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Research Paper

An influence of an exponential anisotropic contribution on the pressure and Cauchy stress tensor into an artery affected by a stenosis

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Abstract: *In this paper we have proposed as a work to highlight influence of the exponential anisotropic contribution on the pressure of a stenosis artery and then on the Cauchy stress components. The geometry of the stenosis has been given, which allowed us to calculate and simulate the elementary invariants and components of the Cauchy stress tensor of a stenosis artery with an application on three potentials in order to see this influence. Through mathematical calculations we also show that the stenosis makes the artery lose its capacity to be incompressible and we determine an exact solution of the internal pressure from certain boundary conditions. The simulation of non zero components of Cauchy stress tensor and pressure accordind to the three models allowed us to show how the fibrous reinforcement from exponential model can influence pressure and stress components of the artery when it has stenosis.*

Keywords: *Artery stenosis, isotrope and anisotrope elementary invariants, compressibility, Cauchy stress tensor, internal pressure.*

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I. Introduction

Cardiovascular diseases or diseases of the circulatory system is the leading cause of death in developed countries. It is for this reason that the study of arterial structures and their prosthetic substitute constitutes an issue of primary importance for biomedical research [1,2]. The description of the anisotropic hyperelastic mechanical behavior of a mechanical cylindrical tube is still useful to better understand the diseases that plague the cardiovascular system [3]. For the achievement, many kinematics translating the geometries of these arteries when they are healthy as well as when they are affected by pathologies have been defined. These studies of many variables and tensors allow many authors to obtain expressions as invariants, stresses and internal pressure with certain condition[3]. To reach their objective in the case of certain biomechanical models, the authors must choose on various strain energy functions which allow to realize such work among which we can quoted the polynomial, exponential, power or logarithmic form [4]. These energy potentials have been established as part of a phenomenological approach that describes the macroscopic nature of arteries and there are functions of elementary invariants [4]. Most of these mechanical studies have different and diverse objectives and one part of this is most often concentrated in the analysis of stresses and pressure in incopressible or compressible, isotropic or anisotropic case [5,6,7]. In an other part there are interested to explains and demonstrates that flow through Venturimeter is comparable to flow through stenotic artery, discarding other complicated physiological factors [8]. As a contribution in the modeling of cardiovascular diseases, we are interested to know how the artery behaves at the level of invariants, stress components and internal pressure when it is reached by stenosis from its defined geometry. We will choose three models of energy potentials in order to determine an exact solution of the internal pressure. The elementary invariants, the non-zero components of the Cauchy stress tensor and the internal pressure of the stenosis artery will be simulated and analyzed in order to highlight the differences that can be observed between these mathematical objects. These simulations will allow us to see the influence of the three models in order to provide better data for the fibrous manufacture and improvement of vacular prosthetic substitutes.

II. Formulation of the problem

Let's consider continuous cylindrical hyperelastic tube which is an artery with stenosis, where a material point occupies the position (R, Θ, Z) before the deformation and the position (r, θ, z) after deformation which is represented as in [9] by the following kinematic

$$r(R,Z) = R - \delta\left(1 + cos\left(\frac{Z}{2}\right)\right) : -2\pi \leq Z \leq 2\pi; \quad \theta = \Theta; \quad z = Z. \qquad (1)$$

It follows the gradient tensor of deformation which is difined by:

$$\mathbf{F} = \begin{pmatrix} \lambda_{rr} & 0 & \lambda_{rz} \\ 0 & \lambda_{\theta\theta} & 0 \\ 0 & 0 & \lambda_{zz} \end{pmatrix}; \qquad (2)$$

where $\lambda_{rr} = \partial r/\partial R$, $\lambda_{\theta} = \frac{r}{R}(\partial\theta/\partial\Theta)$, $\lambda_{zz} = \partial z/\partial Z$ and $\lambda_{rz} = \partial r/\partial Z$.
To measure the tranformation we can then introduce the left Cauchy-Green tensor noted $\mathbf{B}$ and defined by:

$$\mathbf{B} = \begin{pmatrix} \lambda_{rr}^2 + \lambda_{rz}^2 & 0 & \lambda_{rz} \\ 0 & \lambda_{\theta\theta}^2 & 0 \\ \lambda_{rz} & 0 & \lambda_{zz}^2 \end{pmatrix}. \qquad (3)$$

The adjoint of the tensor $\mathbf{B}$ noted $\mathbf{B}^*$ is given by:

$$\mathbf{B}^* = \begin{pmatrix} \lambda_{\theta\theta}^2 & 0 & -\lambda_{rz}\lambda_{\theta\theta}^2 \\ 0 & 1 & 0 \\ -\lambda_{rz}\lambda_{\theta\theta}^2 & 0 & (1+\lambda_{zz}^2)\lambda_{\theta\theta}^2 \end{pmatrix}. \qquad (4)$$

In addition, to take account of deformations in preferred directions, for example in the case of a fibrous reinforcement, a unit vector $\mathbf{M}\,(M_R, M_\Theta, M_Z)$ is introduced representing the fibrous reinforcement in the non-deformed configuration, which gives us the direction of the fibrous reinforcement in deformed configuration denoted $\mathbf{m}$ defined by $\mathbf{m} = \mathbf{FM}$.
We can then calculate the first five isotropic or anisotropic elementary invariants of deformation:

$$\begin{aligned} I_1 &= tr(\mathbf{B}) = \lambda_{\theta\theta}^2 + \lambda_{rz}^2 + 2; \\ I_2 &= tr(\mathbf{B}^*) = \lambda_{\theta\theta}^2 + \lambda_{rz}^2 + 2; \\ I_3 &= det(B) = \lambda_{\theta\theta}^2 \\ I_4 &= \mathbf{m}.\mathbf{m} = \gamma^2 + \lambda_{\theta\theta}^2 M_\Theta^2 + \lambda_{zz}^2 M_Z^2; \\ I_5 &= \mathbf{m}.\mathbf{Bm} = \left[\left(1+\lambda_{rz}^2\right)\gamma - 2\lambda_{rz}M_Z\right]\gamma + \lambda_{\theta\theta}^4 M_\Theta^2 + \lambda_{zz}^4 M_Z^2; \end{aligned} \qquad (5)$$

where $\gamma = M_R + \lambda_{rz} M_Z$.

To verify the incompressible hypothosis which is translated mathematicaly by:

$$I_3 = 1; \qquad (6)$$

we obtain this following second degree equation with cosine:

$$\frac{\delta}{R}cos^2\left(\frac{Z}{2}\right) + 2\left(\frac{\delta}{R} - 1\right)cos\left(\frac{Z}{2}\right) + \left(\frac{\delta}{R} - 2\right) = 0. \qquad (7)$$

An equation which always admits two distinct solutions whatever δ and R because its discriminant Δ is always equal to 4 means greater than Zero.
With the choice of $\delta = 0.1 * R$ and $R = 8mm$ in [10], the previous equation has as solutions $\alpha_1 = -1$ and $\alpha_2 = 19$. As we can see, α_2 can not be a solution of the function cosine. So then the only good solution of the equation gives:

$$Z = \pm 2\pi. \tag{8}$$

As a generalization of this study we can set the condition $\delta = \beta R$ with $\beta \in \left]0; \frac{1}{2}\right[$ because according to our kinematics, $\beta = \frac{1}{2}$ is a limit condition for that the stenosis block the artery, the previous condition of the generalization gives us these following two distinct solutions:

$$\alpha_1 = -1; \qquad \alpha_2 = -1 + \frac{4}{2\beta}. \tag{9}$$

And for $\beta \in \left]0; \frac{1}{2}\right[$ we obtain the following inequality

$$\alpha_2 = -1 + \frac{4}{2\beta} > 3. \tag{10}$$

So α_2 can not be a solution of the trigonometric function cosine.

Remark 1
We can remark about this solution of Z that there is no deformation of the arterial radius at this value. The analysis of the values of the two distinct solutions in the general case shows that there is no good solution of the cosine function when the deformation of the arterial radius caused by the stenosis begins.
So we find our first result of this study by showing that a stenosis artery loses his property of incompressibility because the first solution is obtained for $Z = \pm 2\pi$ ie when there is no deformation and the second solution is not a value of the cosine function.

To characterize the state of stress, that is to say the internal forces brought into play between the deformed portions of a material in mechanics of continuous mediums, the Cauchy stress tensor noted $\mathbf{T}$ is used. So that in the compressible and anisotropic case [11,12], it is given by:

$$\begin{aligned} \mathbf{T} = & \frac{2}{\sqrt{I_3}} \left[W_1 \mathbf{B} + (I_2 W_2 + I_3 W_3)\, \mathbf{1} - I_3 W_2 \mathbf{B}^{-1} \right] \\ & +2 \left[W_4 \mathbf{m} \otimes \mathbf{m} + W_5 \left(\mathbf{m} \otimes \mathbf{Bm} + \mathbf{Bm} \otimes \mathbf{m} \right) \right]; \end{aligned} \tag{11}$$

where $\mathbf{1}$ represents the identity tensor and the $W_i (i = 1, 2, .., 5)$ are given by $W_i = \partial W / \partial I_i$ with W an energy function of deformation.
With the hypothesis of the absence of volume forces, the equilibrium equations are summarized as:

$$div\,(\mathbf{T}) = 0. \tag{12}$$

In a cylindrical coordinate system, the equilibrium equations are reduced to:

$$\begin{cases} \frac{d}{dr}(T_{rr}) + \frac{T_{rr} - T_{\theta\theta}}{r} & = 0 \\ \frac{d}{dr}(r^2 T_{r\theta}) & = 0 \\ \frac{d}{dr}(r T_{rz}) & = 0 \end{cases} \tag{13}$$

where the components of (13) are given by:

$$T_{rr} = \begin{cases} \frac{2}{\sqrt{I_3}} \left[\left(1 + \lambda_{rz}^2\right) W_1 + (I_2 - I_3) W_2 + I_3 W_3 \right] \\ +2 \left[\gamma^2 W_4 + 2\gamma \left(\gamma \left(1 + \lambda_{rz}^2\right) - \lambda_{rz} M_Z \right) \right] \end{cases}$$

$$T_{\theta\theta} = \begin{cases} \frac{2}{\sqrt{I_3}} \left[\left(\lambda_{\theta\theta}^2\right) W_1 + \left(I_2 - \frac{I_3}{\lambda_{\theta\theta}^2} \right) W_2 + I_3 W_3 \right] \\ +2 \left[\left(\lambda_{\theta\theta} M_\Theta\right)^2 W_4 + 2 \left(\lambda_{\theta\theta}^2 M_\Theta\right)^2 \right] \end{cases} \tag{14}$$

$$T_{zz} = \left\{ \begin{array}{c} \frac{2}{\sqrt{I_3}} \left[W_1 + \left(I_2 - I_3 \left(1 + \lambda_{rz}^2 \right) \right) W_2 + I_3 W_3 \right] \\ +2 \left[M_Z^2 W_4 + 2 \left(M_Z^2 - \gamma \lambda_{rz} M_Z \right) \right] \end{array} \right.$$

$$T_{rz} = T_{zr} = \left\{ \begin{array}{c} \frac{2}{\sqrt{I_3}} \left[\lambda_{rz} W_1 + \lambda_{rz} I_3 W_2 \right] + 2\gamma M_Z W_4 + 2\gamma \left(M_Z - \lambda_{rz} \gamma \right) W_5 \\ +2 \left[\left(\left(1 + \lambda_{rz} \right) \gamma - \lambda_{rz} M_Z \right) M_Z \right] W_5 \end{array} \right.$$

For the purpose of determining an expression of the pressure we can choose the boundaries conditions studied in [12] by:

$$T_{rr}(a) = 0; \qquad T_{rr}(b) = p, \tag{15}$$

where p is the internal pressure and a and b two elements of $]0, R]$.
The use of relation $(13)_1$ and the conditions (15) allows us to have:

$$p = \int_a^b \left(T_{\theta\theta} - T_{rr} \right) \frac{dr}{r}. \tag{16}$$

The pressure depends only on the first and second principal components of the Cauchy stress tensor which do not depend on the radius r.

The calculations with all the conditions gives us an exact solution of the pressure which is given by:

$$p = \left(T_{\theta\theta} - T_{rr} \right) log\left(r \right) + p_0; \tag{17}$$

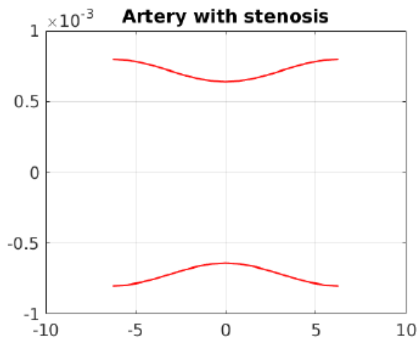
a pressure which is a logarithmic solution where p_0 is the initial internal pressure.

3 Application and simulation

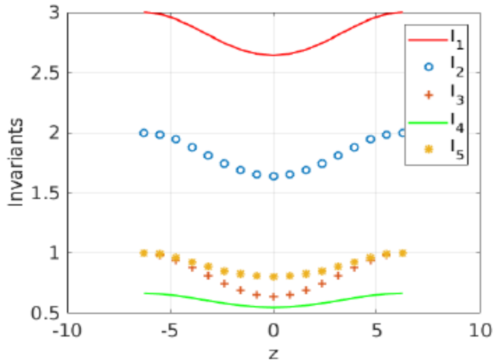
In this paragraph we will make an application on three cases. So then we will consider an isotropic and power anisotropic energy functions defined in [1]. We will also consider an anisotropic energy functions with exponential anisotropic contribution [13]. We will simulate the pressure and Cauchy stress tensor components to see how every model influences the behavior of those components and pressure. Our study will be limited to the case where the fibrous contribution of the fifth invariant will be negligible, which means that our three models will not be functions of I_5.
But before that, we will simulate the elementary invariants to better understand how the stenosis influences these scalars.

3.1 Invariants

For the simulation of the invariants of our kinematic, we choose $R = 0.8mm$ and $\delta = 0.1 * R$ as in [10], that gives us the following geometry:



The geometry that shows how stenosis disease decreases the arterial radius.
And with this geometry, the invariants follow the paces below.



The simulation of the isotropic and anisotropic invariants shows how these latter follows sinusoidal forms in the shape of a hollow as the kinematics of deformation shows. Another observation is that these invariants are all pair functions of axis of symmetry the vertical line of equation $z = 0$.
We note that the isotropic invariants excepted from the fact that they do not have the same values have the same behavior and can be superimposed with the first invariant which records larger values followed by the second invariant which in turn is followed by the third invariant. And when we choose a value of z, we see that the value I_2 remains the average of the two values I_1 and I_3.
The two anisotropic invariants remain close to each other with I_5 which is slightly above I_4. It should be noted that the anisotropic invariants have I_3 between them.

3.2 Pressures and stresses

Firstly let's consider the deformation energy function in isotropic and incompressible case of the Diouf-Zidi model [2] for the modeling of the human carotid

as

$$W = \frac{\mu}{2}\left[(I_1 - 3) + a_1 (I_2 - 3) + a_2 \left(I_3^{1/2} - 1\right)^2\right]. \tag{18}$$

where μ, a_1 and a_2 are material parameters.
The calculation of the partial derivatives of energy function W gives us the following relations:

$$\begin{cases} W_1 = \frac{\mu}{2} \\ W_2 = \frac{\mu a_1}{2} \\ W_3 = \frac{\mu a_2}{2\sqrt{I_3}}\left(\sqrt{I_3} - 1\right). \end{cases} \tag{19}$$

Secondly, we consider the deformation energy function in incompressile of the Diouf-Zidi model [2] for the modeling of the human carotid but this time with a power anisotropic contribution case by:

$$W = \frac{\mu}{2}\left[(I_1-3)+a_1(I_2-3)+a_2\left(I_3^{1/2}-1\right)^2+\alpha(I_4-1)^3\right]. \qquad (20)$$

where μ, a_1, a_2 and α are material parameters. So the partial derivatives of this energy function gives us the following relations:

$$\begin{cases} W_1 = \frac{\mu}{2} \\ W_2 = \frac{\mu a_1}{2} \\ W_3 = \frac{\mu a_2}{2\sqrt{I_3}}\left(\sqrt{I_3}-1\right) \\ W_4 = \frac{3\alpha\mu}{2}(I_4-1)^2. \end{cases} \qquad (21)$$

lately a deformation energy function studied in [13] of an exponential anisotropic contribution for human artery is given:

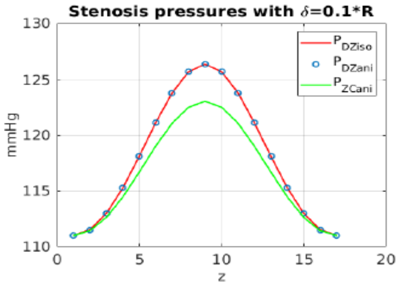
$$W = \frac{\mu}{2}\left[(I_1-3)+a_1(I_2-3)+a_2\left(I_3^{1/2}-1\right)+a_3 ln\left(\sqrt{I_3}\right)\right]+k_0\left[exp\left(k_1(I_4-1)^2\right)-1\right]. \qquad (22)$$

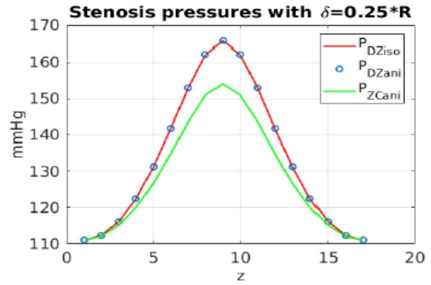
where μ, a_1, a_2, a_3, k_0 and k_1 are material parameters. that yields us these following partial derivatives of this energy function:

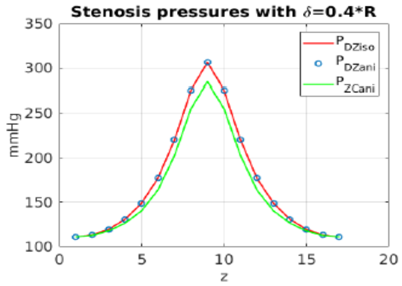
$$\begin{cases} W_1 = \frac{\mu}{2} \\ W_2 = \frac{\mu a_1}{2} \\ W_3 = \frac{\mu}{4\sqrt{I_3}}\left(a_2+\frac{a_3}{\sqrt{I_3}}\right) \\ W_4 = 2k_0k_1(I_4-1)\,exp\left(k_1(I_4-1)^2\right). \end{cases} \qquad (23)$$

And with these previous expressions, we get these following graphs of pressures and stress components:

3.2.1 Pressures







The simulation of pressures from the three models shows increasing sinusoidal bahavior when the stenosis progress with two types of paces: the Diouf-Zidi isotrope and anisotrope models have the same and greatest values for a value of z but the Zidi-Cheref model records the lower values. We can also see that more the stenosi progress, more the diffrence between the both Diouf-Zidi models and the Zidi-Cheref model is biger.

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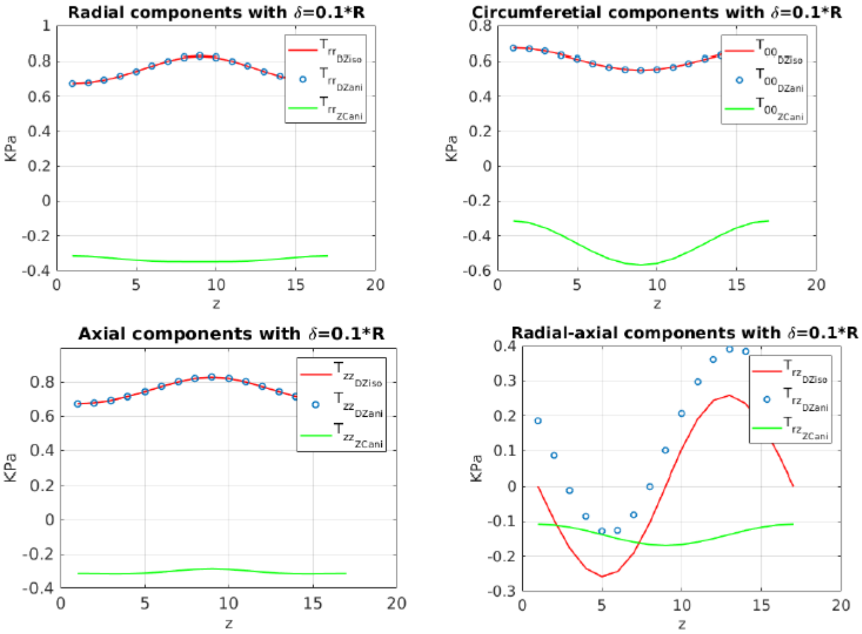
3.2.2 Stresses



The simulation of Cauchy stress tensor components from the three models shows increasing or decreasing sinusoidal bahavior when the stenosis progress. We have the main components of Diouf-Zidi which follow the same pace which is different to that of Zidi-Cheref model. The radial-axial components of the Diouf-Zidi models have the same bahavior with a small difference between them with the anisotrpic biger than the isotropic model and the radial-axial component of Zidi-Cheref model follows a different pace.

Remark 2

The simulation allowed us to see that the pressures of the different models describe the parables with the two Diouf-Zidi models that follow the same values. The same behavior is also observed on the level of the Cauchy stress tensor components of the Diouf-Zidi models excepted the radial-axial component which gives small difference. As an important result, we have shown through these simulations that the exponential anisotrope contribution have an influence on pressure and stress components of an artery affected by stenosis with certain material parameters.

IV. Conclusion

In this paper we have proposed a modelization of mechanical behavior of an artery affected by stenosis. The kinematics of deformation translating the disease of the stenosis is defined. A kinematics which allowed us to determine the mechanical tensors which in turn allowed us to calculate the five isotropic and anisotropic invariants of the stenosis. From these invariants, we have mathematically showed that the stenosis disease forces the artery to lose its compressibility. We defined subsequently the Cauchy stress tensor in compressible and in the anisotropic case. By neglecting the volume forces and with certain boundary conditions, we have determined the different components of the Cauchy stress tensor then give an exact solution of the internal pressure which is a logarithmic form. The application on three models shows that the exponential anisotrope contribution have an influence on pressure and stress components of an artery affected by stenosis. Pressure and Cauchy stress tensor components allowed us to highlight that the exponential model translate a better behavior of an artery affected by stenosis and will be a good tool for the manufacture and improvement of vascular substitutes.

V. Outlooks

As perpectives of our learning in biomechanic, this study can be the begining of a very important subject in the development of a model generalizing the behavior of diseases that deform the artery for the improvement of the vascular substitutes.

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