Quest Journals
Journal of Research in Applied Mathematics
Volume 10 ~ Issue 10 (2024) pp: 53-57
ISSN (Online): 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Review Paper

Thermodynamics analysis of non-interacting Barrow holographic dark energy in Brans-Dicke theory

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Abstract
In the present paper, we study Barrow holographic dark energy in Brans-Dicke theory of gravity for flat Frieddman-Lemaitre-Robertson-Walker metric. We have assumed power law form of Brans-Dicke scalar field in terms of scale factor . The thermodynamics analysis of the model is discussed. It is observed that the model satify generalized second law of thermodynamics for all values of the parameter in past, present as well as in the future universe.

***R**eceived 14 Oct., 2024; **R**evised 24 Oct., 2024; **A**ccepted 27 Oct., 2024 © The author(s) 2024.*
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I. Introduction

The present day universe is in accelerating phase compelled us to think beyond ΛCDM model. The Precision Cosmology has greatly enhanced our understanding of the universe[1]. The first and most accepted model of DE is cosmological constant model popularly known as ΛCDM model. But this model has some issues like fine-tuning and cosmic coincidence problems [2] with it. Recently a very new problem, H_0 tension is diagnosed with ΛCDM model. Therefore, we have to look for some another possibility to retain the universe accelerated expansion scenario. One option can be general relativity(GR) as the underlying gravitational theory, introduces the concept of dark energy[3, 4, 5]. The second possibility involves modified theories of gravity which gives a finer explanation[6, 7]. The DE which is responsible for the universe accelerated expansion does not reflects it's nature and origin. The holographic principle may provide the origin of DE with the application of it's cosmological implications[8, 9]. The dynamical DE models based on the holographic principle , introduced by 't Hooft [10] and further discussed by L. Susskind [11], have earned a lot of attention to explain the accelerated expansion and problems of the ΛCDM model.

The HDE models able to explain the accelerated expansion of the universe and also fits with the observation data as well[12, 13, 14]. Here in the present paper, we use Barrow entropy instead of BekensteinHawking (BH) one. In this context, a comprehensive outlook is proposed by Barrow[15] which relates BH area law[16] in the form

$$S_\Delta = (\frac{A}{A_0})^{1+\Delta/2} \tag{1}$$

where A and A_0 are the black hole horizon, Planck area and Δ be the exponent which calculate the range of quantum-gravitational deformation effects. The quantum effects of gravity are characterized by Barrow exponent $0 = \Delta = 1$. If $\Delta = 0$, it will give rise to BH limit while $\Delta = 1$ corresponds to maximal entropy deformation. BHDE is motivated by concept of HDE based on Tsallis entropy[17], Barrow quantum-gravity corrected entropy[18], Kaniadakis entropy[19], power-law corrected entropy [20] etc. BHDE is the generalization of HDE. In standard HDE, the energy density of the universe is related to horizon area whereas in BHDE the energy density of universe is defined as

$$\rho_D = B\phi^{2+\Delta}H^{2-\Delta}. \tag{2}$$

Here B is constant. Here in the present paper we extend the work of [21] where the authors have studied the BHDE in BD theory without the thermodynamics implications of the model which is an important parameter to discuss the systematic growth of the universe.

In what follows, this manuscript is organized as follows: Section 2. describes model and field equation of BHDE. In Section 3. the mathematical calculations are performed to discuss the thermodynamics analysis of BHDE. We ends in Section 4. with conclusion of the work.

2 Model and field equations

We consider a homogenous and isotropic Friedmann-Robertson- Walker(FRW) metric

$$ds^2 = -dt^2 + a^2(t)\Big(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\Big), \tag{3}$$

, where $a(t)$ is the scale factor of the universe and k is the curvature of the universe i.e flat, open or closed geometry of the universe corresponding to $k = 0, -1, 1$ respectively. Here, we study the BHDE model for flat univese i.e $k = 0$.

The Brans-Dicke field equations for flat FRW line element with pressureless Dark matter(DM) and Dark energy(DE) are given by

$$\frac{3}{4\omega}H^2\phi^2 - \frac{1}{2}\dot{\phi}^{\,2} + \frac{3}{2\omega}H\phi\dot{\phi} = \rho_m + \rho_D \tag{4}$$

$$\frac{-1}{4\omega}\phi^2\Big(2\frac{\ddot{a}}{a} + H^2\Big) - \frac{1}{\omega}H\phi\dot{\phi} - \frac{1}{2\omega}\phi\ddot{\phi} - \frac{1}{2}\Big(1 + \frac{1}{\omega}\Big)\dot{\phi}^2 = p_D \tag{5}$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{3}{2\omega}\Big(\frac{\ddot{a}}{a} + H^2\Big)\phi = 0, \tag{6}$$

where ρ_m is energy density of pressureless DM, ρ_D and p_D are respectively the energy density and pressure of BHDE, and $H = \dot{a}/a$ is the Hubble parameter. Following [21], we use power law form of scale factor as

$$\phi \propto a^n \tag{7}$$

, where a is the scale factor of the universe. Taking time derivative of scale factor with respect to cosmic scale factor t, we get

$$\dot{\phi} = nH\phi \tag{8}$$

,

$$\ddot{\phi} = n^2H^2\phi + n\dot{H}\phi \tag{9}$$

. The conservation equation for non-interacting BHDE are given by

$$\dot{\rho}_m + 3H\rho_m = 0, \tag{10}$$

$$\dot{\rho}_D + 3H(1 + w_D)\rho_D = 0, \tag{11}$$

where $w_D = p_D/\rho_D$ is the Equation of state parameter(EoS) of BHDE. Now, let us define the fractional energy densities corresponding to each energy part as

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{4\omega\rho_m}{3\phi^2H^2} \tag{12}$$

$$\Omega_D = \frac{\rho_D}{\rho_{cr}} = \frac{4\omega\rho_D}{3\phi^2H^2}, \tag{13}$$

where $\rho_{cr} = 3\phi^2 H^2/4\omega$ is the critical density in BD theory. Taking time derivative of equation (2), we get

$$\dot{\rho}_D = (2+\Delta)nH\rho_D + (2-\Delta)\frac{\dot{H}}{H}\rho_D. \tag{14}$$

Now differentiating equation (4) and combining the equation (10), (11), (12), and (13) we get an expression of the form

$$\frac{\dot{H}}{H^2} = \frac{\frac{-9\Omega_m H^2 \Phi_0^2}{4\omega\phi^2 H^2} + (2\omega n^2 - 6n - 3)\frac{n}{2\omega} + nB(2+\Delta)(\frac{\phi}{H})^{\Delta}}{B(\Delta-2)(\frac{\phi}{H})^{\Delta} + \frac{3}{2\omega} - n^2 + \frac{3n}{\omega}} \tag{15}$$

3 Thermodynamics Analysis

Jacobson was the first who introduced the relation among gravity and thermodynamics with Einstein's field equations using only thermodynamical considerations [22]. Padmanabhan [23] derived the first law of thermodynamics starting from the Einstein's field equations for a static and spherically symmetric space-time. The generalized second law of thermodynamics(GSL) is one of the important study of laws of thermodynamics. According to GSL, total entropy of the universe (horizon entropy and entropy of the matter inside the horizon) is an increasing function of the time. The total entropy of the universe is given by

$$S_{tot} = S_h + S_{in}, \tag{16}$$

where, S_{tot} denotes total entropy, S_h denotes horizon entropy and, S_{in} denotes entropy of total fluid inside the horizon. The rate of change of total entropy S_{tot} can be obtained as

$$\dot{S}_{tot} = \dot{S}_h + \dot{S}_{in}, \tag{17}$$

where dot represents the time derivative. The entropy of the apparent horizon takes the form as $S_h = \frac{8\pi^2}{H^2}$ and it's rate of change is given by

$$\dot{S_h} = -16\pi^2\frac{\dot{H}}{H^3}. \tag{18}$$

The Gibbs law of thermodynamics for fluid inside the horizon produces the relation

$$T_{in}dS_{in} = dE_{in} + p_t dV_h, \tag{19}$$

where the subscript t denotes the total quantity and the volume $V_h = \frac{4}{3}\pi R_h^3$. Now, we can obtain the rate of change in entropy of the fluid inside the horizon as

$$\dot{S}_{in} = \frac{(\rho_t + p_t)\dot{V}_h + \dot{\rho}_t V_h}{T_{in}}. \tag{20}$$

If we consider that the fluid is in thermal equilibrium with the horizon then temperature of fluid inside horizon (T_{in}) and temperature of dynamical apparent horizon (T_h) are same and given by

$$T_h = \frac{2H^2 + \dot{H}}{4\pi H}. \tag{21}$$

Now, the rate of change of entropy of fluid inside the horizon can be obtained as

$$\dot{S}_{in} = 16\pi^2\frac{\dot{H}}{H^3}\Big(1 + \frac{\dot{H}}{2H^2 + \dot{H}}\Big). \tag{22}$$

We can also get the rate of change of total entropy as

$$\dot{S}_{tot} = \frac{\left(\frac{4\pi\dot{H}}{H^2}\right)^2}{H\left(\frac{\dot{H}}{H^2} + 2\right)}. \tag{23}$$

Using the value of equation (15) in (23), we can easily get the value of $\dot{S}_{tot}$ i.e. the thermodynamics equation of BHDE. Let us discuss the plot of thermodynamics analysis of BHDE to understand the universe. We have plotted the graph of $\dot{S}_{tot}$ against the scale factor a for fixed values of $\omega = 150$, $\Omega_m = 0.3$, $H = 70$, $n = 0.5$ and various values of parameters Δ, B and ϕ_0 in Figure 1.. The GSL is satisfied if $\dot{S}_{tot} > 0$ otherwise the model violate the thermodynamics law of the universe. It is observed from all the trajectories in Figure 1. that our model satisfy GSL for all values of model parameter for past, present and future universe. It is observed from the figure that for small values of parameter Δ and large values of B and ϕ_0 the trajectories shows constant behavior and satisfy GSL of thermodynamics.

4 Conclusion

The laws of thermodynamics explains some fundamental physical quantities like energy, temperature and entropy and shows how these quantities behave under different situations. The GSL states that the entropy of the accelerating universe increases as we go with time. In the present study, we have analyzed GSL of BHDE model. It is observed from all the trajectories that our model satisfy GSL for all values of model parameter for past, present and future universe. It is observed from the figure that for small values of parameter Δ and large values of B and ϕ_0 the trajectories shows constant behavior and satisfy GSL of thermodynamics

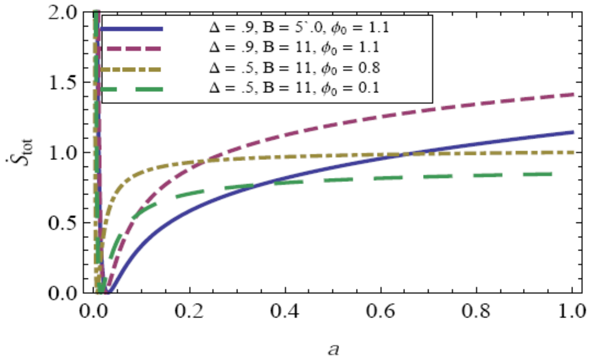

Figure 1: To plot the $\dot{S}_{tot}$ against scale factor a, we have taken $\omega = 150$, $\Omega_m = 0.3$, $H = 70$, and $n = 0.5$. The trajectories are plotted for various values of parameters Δ, B and ϕ_0.

References

[1] J.R. Primack, Nucl. Phys. B Proc. Suppl. **173**, 1 (2007).

[2] E.J.Copeland, M.Sami, S.Tsujikawa, Int. J. Mod Phys. D, **15** (2006), 1753.

[3] Y.-F. Cai, E.N. Saridakis, M.R. Setare and J.-Q. Xia, Phys. Rept. **493**, 1(2010) .

[4] K. Bamba, S. Capozziello, S. Nojiri and S.D. Odintsov, Astrophys. Space Sci. **342**, 155(2012).

[5] S. Nojiri and S.D. Odintsov, Phys. Rept. **505**, 59(2011).

[6] S. Capozziello and M. De Laurentis, Phys. Rept. **509**, 167(2011).

[7] Y.-F. Cai, S. Capozziello, M. De Laurentis and E.N. Saridakis, Rept. Prog. Phys. **79**, 106901(2016).

[8] W. Fischler and L. Susskind, hep-th/9806039.

[9] D. Bak and S.-J. Rey, Class. Quant. Grav. **17**, L83(2000).

[10] G. 't Hooft, arXiv:gr-qc/9310026 (1993).

[11] L. Susskind, J. Math. Phys. **36**,6377 (1995).

[12] X. Zhang and F.-Q. Wu, , Phys. Rev. D **72** 043524(2005).

[13] M. Li, X.-D. Li, S. Wang and X. Zhang, JCAP **06** 036(2009).

[14] C. Feng, B. Wang, Y. Gong and R.-K. Su, JCAP **09** 005(2007).

[15] E.N. Saridakis, Phys. Rev. D **102**, 123525 (2020).

[16] F.K. Anagnostopoulos, S. Basilakos, E.N. Saridakis, Eur. Phys. J. C **80**, 826 (2020).

[17] C. Tsallis, J. Statist. Phys. **52,** 479(1988).

[18] J.D. Barrow, Phys. Lett. B **808**, 135643(2020).

[19] G. Kaniadakis, Phys. Rev. E **66**, 056125(2002).

[20] S. Das, S. Shankaranarayanan and S. Sur, Phys. Rev. D **77,** 064013(2008).

[21] S. Ghaffari et al., Eur. Phy. J. C **82,** 138(2023).

[22] T., Jacobson, Phys. Rev. Lett. **75,** 1260 (21)

[23] T., Padmanabhan, Class. Quantum Grav. **19**, 5387 (2002), DOI 10.1088/0264-9381/19/21/306