

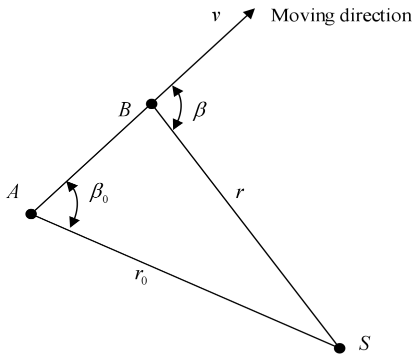

Figure2: Geometric measurement relationship of moving target

2.3 Motion radius vector

From the perspective of geometric analysis, the expression form of the motion radius vector of the two detection methods is the same whether it is the detection of the stationary target by a moving platform or the detection of the moving target by a fixed detection station.

Let the radial distance r_0 at the initial moment be a certain value. If, according to the geometrical relationship in Figure 2, using the law of cosines, the slope distance r when arriving at point B from point A through the travel time Δt is

$$r = \sqrt{r_0^2 + (v\Delta t)^2 - 2r_0 v\Delta t \cos\beta_0} \qquad (2)$$

Where: v can be either the moving speed of the moving platform or the flight speed of the target.

III. DOPPLER SHIFT IN TWO DIMENSIONS

3.1 The radial velocity of the motion radius vector

The moving velocity of the target in the radial direction can be obtained by the velocity definition of the moving point

$$v_r = -\frac{dr}{dt} = -\frac{v^2\Delta t}{r} + \frac{vr_0\cos\beta_0}{r} \qquad (3)$$

If we use the law of sines, we get the following formulas

$$\frac{v\Delta t}{r} = \frac{\sin\Delta\beta}{\sin\beta_0}$$

$$\frac{r_0}{r} = \frac{\sin\beta}{\sin\beta_0}$$

Substitute them into Equation (3) and finally use the radial velocity equation

$$v_r = v\cos\beta$$

It can verify that the deduced equation (3) is valid.

3.2 Kinetic equation method

Based on the relationship between the rate of change of radius vector and the radial velocity

$$\frac{\partial r(t)}{\partial t} = -v_r$$

And the basic relationship between the radial velocity and the Doppler shift function

$$v_r = -\lambda f_d$$

It can be obtained from Equation (3)

$$\begin{aligned} f_d &= -\frac{v_r}{\lambda} = -\frac{r_0}{r}\cdot\frac{v\cos\theta_0}{\lambda} + \frac{v^2\Delta t}{\lambda r} \\ &= \frac{r_0}{r} f_{d0} + \frac{v^2\Delta t}{\lambda r} \end{aligned} \quad (4)$$

Where, f_{d0} is the Doppler frequency corresponding to the radial distance r_0 at the previous moment.

If the velocity at the end point of the current radius vector r is orthogonal decomposed, we can get

$$\begin{aligned} f_d &= \frac{r_0}{r} f_{d0} + \frac{\left(v_r^2 + v_t^2\right)\Delta t}{\lambda r} \\ &= \frac{r_0}{r} f_{d0} + \left(\frac{v_r^2}{v_t^2} + 1\right)\frac{v_t^2\Delta t}{\lambda r} \end{aligned} \quad (5)$$

Owing to

$$\dot{f}_d = \frac{v_t^2}{\lambda r}$$

$$\frac{v_r}{v_t} = ctg\beta$$

$$\frac{r_0}{r} = \frac{\sin\beta}{\sin\beta_0}$$

Substituting the above equation into Equation (5), we get

$$\begin{aligned} f_d &= \frac{\sin\beta}{\sin\beta_0} f_{d0} + \left(1 + ctg^2\beta\right)\dot{f}_d\,\Delta t \\ &= \frac{\sin\beta}{\sin\beta_0} f_{d0} + \frac{\dot{f}_d\,\Delta t}{\sin^2\beta} \end{aligned} \quad (6)$$

3.3 Trigonometric function method

Starting from the geometrical relation shown in Fig. 1, using the relation of internal and external angles: $\beta = \beta_0 + \Delta\beta$, in the direction of radius vector r, there is

$$\begin{aligned} \lambda f_d &= -v\cos\beta = -v\cos(\beta_0 + \Delta\beta) \\ &= -v\left[\cos\beta_0\cos\Delta\beta - \sin\beta_0\sin\Delta\beta\right] \end{aligned}$$

Owing to: $\cos\Delta\beta \to 1$, $\sin\Delta\beta \approx \Delta\beta$. So

$$\lambda f_d \approx -v\cos\beta_0 + v\sin\beta_0\cdot\Delta\beta$$

Owing to: $\omega \approx \Delta\beta/\Delta t = v_t/r$. So

$$\begin{aligned} \lambda f_d &= -v\cos\beta_0 + v\sin\beta_0\cdot\frac{\Delta\beta}{\Delta t}\Delta t \\ &\approx -v\cos\beta_0 + v\sin\beta_0\cdot\omega\cdot\Delta t \\ &= \lambda f_{d0} + \frac{v_{t0}v_t}{r}\Delta t \end{aligned}$$

Assume $v_{t0} \approx v_t$, and according to the definition of radial acceleration: $a_r = v_t^2/r$, we can be obtained

$$\begin{aligned} \lambda f_d &\approx \lambda f_{d0} + \frac{v_t^2}{r}\Delta t \\ &= -v_{r0} + a_r\Delta t \end{aligned} \quad (7)$$

3.4 Compare

Compared with the kinetic equation method, the trigonometric expansion method uses a more moderate approximation, so the result should be more accurate.

For the kinetic equation method, if the tangential velocity is much greater than the radial velocity, that is, when $\beta \to 90^0$, the detection station is looking sideways at the target. In the case that the detection time interval is short and the two radial distances are approximate to each other, approximately

$$\sin\beta \approx \sin\beta_0$$

Then we get

$$f_d \approx f_{d0} + \dot{f}_d \Delta t \qquad (8)$$

Obviously, this is consistent with the mathematical definition of the Doppler rate of change

$$\dot{f}_d = \frac{f_d - f_{d0}}{\Delta t}$$

The trigonometric expansion makes it more clear that the Doppler shift corresponding to the radius vector of the motion can always be decompositionalized into the sum of two terms, one related only to the initial radial velocity and the other to the current radial acceleration.

The derived expression is also suitable for the recursion of Doppler frequency shift in two dimensional plane[3-5]. That is, the Doppler frequency shift of the current time can be obtained by recursion of the parameter values such as the Doppler frequency shift and Doppler rate of change at the previous time.

IV. DIFFERENTIAL TREATMENT OF RADIAL VELOCITY

4.1 One dimensional transformation

According to the relationship between Doppler frequency shift and the rate of change of radial distance

$$\frac{\partial r(t)}{\partial t} = \lambda f_d$$

Assuming that the change of time is short, the difference calculation method can be used to convert the differential of distance to time into

$$\frac{\partial r(t)}{\partial t} \approx \frac{\Delta r}{\Delta t} \qquad (9)$$

Where, Δr is path difference. Δt is the time difference when the detection platform is moving, or the time difference when the detected target is moving from position A to position B .

Thus, the transformation relationship between frequency shift and path difference is obtained, and the virtual path difference based on Doppler frequency shift or radial velocity measurement is

$$\Delta r = \lambda f_d \Delta t = |v_r| \Delta t \qquad (10)$$

Since the path difference is generally positive, the radial velocity is appended with the sign of the absolute value. Equation (10) is a very intuitive explanation of the general physical meaning of path difference.

For a moving single station, when the moving distance of the platform is d , the time difference experienced by forming the path difference is

$$\Delta t = \frac{d}{v} \qquad (11)$$

Thus, the expression of virtual path difference independent of the measurement of time difference is obtained

$$\Delta r = \frac{\lambda d}{v} f_d = \frac{|v_r|}{v} d \qquad (12)$$

4.2 2D transformations

If the Doppler frequency shift expression (8) on the two-dimensional plane is substituted, the virtual path difference is

$$\begin{aligned} \Delta r &= \lambda \Delta t \left(f_{d0} + \dot{f}_{d0} \Delta t \right) \\ &= \lambda f_{d0} \Delta t + \lambda \dot{f}_{d0} \Delta t^2 \end{aligned} \qquad (13)$$

or

$$\Delta r = \left| -v_{r0} \right| \Delta t + a_{r0} \Delta t^2 \qquad (14)$$

The virtual path differential is composed of two parts in two - dimensional plane. One is caused by the initial Doppler shift, which represents the effect of the initial velocity. The other part is generated by the current Doppler rate of change and represents the effect of the acceleration.

V. CONCLUSION

If only from the physical level, in order to obtain a true path difference in the two-dimensional plane usually requires two receiver arrays separated by d . Therefore, in other words, if the equivalent Doppler shift is to be solved based on the measurement of the path difference, two receiver arrays are actually required.

If the path difference is solved based on Doppler frequency shift, a virtual single-base array is actually formed by differential processing of the rate of distance change. The geometry of the virtual single base array will be determined by the velocity and time of the detection platform, as well as the Doppler frequency shift obtained by detection. On the one hand, it is very intuitive to illustrate the basic physical meaning of the path difference: the distance is the product of speed and time. On the other hand, it is more intuitive to reveal that Doppler frequency shift represents a distance distributed in a two-dimensional plane.

New concepts always imply potential new applications. Literature [2] has shown an application of virtual path difference based on Doppler shift measurement in passive location. It can be expected that the new concept proposed in this paper will provide more and newer solutions to the existing passive detection.

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