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Review Paper

Wave Function Collapse Induced By Electromagnetic Radiation

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ABSTRACT
Here we propose the possibility that the pushing effect induced by electromagnetic radiation (EMR), i.e. the light pressure, can explain the intimate physical mechanism (at the moment still unknown) through which the so-called Wave Function Collapse of the hit particle occurs, whereby the particle passes instantly from a wave behavior to a corpuscular one.

In other words, the interaction of a single light quantum with a subatomic particlelocalises it in that instant, while inducing thecollapse ofitswave function(WFC).

As it is known, indeed, the observation of the microscopic world, that is the measurement of a quantum object, inexorably modify the physical system we want to examine.

According to Feynman, if we want to detect, observe, measure an electron, we need to light it, we need to point on it an electromagnetic wave with the same or shorter wavelength.

Hence, a possiblelink between measurementand EMR seems to come out.

In short, it seems likely that it is the momentum of the light quantum to be transferred to the struck particle, exerting a force on it, a pressure (the so-called Radiation Pressure) enough to induce the WFC of the measured quantum object.

Keywords:*Electromagnetic Radiation (EMR); Wave Function Collapse (WFC); Quantum Mechanics (QM); quantum objects (QO);measurement (M).*

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I. INTRODUCTION

According to de Broglie, Quantum Mechanics (QM) gives particles, that is quantum objects (QO), a their own *wave function*, indicated with Ψ(x), or simply Ψ [1].

The *wave function* (WF) is a mathematical function which depends on time (t) and on the position (x) of the particle it is referred to.And 'interesting to emphasize ttatthe WF(Ψ) of a QO describes both its wave and particle character [1].

It is how to tell that when a particle is not disturbed, i.e. before we search it, before we *measure* it,the QO *lives* just on its own, probably travelling as a wave and wide.

During this phase, defined "*linear evolution phase*, or *unitary phase* (***U*** *phase*)" [2], any particle, represented by its WF, is *not determined* and even doing our best we can never have information about its structure.

In agreement with Feynman, supposedly the particle is spread throughout the employable space, as if for each point there was associated a precise value of *probability density* we have to find it [3]. To be precise, He writes:

"The *probability*(P) of an event in an ideal experiment is given by the square of the absolute value of a complex number, ψ, which is called the *probability amplitude*"[3]:
$P = |\psi|^2$(1)
In short, as Zeilinger reminds us, "We are not able to say that a quantum system, before being observed, has well defined properties, since we cannot know them"[4]. In fact, the properties and characters of a quantum system, such a subatomic particle,before the *measurement* (M) are not well outlined. We can just presume approximately the structure and behaviour of a QO, but we have no
certitude.

II. MATERIAL and METHODS

2.1 WAVE FUNCTION EQUATION

The*linear **U** phase*of a QO has been brillantly described by the Schrödinger Equation [5] inherent to the*electron wave function* (WF).
To this purpose the first difficulty Schrödinger found, was that the WF was as a *function* of time. How to add the difference from the time (t)? As known, indeed, the classical Hamiltonian ($\boldsymbol{H}$), representing the total energy of the examined physical system, is independent by the time.
In agreement with Penrose, in the Hamiltonian representation the generalised condition positions (x1 ,.....xN) are associated to the conjugated momenta($\boldsymbol{p}$1,....$\boldsymbol{p}$N), so the *momentum* ($\boldsymbol{p}$) of a free particle is given by the velocity ($\mathbf{v}$) of the particle, times its mass (m):
$\boldsymbol{p} = \mathrm{m}\,\mathbf{v}$(2)
Thus, according to the Hamiltonian formalism, aiming to describe the total energy of the physical system we are examining, independently by the time, but by momenta and positions, we have the Hamiltonian function ($\boldsymbol{H}$):
$\boldsymbol{H}=\boldsymbol{H}\,(\boldsymbol{p}1,\ldots.\boldsymbol{p}\mathrm{N};x1,\ldots..x\mathrm{N})$ (3)
As we know, in agreement with the mathematical formalism of QM, $\boldsymbol{p}$ can be identified by a Heaviside *differential operator* ($\boldsymbol{D}$):
$\boldsymbol{D} = \frac{d}{dx}$(4)
In this identification, between $\boldsymbol{p}$ and $\boldsymbol{D}$, with the QM we have the quantum *momentum* ($\boldsymbol{p}_{\mathrm{a}}$):
$\boldsymbol{p}_{\mathrm{a}}=\frac{i\hbar d}{dx^{a}}$(5)
inherent to the *momentum* associated todx^{a} [2].
In Eq.(5) *i*indicates the *imaginary unit (*equal to√-1) and $\hbar$ the Planck's constant divided by 2π.
As Penrose reminds us, "the *method* adopted by Heaviside was to treat the operator $\boldsymbol{D}$ = d/dxas if it were a common number.The new *momentum* operator ($\boldsymbol{p}_{\mathrm{a}}$), typical of the quantum formalism, substitutes the classical *momentum* ($\boldsymbol{p}$) in the Hamiltonian classical function, see Eq.(3), according to the process known as *canonical quantization*"[2].
The $\boldsymbol{p}_{\mathrm{a}}$ in Eq.(5) was used by Schrödinger in his WF equation, occupying all the first member, adding the quantum state of the entire particle (Ψ), which varies according to the time (*t*):
$i\hbar\,\frac{\partial\Psi}{\partial t}= \boldsymbol{H}\Psi$(6)
The second member of the equation (6) expresses the energy of the examined system, that is of its WF, indicated by Ψ. With reference to Penrose [2], this energy is represented, as in the classical form, by the Hamiltonian ($\boldsymbol{H}$), but in that case it is a quantum Hamiltonian function, as:
$\boldsymbol{H}=\boldsymbol{H}\,(i\,\hbar\ d/dx1,\ldots.i\,\hbar\ d/dx\mathrm{N};x1,\ldots..x\mathrm{N})$ (7)
The Wave Functions (WF$_{\mathrm{s}}$) which can be normalised (that is the particles) constitute a complex vectorial space (an underspace of the state spaces **W**) known as 'Hilbert space', which we indicate with **HS**, to make a difference from $\boldsymbol{H}$ of the Hamiltonian. The **HS** is represented by the symbol |...⟩. As it is known,the complex number ⟨Ψ|ϕ⟩is the *conjugated complex* of⟨ϕ|Ψ⟩.
The action on |ψ⟩ from a *linear operator* $\boldsymbol{L}$, is written $\boldsymbol{L}$|Ψ⟩, and the scalar product of the ket|ϕ⟩, with $\boldsymbol{L}$|Ψ⟩ [2], is written:
$\langle\phi|\,\boldsymbol{L}|\Psi\rangle$(8)
where as it is known, when the *ket* and *bra* symbols are used, the equation is read from right to left.

In the Schrodinger evolution, $\langle\phi|\Psi\rangle$is constant in time, that is:
$\frac{d\langle\phi|\Psi\rangle}{dt} = 0$ (9)
Thus $\langle\phi|\Psi\rangle$remains unchanged in time.Let's analyse some evolution modalities of a quantum state. Let us suppose we have, at time $t = 0$, the *quantum states* $|\phi\rangle$ and $|\Psi\rangle$, and make them evolve, in line with the Schrödinger description, till time T, when the *states* become respectively
$|\phi\rangle\rightarrow|\phi_T\rangle$(10)
and:$|\Psi\rangle\rightarrow|\Psi_T\rangle$(11)
where, according to Penrose[2],Eqs.(10) and (11)indicate as a *quantum state*will evolve over time.Then:$\langle\phi|\Psi\rangle = \langle\phi_T|\Psi_T\rangle$ (12)
Therefore the Eq.(6), or *Schrödinger equation*, or *Wave Function*(WF) *Equation*, is an equation of temporal evolution indicating how the considered physical system, the particle, represented in its *quantum state* or *wave function* (WF), can develop in time.
WF Equation expresses the phase of *linear evolution* of the considered particle and it is called '***U** phase*' since it is the process of *Unitary evolution* [2].
In this regard, it could say that this ***U** evolution*indicates a particle when it is not troubled, but it develops *linearly*, normally, according to the *need*of the particle itself and its parameters.
This situation persists in time till we observe it, till we make a *measurement* (*M*), or till it interacts by chance with another particle or physical system[6].

2.2 MEASUREMENT(*M*)of a QUANTUM OBJECT(QO)

Thus, the QO we are examining is something and shows a its own property only after the *M*. In agreement with Quantum Mechanics (QM), the probable undulating aspects of a particle, of its WF, remain such until we decide to carry out a *M* in order to detect and find the particle.
In other words, we have that before the *M*, the QO is *delocalised* and behaves almost as a wave.
Briefly, according to QM, before the *M*, the particle may be represented by a combination of quantum states more or less superimposed. However it is thought that the *M* itself makes it pass to a particular state. Therefore, if we consider that an electron is localized in this or that point, the QM tells us that it can accumulate the 2 possibilities, the 2 possible states, and become the sum of an electron which is in this or that point: with the opportunity then to pass through 2 close splits in the same time, until we don't observe it.In fact, in the *experiment of the two holes* (if we do not make any *M*) the QO leaves on the screen an interference figure, typical of waves.
However, in the same experiment, if we try to see where the QO passes (as saying that we take a *M*), we have that it is localized and leaves on the screen only point like marks, as a corpuscle [7].
In short, when we make a *M*, we are going to interact with the whole particle,i.e.wih the whole QO. In other words, the*M* interferes with everything is in thespace relative to the observed particle, which, according to Penrose, is known as Hilbert Space(**HS**) [2].

2.3 MEASUREMENT's MATHEMATICAL FORMALISM

In the mathematical formalism of QM the*M* of a quantum system must be represented by a certain kind of operator ***Q***, called *observable*. Examples of observables are the 'dynamic variables': i.e. the *momentum*(***p***) and the *position*(*x*) of the considered particle.
As Penrose reminds us, the theory requires that an *observable* ***Q*** is represented by a linear operator ***L***, so that its action in **HS** is to make a linear transformation of **HS.** The linear operator ***L*** must be considered as space-time *density.*It is useful to underline that a primary request for the *quantum observables* is that their eigenvectors cover the entire **HS**. That is, the *eigenvectors* of the particle we wish to observe (its quantum superimpositions fluctuating inside the space occupied by the particle itself)must move inside the **HS**. In other words the requirement of QM leads the *real* space occupied by the particle to coincidewith the **HS [2]**.
According to these considerations, the **HS** should beconsidered a *real* space, not only hypothetical.
In QM the **HS** coincides with the *phase space* of classical physics.
The gravitational Lagrangian of Hilbert, indicated with *S*, consists essentially of the scalar curvature(*R*) divided by the constant $-16\pi G$ (where *G* is the gravitational constant) and multiplied by ε:

$S = \int_{\nu} \frac{R}{-16\pi G}\, \varepsilon$(13)

where, in agreement with Penrose,ν indicates the four-dimensional (complete) volume of space-time andε represents the quantity normally expressed as:

$\varepsilon = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \sqrt{(-\det \boldsymbol{g}_{ij})}$(14)

where, of course, x^0 is the temporal coordinate and x^1, x^2 and x^3 are the spatial coordinates [2].

In other words, the **HS**should correspond with the volume of space occupied by a particle till it is not troubled, observed, *measured*,that is during the time, the phase in which the particle is undetermined, *not localized*: the so-called ***U** phase*, which corresponds to the "*Process 2*" described by von Neumann [8].

In compliance with the rules of QM, the result of a *M*, related to an operator ***Q***, is always one of the two eigenstates: this is the *jump* of the quantum state, or Wave Function Collapse (WFC), which occurs with the *Reduction Process*(***R** Process*),or *Amplitudes Reduction* inherent to the WF of the measured QO [2]. The ***R** Process* corresponds to the "*Process 1*" described by von Neumann [8].

In other terms,"whatever the state before the *M*, it *jumps* in one of the ***Q**eigenstates*, as soon as the state (that is the particles in exam) is *measured.* After the *M* the state gets a definite value for the *observable **Q***, namely the *eigenvalue q*. If the *M* is repeated, the second*M* will give the same *eigenvalue*, that is the same result we got with the first *M*"[2].

In short, when the *observable **Q*** is measured on the state $|\Psi\rangle$, the rule is that the *probability* tells us that the state *jumps* from $|\Psi\rangle$ to the *eigenstate* $|\phi\rangle$of***Q***[9]**.**

This *probability*is mathematically represented as follows:

$|\langle\Psi|\phi\rangle|^2$(15)

Therefore, in agreement with Penrose, the *jump* of the WF(Ψ), or Wave Function Collapse (WFC), induced by any kind of *Measurement* (*M*), is just represented by the formula $|\langle\Psi|\phi\rangle|^2$[2].

2.4 WAVE FUNCTION COLLAPSE

To this purpose, as we all know, the *M* leads to the *collapse* of WF(WFC) of the observed particle, working in the **HS** relative to the same particle. Hence, when we make a *M*, we work on the particle, i.e. on the QO, not only interacting with its more external region, but also and more interacting with its internal structure, by altering violently its inner configuration, its internal space, and so the arrangement and positioning (probably fluctuating) of quantum superimpositions that characterize the particle.

It is thought, indeed, that before the *M* the electron could be found potentially in one of the several points of its wave volume, each corresponding to a *probability amplitude*, to a probability density. It really seems that when the electron (or another particle) is not disturbed, that is no *M* is carried out, it stays in its natural state: it *lives* as a QO.

Thus it occupies a volume, it is spread in the space which is allowed to it (it is *delocalized*), and it is represented by superimposed quantum state: it tend to behave as a wave. On the contrary, after the *M* of the QO, the particle is detectable: now it is *localized* in a very small space.

In this respect, indeed, Feynman points out: "The WF for a single particle is a 'field', in the sense that it is a *function of position*"[3]. To this purpose,Miller writes "In agreement with QM the falling electron can be in any position, since its WF is diffused throughout the space" [10].

Feynmanadds:"The function $\Psi(x)$ is usually called '*the Wave Function*' because it more often than not has the form of a *complex wave* in its variables" [3].

Thus, in conformity with QM, the WF has all the properties of *de Broglie associated wave* related to the particle itself [1], in fact it can also be indicated as *de Broglie wave*.

In short, the *M* induces the collapse of the WF of the particle we want to examine, so it will pass from a wave behaviour to a corpuscular aspect.At this regard, Penrose argues: "It is clear that the WF is something more *real* than a simple probability wave. Schrödinger equation gives us this entity (both charged and uncharged particles): a precise evolution in time, an evolution which depends critically on how the *phase* changes from a point to another"[2].

2.4.1 COPENAGHEN INTERPRETATION

As known, physicists wondered what was the role of the observer in the *M* process of a physical system. In other words, does the chance have a role, or it doesn't, in determining the results of the *M*? To this purpose, in agreement with Bohr we cannot talk about a particle without taking in account the interaction we, observers, can have with it (in contrast with classical physics).

Bohr asserts: "The finite interaction between object and measuring agencies conditioned by the very existence of the quantum of action entails –because of the impossibility of controlling the reaction of the object on the measuring instruments if these are to serve their purpose– the necessity of a final renunciation of the classical ideal of causality and a radical revision of our attitude towards the problem of physical reality[11]". As to say that a physical theory can describe physical phenomenon only if it includes an experimental content, the observation, the *M*, which make these phenomena show (though there are modified). In this respect, Prigogine replies: "The cosmic microwave background radiation, distributed in the cosmos at 3° Kelvin, is witness to the beginning of the universe. But the idea that such radiation would be the result of *M* is absurd: in fact, who could or should measure it? It is therefore necessary in QM to have an intrinsic mechanism that leads to the observed statistical aspects: this mechanism is precisely instability, *chaos*"[12].

In short, Stewart reminder, "According to *Copenagen Interpretation*,the *M* process, in some unspecified and unkown way (don't try to ask what it might be), collapses the complex superimposed *wave function* into a single component eigenfunction"[13].

At this regard, Mermin coined the phrase "Shut up and calculate!"[14], just to summarize Copenhagen-type views.

Hence, in keeping with *Copenagen Interpretation*, you don't know the real physical instrument to make a *M* and it is useless to worry about looking for it.

2.5 POSSIBLE *MEASURING* INSTRUMENT

To this purpose, Miller points out: "It doesn't have any sense to wonder where the electron is, until a *M* is carried out, i.e. taking a picture of it: in this case **we need to light it up**, **at least with a light's quantum**, which becomes part of the *measured* system" [10] and at the same time it

induces a peculiar phenomenon of the QM : the *Wave Function Collapse* (WFC).

Agree with Penrose the WFC, induced by *M*, could represent a *real* (not only hypothetical) event,

occurring completely in the reality, so that also the space in which the *collapse* happens could be probably a real space, not imaginary: *objective* WFC.

And which is the space where the WFC occurs? It is of course the volume of space occupied by the particle before *M*, the space where the superpositions of quantum states of the observed particle move. And this space could correspond to theHilbrt Space(**HS**)[2].

Moreover, considering "the WFC a *real* event"[2], it is logical to expect that the *measuring* instrument must also be something real, a concrete object, a real physical system.

And what can it be? Let's try to think about it.

At this regard, according to Miller,we should keep in mind that "the **interaction of the single light quantum with the electron, *localises* it in that instant**"[10](while inducing its *collapse*, that is the WFC of the *observed* particle).

As we all know, indeed, what is particularly relevant is that to carry out a *measurement* (*M*), to *observe* anything in the Universe, any macroscopic object or particle, it is necessary to use an electromagnetic radiation (EMR) having a wave length shorter or equal to the diameter of the object to be observed.

To this purpose, in fact, Feynman points out: "Of course, the smaller the object or particle to be examined, the smaller has to be the wave length (λ) of the EMR used, thus bigger its *energy*"[3].In this way the EMR hits the object and, bouncing back partially towards us, will give us the information about the object examined.

In fact, to detect a subatomic particle, we have to hit it with the light, yet at the same time we deviate and modify its trajectory and morphological configuration.

Indeed, the Quantum Mechanics (QM) teaches us that the *observation* of the microscopic world, the *M* of a QO inexorably modify the physical system we want to examine. In fact,to observe electrons, we need a light because the light rebounding on electrons make them visible.

Nevertheless the light affects the result because “when light is shining on a charge and it is oscillating in response to that charge, there is a driving force in the direcction of the light beam”[15].

Hence, if we want to detect, *observe*, *measure* an electron, we need to light it, we need to point on it an EMR with a short λ. Thus, a *link* between *M* and EMRseems to come out.

In other terms, it seems that the *main character*in thisunsolved enigma, inherent to the *M*’s Process, is theEMR.

Why? The main reason is that in order to observe, to see, or make a *Measurement* (*M*), we always need to use the light. It is the only physical mean which allows us to detect a particle, analyse and study the physical system we are interested in.Only using the EMR we can acquire the information about the state and the property of the objects of the subatomic world.In short, the EMR seems likely to be the *wire* which links the observer to the physical system to be observed.

This wire allows us to get the *M* of the particle we are interested in. Without this wire we wouldn’t have any information of the subatomic world.

In other words, when we make a *M*, when we try to see and study an electron, and we shoot against it even a single light quantum (the minimum quantity of energy to be able to see it), what happens is that the electron is hit by a corpuscle with a *dynamic-mass* bigger than its rest-mass, most likely succumbing under its mechanical effect, under such a shot, thus it *collapses*.

So, every time a *M* is carried out, always using the light quantum, the *Planck’s grain* [16], theobserved particle undergoes a probabilistic *reduction of the state vector*, indicated as *Reduction Process*, or ***R** Process*. According to Penrose, with the ***R** Process* the state vector, represented by |Ψ⟩, *jumps* to another state vector, let’s say |ϕ⟩, which represents one out of two or more orthogonal alternative possibilities: the other can be |q⟩, |X⟩, etc..., which depend on the kind of observation, the kind of *M* carried out [2].

So, with the *M* we move immediately from the *phase **U***(or ***U**nitary phase*)to ***R**phase* (or ***R**eduction process*) and the induced *jump* of the quantum state is known as WFC.

It seems right to us to underline that this peculiar phenomenon is always related to the use of thePlanck’s quanta [17].

Therefore, to observe a QO we cannot do it without using a*light’s quantum*. In our opinion, more than a mere and non-specific energetic effect, to induce the WFC it is a real mechanical action exerted by the *dynamic-mass*, by “the force, the *pushing momentum*, that is delivered per second by the light. *In any circumstance where light is being absorbed, there is a pressure*”[15].

Hence, let’s analyse briefly the nature of such a radiation.

2.6 On theCONSTITUTION of RADIATION

In September 1909, as Nachel*et al*. remind us [18], Planck invited Einstein to talk to the *eighty-firstMeeting* of the “Gesellschaft Deutscher Naturforscher und Arzte”,in Salzburg, where Einstein presented a new research “On the Development of Our Views Concerning the Nature and Constitution of Radiation”, maintaining that, as an electron, every *quantum of radiation* propagates in a specific direction: technically the *quantum* has a *momentum*. Moreover, the elimination of the *ether* implies that light propagating through empty space consists of electromagnetic fields behaving as "independent structures" ("selbständigeGebilde")[18]. Furthermore, "according to the theory of relativity, light has the characteristic in common with corpuscular theory of transferring *inertial mass* from the emitting to the absorbing body"[18]. In view of the presence of both wave and corpuscular terms in fluctuations of black-body radiation, Einstein argued that a new "mathematical theory of radiation" ("mathematischeTheorie der Strahlung") was needed, which "can be considered as a sort of fusion of the wave and the emission theory of light"[18].

At this regard,as Farmelo reminds us, “for the first time Einstein suggested in public that the radiation is made of particles”[19], that is to say corpuscles, in full accordance with Newton[20].

As described first by Planck and later by Einstein, the energy(*E*) of each single “*elementary quantum of action*” [21], or *light quantum* [22],is expressed by the formula:

$E = h\, \nu$(16)

where $\nu$is the oscillation frequency of thelight quantum and *h* is the Planck’s constant.

In this respect, Feynman says: “The energy of a light-particle is a constant times the frequency:E= $h\nu$. We now appreciate that light also carries a *momentum* equal to the energy divided by *c*, soit is also true that these effective particles, these photons, carry a *momentum*”[15].Fermi adds: “The photon too, as other particles, is a corpuscle, a light's quantum and has an itsown *momentum*, through which transfers all its energy to the hit particle"[23].

As it is known, the energetic values of each light quantum, or photon, without considering its oscillatingfrequency, corresponds to the Planck constant (*h*), which is just an energetic value, corresponding to $6.626 \cdot 10^{-27}$ [erg ·sec].

Let's now analyze the *momentum* values of the light quanta.

2.6.1 On the *MOMENTUM* of PHOTON

In Newtonian Mechanics the *momentum* (*p*), or quantity of motion, is thus represented:

$\vec{p} = m \cdot \vec{v}$(17)

where *m* is the mass and v the velocity of the involved particle[24].

In Quantum Mechanics(QM), in its turn, *p* is described by the de Broglie formula:

$$\boldsymbol{p} = \frac{h}{\lambda} \quad (18)$$

where λ is the wavelength of the considered quantum radiation, or other quantum object (QO) and *h* indicates the Planck constant.

As known, indeed, de Broglie suggested to give particles the sameproperty as waves. He gave each particle a its own wave length depending only on the *momentum* (***p***) of the particle itself[1].

In agreement with de Broglie, any QO (i.e. any particle) with a*momentum*(***p***) seems to be something periodic, oscillating as a wave, with an universal relationbetween the wave length of the particle, indicated by λ, and modulus ***p*** of its *momentum*.

As Weinberg reminds us, the mean wave length of a photon in the optical bandcorresponds toabout $5 \cdot 10^{-5}$[cm][25] and in line with *de Broglie formula* its ***p*** is:

$$\boldsymbol{p} = \frac{h}{\lambda} = \frac{6.626 \cdot 10^{-27} [erg \cdot s]}{5 \cdot 10^{-5} [cm]} \quad (19)$$

$$\boldsymbol{P} = \frac{6.626 \cdot 10^{-27} [\mathrm{g} \cdot \frac{\mathrm{cm}^2}{s}]}{5 \cdot 10^{-5} [cm]} \quad (20)$$

$$\boldsymbol{p} = 1.325 \cdot 10^{-22} [\mathrm{g} \cdot \frac{cm}{s}] \quad (21)$$

As it is clear, from Eq.(21) it is evident that the *momentum* (***p***) of a visible photon should carry out an *hiddendynamic-mass*.Moreover, this hidden *dynamic-mass*carried by the *momentum* of an optic photon is bigger than the *rest mass* of 100 protons. No surprise! To this purpose, Feynman asserts: "The *momentum*, as a mechanical quantity, is difficult to hide. Nevertheless, momentum *can* be hidden –in the electro-magnetic field, for example. This case is another effect of relativity"[15]. He goes on: "In the Einstein Relativity Theory, anything which has energy has mass, mass in the sense that it is attracted gravitationally. **Even light, which has en energy, has a *mass***"[15].

It is like saying that the *momentum* carries, albeit *hidden*, a dynamic-mass.

III. DISCUSSION

As Hawking reminds us,"In accordance with Einstein's equation ($E=mc^2$), the energy is proportional to the mass" [26] and, in agreement with Relativity itself, as Feynman tells us: "**To every form of energy corresponds a mass**" [15].Therefore, it should be logicalto assume that there should not be real particles, having any energy, with a zero mass[27]. If there are, in agreement with Chandrasekhar,they could *subtend* a tiny mass, a *Zero Point Mass*[28],[29].

At this regard, in fact, Chandrasekhar points out: "It is useful to consider a fundamental consequence of the *quantum nature of the matter*: the lowest energy possible for a system cannot be null, that is zero, but it needs to have a value different from zero, it is called Zero PointEnergy(ZPE)" [29].Hence, in our opinion and in total agreement with Einstein Relativity Theory, it may be incongruous to say that a particle with energy does not have an*equivalent mass* [30],[31].

To this purpose, in fact, it is just the Mass-Energy Equivalence Principle (MEEP) equation ($E=mc^2$)
[31]to show that an energetic particle should also carry a mass value, otherwise the MEEP
equation would be null, the result would be zero.

In other words,even in the subatomic world to a very small energy, as in the case of a quantum light, a photon, should correspond a very small mass, however $\neq 0$ [32].

In this respect, one could argue: what can the *photon's dynamic-mass* be represented by?

As known, in order to obtain his formula, see Eq.(16), Planck was forced to admit that the energyof the oscillators (i.e., the electromagnetic source: an electron, for instance) can coincide only with discrete values, that is discrete quantities defined as electromagnetic(EM) energy quanta [21]. In this regard, Planck wrote: "Considering that — and this is the crucial point of the whole calculus — the energy (ε), oscillator energy, is made of a defined number of finished and same parts, we can use to this purpose the natural constant $h = 6.55 \cdot 10^{-27}$ [erg·sec]. If this constant is multiplied for the normal oscillators' oscillating frequency, (υ), we get the Energieelement (the element of energy), ε, expressed in erg·sec"[21].

Planck revealed, in fact, that he had been able to infer his formula relating to the distribution mode of the EM radiation (EMR) emitted by the *black body*, only by admitting that the EM source emits or absorbs energy only in the form of packets of energy (ε) proportional to their oscillation frequency [33]. To this purpose, He enunciated: "The essential point is to consider *energy*, at each frequency, as made of a certain number of *energieelements*, all equal to each other, indistinguishable and indivisible" [33].

In short, according to Planck, each of them represents an "elementary quantum of action"[21] corresponding to the Planck's constant: *h*. Consequently, as Kumar states, "Planck was forced to divide the energy (ε) into blocks of units (packets) $h \cdot v$"[34].

Thus, in our opinion, it is just the Planck constant,*h*, oscillating a certain number of times per second, that may represent the *hidden mass*,the *dynamic mass*carried by the quantum light (or photon) [35].It is like saying that the Planck constant is the *soul* of the quantum of radiation, that is *h* represents the intimate essence of the *Planck' grain*.

In our comfort, we can read from Barrow: "The non-null value of the Planck constant (*h*) is important for the stability of the matter. In the impacts between the atoms and the electromagnetic radiations, the value of *h* is large enough to take a rather strong 'stroke' to push the electrons to the immediately higher permissible level. *h* identifies with Planck'grain, with the quantum of light, that is with a photon. **And yet, a massless photon is capable of inferring such a stroke, besides giving *stability to the matter***!"[36].

This makes us think about what Hawking writes:" When an electron moves from an orbit to one closer to the nucleus, it will emit a real photon, observed as visible light, so if a (real) photon collides with an atom, it will move an electron on a more external orbit. This movement uses the energy of the photon"[26].

Hence, why cannot we suppose that at the bottom of this phenomenonthere is a strictly *mechanic action of the photon*, as to say the Planck's grain, which with its energy-mass would raise the kinetic energy of the orbiting electron from which it was absorbed? In other words, this phenomenon should not depend on a merely energetic effect, but also on a specifically *mechanic effect*, as a consequence of the probable dynamic-mass carried by the Planck quantum [37].

Similarly, it could be a mechanical effect, exerted by Planck's grains, that is by "*radiationpressure*"[15], to induce the Wave Function Collapse(WFC) of a *measured* particle.

On the other hand, the concept of a mechanic action induced by the*radiation pressure* is well known: it was highlighted over 4 centuries ago.

For the accuracy, it was first pointed out by Johannes Kepler in 1619 the concept of *Radiation Pressure* to explain the observation that a tail of a comet always points away from the Sun [38].

Moreover, a well-known mechanical action excerted by light is represented by the *photo-electric effect*:it was carefully described by Lenard, in 1902 [39]. At this regard, as Asimov reminds us,it is important to mention that at the beginning of last century Lenard had discovered that when the light hit certain metals it caused the emission of electrons from their surface, just as the light had the power to push out the electrons from the atoms. When physicists started to make experiments on this phenomenon (*photoelectric effect*) they realized, with great surprise, that if they raised the light intensity, the energy of the emitted electrons did not increase [40].

What influenced them instead, were the different colours of the wavelength of the light used: for instance, "the blue light gave the electrons a bigger speed than the yellow light. A blue weak light caused the emission of fewer electrons than an intense yellow light, however the few electrons pushed out by the blue light had a bigger speed than any electron pushed out by the yellow light. A red light of any intensity did not cause at all the emission of electrons in certain metals"[40].

According with Asimov, none of these phenomena could be explained by the old theories of light. Why ever the blue light was able to do something which the red light was not able to do? Einstein found the answer: an electron had to be hit by a quantum of energy higher than a minimum value in order to absorb enough energy to abandon the surface of the metal[40], that is higher than the energy which keep the electron linked to the atom: ‘*threshold value* or *shearing value*’.

Asimov points out: “Anyway, the higher the energy of the quantum, the higher was also the speed of the electron pushed out from the metal”[40].

Therefore, what we learn from the Lenard’s experiment? We learn that the EM radiations (EMR_s)having a greater frequency of oscillation (v), that is the more energetic radiations, transmit agreater speed to the hit particles, compared to what the less energetic EMR can do [41].

In short, the more energetic EMR gives a greater and faster thrust to the particles it hits.

Thus, the Lenard’s experimental demonstration represents an enormous leap, which gives a significant turning point to Mechanics and assumes a very important role for the purpose of our work. In our opinion, in effect, it is still a mechanical action, exerted by EMR on the electron, to induce the WFC of the affected particle.

This is the key, in other words, to trying to understand the intimate mechanism that can be thebasis of the peculiar mechanical phenomenon represented by the WFC.

On the other hand, the strength exerted by light should not be understimated.

To this purpose, Feynman says: “I want to emphasize that light comes in this form: particles. It is very important to know that light behaves like particles. Light is made of particles”[42].

He points out that when light hit a particle there is a driving force in the direction of the light beam:“This forceis called *Radiation Pressure* or *Light Pressure*”[15].

At this regard, Feynman adds:”Let us determine how strong the *Radiation Pressure* is. Evidently it is that the light’s force (F) on a particle, in a magnetic field ($\boldsymbol{B}$),is given by:

$$F = qv\boldsymbol{B} \quad (22)$$

where q is the charge and $\mathbf{v}$ the velocity. As known, associated with an electric field ($\boldsymbol{E}$) is a magnetic field ($\boldsymbol{B}$), always at right angles to the electric field and at right angles to the apparent direction of the source. Since everything is oscillating, it is the time average of this*Force*,$\langle F\rangle$. We know that the strength of the magnetic field is the same as the strength of the electric field ($\boldsymbol{E}$) divided by c (the velocity of light in vacuum), so we need to find the average of the electric field, times the velocity, times the charge, times $1/c$:

$$\langle F\rangle = q\,\frac{\boldsymbol{vE}}{c} \quad (23)$$

But the charge q times the field E is the electric force on a charge, and the force on the charge times the velocity is the *workdW/dt* being done onthe charge! Therefore the *force*(F), the ***Pushing Momentum****, is a Pressure.* The *momentum* that the light delivers is always equal to the energy (W) that is absorbed, divided by c:

$$F = \frac{\frac{dW}{dt}}{c} \quad (24)$$

That light carries energy we already know. We now understand that it also carries *momentum,* and further, that the *momentum* carried is always $1/c$ times the energy”[15].

In short, there are good chanches thatit may be the “*pushing momentum*”[15] of the photon toinduce the various mechanical phenomena described.

This known mechanical action exerted by the *Radiation Pressure* [15],[38] should also apply to the Wave Function Collapse (WFC) of a particle.

In accord with de Broglie formula, as evidenced by Eq.(18), the *momentum* ($\boldsymbol{p}$) of an optic photon corresponds to $1.325\cdot10^{-22}$ [g·cm/s], as shown by Eq.(21).

Thus, it is really a very remarkable impact force that the *measured* particle undergoes.

All this may result even more surprising if we consider that the tremendous impact that generates the WFCcould be induced by a massless particle!

Unless we start to consider the possibility that the *momentum* ($\boldsymbol{p}$) of light quanta can also carry an *hiddendynamic-mass,* since it is well-known that it also carries grams per centimeter per second: as it is clear from Eq.(21).To this purpose, in our comfort we can avail of the prestigious supportof Richard Feynman, one of the most expert in the secrets of light, who says: "*momentum*(***p***) can be *hidden* in the electromagnetic field " [15].

Nevertheless, you may be wondering: why would this supposed mass be *hidden*?

In order to respect the well-known Bohr Complementarity Principle [43], according to which a particle can show itself only with one of its two “aspects”: wave or particle, but never simultaneously! These parameters are “complementary”, similarly to the complementary parameters of the Heisenberg Uncertainty Principle[44]: energy-time, or position and *momentum* of a particle.

According to Heisenberg [44], indeed, the more accuracy we have in knowing a parameter, the more uncertain the measure of the complementary corresponding parameter will be.

Similarly, the more information we have about the wave aspect of the light quantum, the less, in the same instant, we have of its particle aspect[43],[44],[45].

Hence, in agreement with the Complementarity Principle [43], if the Planck grain (or any otherparticle) is in motion, we can only catch its kinetic energy, adding it to its main base energy, but wewill never be able to have news, simultaneously, about its corpuscular characteristics.

In other words, from thePlanck quantum in motion (wave-like aspect) we can only have informationon its energy values, but we cannever check its eventual mass, since it is probable that

the mass is *hidden*, *masked*by the wave behavior assumed by themoving quantum object(QO).

Whereas, when the light’s particle interacts with another particle,just in that very short moment it will cease to show its wavelike appearance and will showus its corpuscular one, allowing us to determine its mass (in case it has some!). It’s like saying thatthe photon *wears* its corpuscular aspect, only in the very brief instant in which it interacts.

At this regard, we may avail of the very prestigious endorsement provided bySir Roger Penrose,

who writes: “The particle aspect of the wave-particle object shows itself only to the detector, when the *measurement* (*M*) is finally performed. The *M* makes clear the holistic nature of the Wave

Function, in the sense that the particle always appears and only in one point”[2].

In short, onlywhen the motion almost stops (and its wave aspect disappears) the light quantum will be ableto show its corpuscular aspect. We find very important to emphasize that only in these circumstances, as a corpuscle, the photon will show us, at last, itseventual mass: but always indirectly, showing us its probable mass-effects or mechanical effects[46].

IV. CONCLUSIONS

Concluding, in additional support of what we claim, there are several examples of *mass-effect*, or mechanicalactions,elicited by light quanta.

To this purpose, it is possible to mention the photo-electric[39][47], Compton[48] and Raman [49] effects: these mechanical effects induced by EMR represent unequivocal experimental evidences.

Furthermore, at this regard, we can read from Klein: “Physicist have begun to understand that in the energy balance of any physical process you have to keep in mind that **every body, even at rest,contains a *certain mass energy***”[50]. At this regard, we think that a *certain mass energy* is also transported by the photon and that the light quantum is not so evanescent.

In fact, what we get from our calculations is that a single luminous photon hits the electron with an impact force over 100,000 times bigger than the rest mass of the electron itself [51];according to Fermi it expresses the work-force of a light particle which, interacting with another particle, transfers to it its *energy*, its *momentum* (***p***) [52].

Thus, every time a *M* is carried out (using the EMR), the observed QO undergoes a probabilistic reduction of the state vector, indicated as “*Reduction Process*, or *R Process*”, quoting the words of Penrose [2]. With the *R Process* the state vector, represented by $|\Psi\rangle$, jumps to another state vector, where saying$|\phi\rangle$, which may represent one of the orthogonal alternative probabilities.

In sum, maybe now we can try to understand why a light quantum, apparently immaterial, can induce the WFC of the struck quantum object (QO) in the *Measurement* (*M*) processes.

In fact, when we wish to carry out a *M* of a QO we need to illuminate it; however in this circumstance we modify the physical system we are examining.

The electron, for instance, does not remain indifferent to the *energy* carried out by light’s quanta.

To this purpose, Feynman points out: "T**o observe electrons, we need a light** because the light rebounding on electrons make them visible. Nevertheless the light affects the result, because the result of *light on* is different from that of *light off*" [53].

In short, likely it is a typically mechanical effect, carried out by the *momentum*(***p***) of the light quantumwhich is transferred in full to the struck electron [52] exerting on it a force, a pressure (the so-called *RadiationPressure*), that is, a purely mechanical action enough to induce the WFC of the *measured* particle.

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