

Design and Analysis of a 3-Loop Autopilot for Missile Interceptor Guidance with PN Guidance Law in Simulink

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Abstract—The missile's control system plays a pivotal role in ensuring that the missile meets its performance criteria by executing commands generated by the guidance law. These commands dynamically change throughout the various phases of flight. The control system represents a vital component within the overall system architecture, orchestrating the missile's path to align with the desired trajectory. This paper comprehensively explores the multifaceted role of the control system, presents an in-depth system model, formulates a tailored guidance law, and demonstrates the practical implementation within the Simulink environment.

Keywords—missile, proportional guidance, control, autopilot

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I. INTRODUCTION

The control system constitutes a pivotal component within the homing loop, as illustrated in Figure 1, configured specifically for the terminal phase when the missile is in close proximity to the target. During this phase, the control system effectively combines the inertial motion of the missile with the target's motion, establishing a relative geometric relationship. Precision sensors, including RF and IR seekers, measure the angle between the missile and the target's line of sight, denoted as the Line-of-Sight (LOS) angle. Additionally, an estimator such as the Kalman filter is utilized to estimate the rate of change of the LOS angle. Subsequently, the guidance law generates imperative commands, serving as inputs to the control system. Leveraging missile effectors such as aerodynamic tail surfaces, the control system adeptly directs the missile to follow the steering commands, ultimately achieving the objective of intercepting the target. In this paper, a conventional approach is employed to formulate a PI controller that tracks the normal acceleration command generated by the guidance law. Various contemporary control techniques, such as H infinity [3-5], dynamic scheduling [9-10], and μ -synthesis [7-8], can also be applied to achieve the desired trajectory. The paper is structured into five sections for comprehensive coverage. Section 2 delves into the explanation of the guidance law in use. Section 3 provides an overview of the flight control system's architecture and its constituent elements. Section 4 offers insights into the transfer functions of the actuator, sensors, and airframe. Lastly, Section 5 outlines the implementation of all components within the homing loop using Simulink.

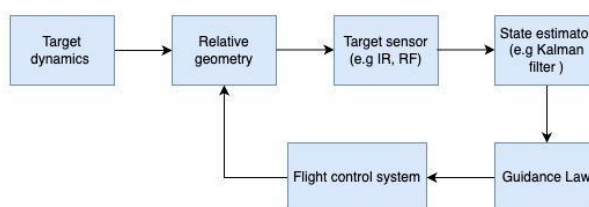


Figure 1: Missile Homing Loop

II. GUIDANCE LAW

The control system takes the guidance law outputs required for an interception, as its inputs. The specific configuration of the guidance law is contingent upon the application. To develop an effective guidance law, the initial consideration must be given to the airframe dynamics. These dynamics are rigorously governed by the equations of motion, derived from Newton's second law. For visualization, refer to Figure 2, which

illustrates the missile's movement within a 2D space. Herein, the angle between the inertial horizontal axis and the missile's velocity vector is denoted as the flight path angle, referred to as γ . The angle between the velocity vector and the missile's body x-axis is identified as the angle of attack (AOA) denoted as α . Moreover, the angle from the inertial x-axis to the missile's body x-axis is represented θ . The acceleration perpendicular to the missile's path is labeled as A_z . Additionally, the rate of change of the flight path angle can be attributed to the component of missile acceleration that is perpendicular to the missile's velocity vector, divided by the magnitude of the velocity vector. Assuming a small AOA angle, the flight path angle rate is

$$\frac{d\gamma}{dt} = \frac{A_z \cos(\alpha)}{v} \approx \frac{A_z}{v} \quad (1)$$

The resulting guidance law can be represented as

$$A_z = NV \frac{d\gamma}{dt} \quad (2)$$

and is called Proportional Navigation guidance law, where N is the PN law constant. The choice of N determines the speed of convergence and depends on how much control effort we want to put in. This command for A_z goes into the control system and then the control system forces the missile to track this command.

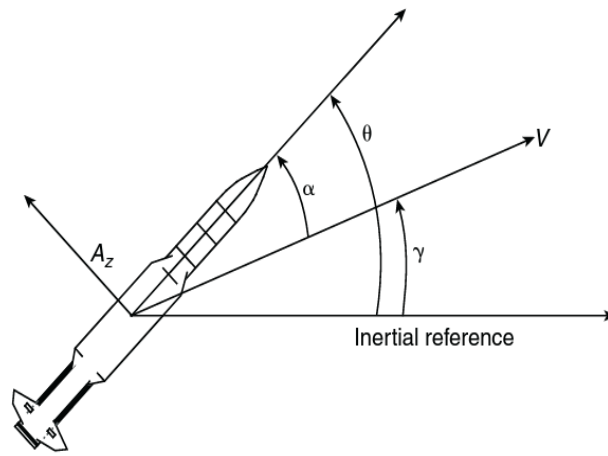


Figure 2: Missile dynamics in the pitch plane

III. FLIGHT CONTROL SYSTEM

Figure 3, illustrates the fundamental components of the flight control system, which plays a role within a larger feedback loop for guiding the missile. The inertial measurement unit (IMU) is responsible for gauging the missile's translational acceleration and angular velocity, sensing its inertial movement. This information is integrated with the guidance law outputs within the autopilot to generate a directive signal for the control actuator. Subsequently, the actuator translates this directive into the actual physical movement of the control effector, thereby altering the dynamics to follow the guidance command.

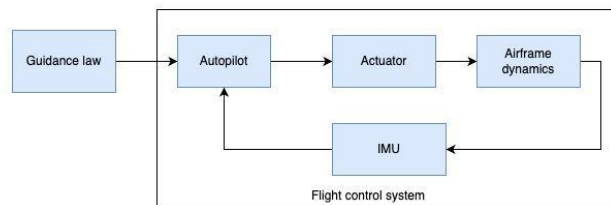


Figure 3: Flight Control System

A. Acceleration control system

In this paper, the control system is designed to track commanded acceleration perpendicular to the missile's longitudinal axis. This block diagram in Figure 4, provides an illustration of a conventional approach for designing an acceleration control autopilot system. It starts by computing the difference between the scaled input acceleration gain command and the actual measured acceleration. This difference is then multiplied by a proportional gain, K_a resulting in the generation of a pitch rate command.

Next, the difference between this derived pitch rate command and the measured pitch rate is calculated and subsequently multiplied by an integral gain K_i . This value is integrated over time. The resulting integral is then subtracted from the measured pitch rate and further multiplied by a proportional gain K_p , yielding the control effector command, such as the desired tail-deflection angle.

The inclusion of the gain K_{dc} on the input acceleration command serves to ensure zero steady-state error when the system is subjected to constant acceleration command inputs. It's important to note that this foundational structure can be further expanded and refined in the final autopilot design, which may incorporate features like noise filters and actuator command limits. This fundamental structure is commonly referred to as the "three-loop autopilot."

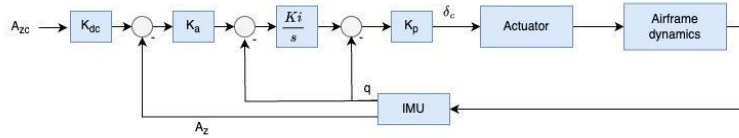


Figure 4: Autopilot Design

C. Equations

The missile dynamics presented in the paper [1], have been linearized around a nominal operating condition of a 10 deg AOA(α) and missile speed of Mach 3. The transfer function representing the decoupled equation is shown below,

$$\frac{A_z(s)}{\delta(s)} = \frac{0.2038(s^2 - 34.3^2)}{s + 0.56 \pm 9.32j} \quad (3) \quad \frac{q(s)}{\delta(s)} = \frac{-131(s + 1)}{s + 0.56 \pm 9.32j} \quad (4)$$

The transfer function in eq. 3 shows the relationship of normal acceleration with the change in the tail fin actuator deflection angle. The transfer function in eq. 4 shows the relationship between the pitch rate (q) with the actuator deflection angle.

The dynamics of the tail fin actuator is assumed to be a second-order transfer function represented in eq 5, having natural frequency (w_N)= 150rad/s and damping ratio (ζ) = 0.7.

$$\frac{\delta_c}{\delta} = \frac{22500}{s^2 + 210s + 22500} \quad (5)$$

The sensor dynamics for IMU consist of both gyroscope and accelerometer is considered to be a second-order transfer function (e.g. Eq. 6) with a w_N = 10rad/sec frequency and 0.8 damping ratio(ζ).

$$\frac{y_m(s)}{y(s)} = \frac{w_m^2}{s^2 + 2\zeta_m w_m s + w_m^2} \quad (6)$$

IV. SIMULINK IMPLEMENTATION

The comprehensive architecture implemented within Simulink is visually presented in Figure 5. It encompasses several integral components, including a module responsible for generating target position, a function dedicated to computing and visualizing missile position, a PN (Proportional Navigation) guidance law function, and the missile autopilot.

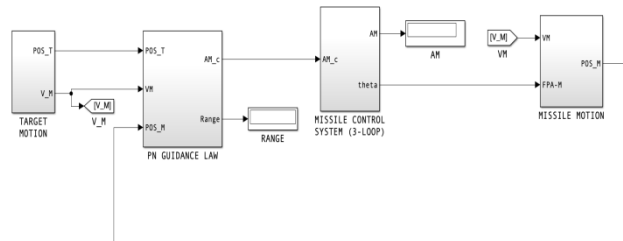


Figure 5: Missile homing loop implementation

A. Target Motion

Initial values for critical parameters like target velocity, flight path angle, and position are defined within a Matlab script. In the system shown in Figure 6 target's position is calculated by utilizing the provided initial target velocity (V_T) and the initial flight path angle (FPA).

The launch sequence logic is configured as follows: At the instant $t = 0$, the target is positioned behind the missile and is approaching it. The missile initiates its launch sequence when the target is positioned 200 meters behind it.

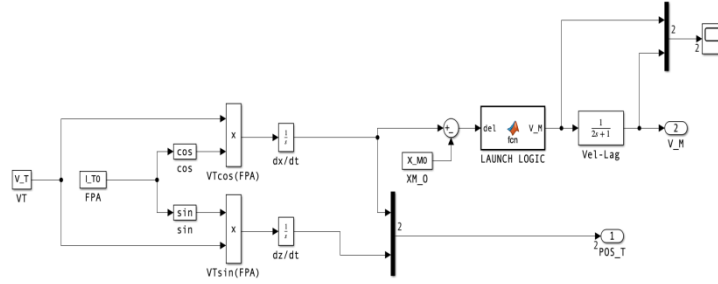


Figure 6: Target Position subsystem

B. PN Guidance law

The positional information of both the target and missile from the target position subsystem is then directed into the PN guidance law function block shown in Figure 7. This function accomplishes two primary tasks. Firstly, it computes the range between the target and the missile and identifies the moment when the derivative of this range becomes positive. This factor is used to stop the simulation. Secondly, it generates the acceleration command by implementing the PN guidance law discussed in section 2. The PN guidance constant, N is empirically determined through a trial-and-error process and is set to a value of 3 to achieve the desired system performance in this paper.

To maintain a controlled acceleration within specified limits, an essential saturation block is incorporated. This saturation block ensures that the acceleration remains within the range of -0.10 to 0 .

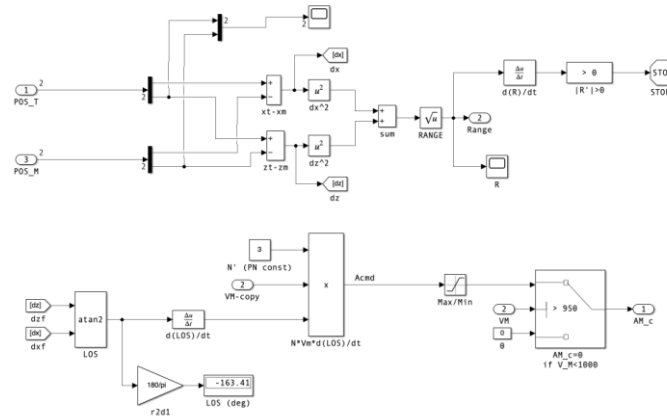


Figure 7: PN guidance law subsystem

C. Missile Autopilot

The commanded acceleration acts as the reference signal for a 3-loop autopilot control system shown in Figure 8. The autopilot gains are carefully chosen to ensure a time constant of less than 0.2 seconds while minimizing overshooting. The process of tuning these control gains involves utilizing root locus and bode plot analysis techniques. Initially, the inner pitch loop gains are adjusted, starting with the proportional gain K_p followed by the integral gain K_i while keeping all other parameters constant. Once a satisfactory frequency response is achieved, the gains for K_a and K_{dc} are fine-tuned. The resulting values for the autopilot gains are as follows: $K_{dc} = 1.1$, $K_a = 4.5$, $K_i = 14.3$, $K_p = -0.37$.

In addition to the aforementioned control components, F1 and F2 represent the transfer functions discussed in section C, while F3 accounts for the dynamics of the actuator. This particular block yields the output of the controlled missile acceleration.

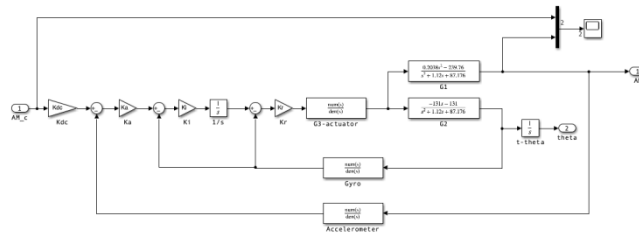


Figure 8: Autopilot subsystem

D. Missile position subsystem

This block takes the updated pitch angle from the autopilot block and calculates the real-time position of the missile as shown in eq. 8-9. The resulting current velocity of the missile is fed back into the guidance law block to update the acceleration command in order to track the target.

$$\frac{dx}{dt} = V \cos(\theta) \tag{8}$$

$$\frac{dz}{dt} = V \sin(\theta) \tag{9}$$

V. RESULTS

The successful interception of the missile with the target is vividly depicted in Figure 9. During this interception sequence, the missile consistently exhibits stable behavior and adeptly adheres to the desired trajectory. The culmination of this precise maneuver results in the interception event occurring approximately 30 seconds into the mission. The stability and robustness of the missile are analyzed using the Bode plot [11].

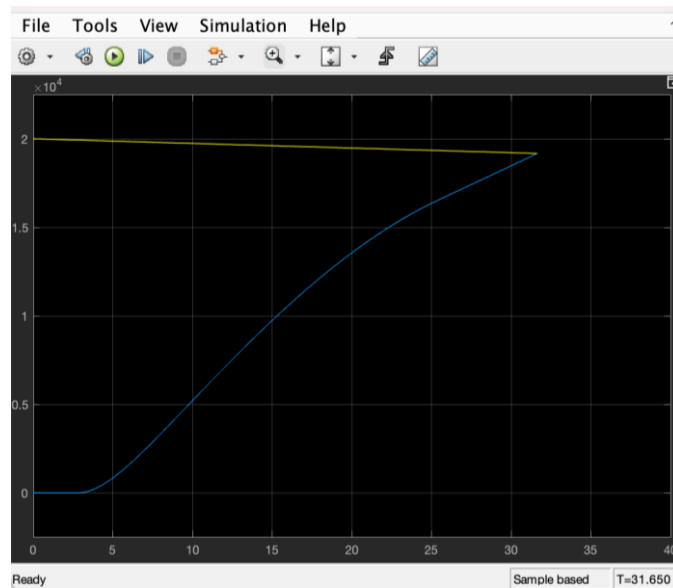


Figure 9: Simulation plot showing interception

VI. CONCLUSION

The missile's flight control system represents a crucial and ever-evolving component in the realm of modern weaponry. This sophisticated subsystem is not static; instead, it constantly undergoes refinements and enhancements to adapt to the ever-changing landscape of combat and technology. The dynamic nature of this system is essential, as it plays a pivotal role in ensuring the missile's accuracy, reliability, and overall effectiveness in various operational scenarios.

The design of this subsystem is a continuous journey of improvement and innovation. It's not a one-time effort but an ongoing process that seeks to maximize the missile's performance potential. The primary goal is to fully harness the capabilities of the missile's airframe, enabling it to operate at the highest level of efficiency and precision.

In the face of evolving high-stress threats and challenging scenarios, the flight control system's adaptability and progress are of paramount importance. As adversaries develop more sophisticated defense mechanisms and strategies, the missile's flight control system must keep pace to effectively engage and neutralize these threats. The ongoing refinement of this subsystem ensures that it remains at the forefront of technology, enabling the missile to tackle a wide range of mission profiles and deliver precise, timely, and reliable results.

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