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Review Paper

k-Total Geometric Mean Cordial Labeling of Some Graphs

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Abstract:
Let G be a (p, q) graph. Let $f : V(G) \rightarrow \{1, 2, 3,\ldots,k\}$ be a function where $k \in N$ and $k>1$. For each edge uv, assign the label $f(uv) = \left\lceil \sqrt{f(u)f(v)} \right\rceil$. f is called k-Total geometric mean cordial labeling of G if $|t_{mf}(i) - t_{mf}(j)| \leq 1$, for all $i, j \in \{1, 2, 3,\ldots,k\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labeled with x, $x \in \{1, 2, 3,\ldots,k\}$. A graph that admits the k- total geometric mean cordial labeling is called k-total geometric mean cordial graph.

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Keywords
k- total geometric mean cordial labeling, k-total geometric mean cordial graph, Path, Y_n -tree, F_n - tree, Cycle, Comb and Star graph.

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I. Introduction

Finite, simple and undirected graphs are considered here. Cordial labeling was introduced by Cahit [1]. For notations and terminology we follow [2]. The concept of Geometric mean labeling has been introduced in [3]. Geometric mean cordial labeling of graphs was introduced in [4]. k- total mean cordial labeling of graphs was introduced in [5] . In this paper we introduced k-total geometric mean cordial labeling of some graphs and investigated 4-total geometric mean cordial labeling behaviour of cycle, comb, star graph.

Definition 1.1.

Let G be a (p, q) graph. Let $f : V(G) \rightarrow \{0, 1, 2, 3, \ldots, k-1\}$ be a function where $k \in N$ and $k > 1$. For each edge uv, assign the label $f(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called k-total mean cordial labeling of G if $|t_{mf}(i) - t_{mf}(j)| \leq 1$, for all $i, j \in \{0, 1, 2, 3, \ldots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labeled with x, $x \in \{0, 1, 2, 3, \ldots, k-1\}$. A graph that admits a k-total mean cordial labeling is called k-total mean cordial graph.

Definition 1.2.

A Y_n - tree is obtained from a path P_n by attaching a pendant vertex to the $(n-1)^{th}$ vertex of P_n . A tree on n+1vertices is denoted by Y_n.

Definition 1.3.

A F_n - tree on n + 2 vertices denoted by F_n is obtained from a path P_n by attaching exactly two pendant vertices to $(n-1)^{th}$ and n^{th} vertex of P_n .

II. Main Results

Motivated by the concept of k- total mean cordial labeling, we introduce a new labelingcalled k- total geometric mean cordial labeling as given below.

Definition 2.1.

Let G be a (p, q) graph. Let f : V (G) → {1, 2, 3, …,k} be a function where k∈N and k>1. For each edge uv, assign the label f (uv) = $\left\lceil \sqrt{f(u)f(v)} \right\rceil$. f is called k-Total geometric mean cordial labeling of G if | tmf (i) – tmf (j) | ≤ 1, for all i, j∈{1, 2, 3,…, k} where tmf (x) denotes the total number of vertices and edges labeled with x, x∈{1, 2, 3,…, k}. A graph that admits the k- total geometric mean cordial labeling is called k-total geometric mean cordial graph.

Theorem 2.2.

Any path P_n is k-total geometric mean cordial graph.

Proof:

Let P_n be a path on n vertices $u_1, u_2, u_3, \dots, u_n$ and $n = kt + r, 0 \le r < n$

Consider the vertices $u_1, u_2, u_3, \dots, u_t$. Assign the label k to the vertices $u_1, u_2, u_3, \dots, u_t$. Next assign the label $k-1$ to the vertices $u_{t+1}, u_{t+2}, u_{t+3}, \dots, u_{2t}$. Now we assign the label $k-2$ to the vertices $u_{2t+1}, u_{2t+2}, u_{2t+3}, \dots, u_{3t}$ and assign the label $k-3$ to the vertices $u_{3t+1}, u_{3t+2}, u_{3t+3}, \dots, u_{4t}$. Proceeding like this assign the label 1 to the vertices $u_{(k-1)t+1}, u_{(k-1)t+2}, u_{(k-1)t+3}, \dots, u_{(k)t}$ and u_{kt+1}. Now we consider the vertices $u_{kt+2}, u_{kt+3}, u_{kt+4}, \dots, u_{kt+r}$.

Case (i) : k is even

"We assign the label to the vertices in the following manner".

If all the even numbers are $\le k$, then we assign all such even integer labels as $k, k-2, k-4, k-6$ .... to the vertices $u_{kt+2}, u_{kt+3}, u_{kt+4}, \dots$.

If all the even numbers are $\le k$ are fully exhausted, then we assign the label 1 to the next vertices $u_{k+\frac{k+4}{2}}, u_{2k+\frac{k+4}{2}}, u_{3k+\frac{k+4}{2}}, \dots$.

Now, we assign the odd integer labels as $k-1, k-3, k-5, \dots$ to all the remaining non labeled vertices.

In this case we observe that | t_{mf} (i) – t_{mf} (j) | ≤ 1, for all i, j∈{1, 2, 3,…,k} and hence f is k-total geometric mean cordial labeling and P_n is k-total geometric mean cordial graph.

Case (ii) : k is odd

If all the odd numbers are $\le k$, then we assign the odd integer labels as $3, 5, 7, \dots$ to the vertices $u_{kt+2}, u_{kt+3}, u_{kt+4}, \dots$

If all the odd numbers $\le k$ are fully exhausted, then we assign the label 1 to the next vertices $u_{k+\frac{k+3}{2}}, u_{2k+\frac{k+3}{2}}, u_{3k+\frac{k+3}{2}}, \dots$

Now we assign the even integer labels as $k-1, k-3, k-5, \dots$ to all the remaining non labeled vertices.

We observe that | t_{mf} (i) – t_{mf} (j) | ≤ 1, for all i, j∈{1, 2, 3,…,k} and hence f is k-total geometric mean cordial labeling and P_n is k-total geometric mean cordial graph.

Theorem 2.3.

Any Y_n tree, $n \geq 3$ is a k- total geometric mean cordial graph.

Proof :

Let Y_n be a graph of n+1vertices and n edges. Let P_n be a path $u_1\ u_2\ u_3\ \dots u_n$ and v_1 be the pendant vertex and $n = kt + r, 0 \leq r < n$

Consider the vertices $u_1, u_2, u_3, \dots, u_t$. Assign the label k to the vertices $u_1, u_2, u_3, \dots, u_t$. Next assign the label $k-1$ to the vertices $u_{t+1}, u_{t+2}, u_{t+3}, \dots, u_{2t}$. Now we assign the label $k-2$ to the vertices $u_{2t+1}, u_{2t+2}, u_{2t+3}, \dots, u_{3t}$ and assign the label $k-3$ to vertices $u_{3t+1}, u_{3t+2}, u_{3t+3}, \dots, u_{4t}$. Proceeding like this assign the label 1 to the vertices $u_{(k-1)t+1}, u_{(k-1)t+2}, u_{(k-1)t+3}, \dots, u_{(k)t}$ and u_{kt+1}.

Now we consider the vertices $u_{kt+2}, u_{kt+3}, u_{kt+4}, \dots, u_{kt+r}$.

We assign the label to the vertices in the following manner.

Case (i) : k is even

Assign the even number to the vertices $u_{kt+2}, u_{kt+3}, u_{kt+4}, \dots$ with the condition that even numbers are $\leq k$. Assign the odd number to the vertex v_1 , which is less than k.

Case (ii) : k is odd

Assign the odd number to the vertices $u_{kt+2}, u_{kt+3}, u_{kt+4}, \dots$ with the condition that odd numbers are $\leq k$.

The labeling pattern of v_1 is

Subcase:

(i) k = 3, assign the odd number to the vertex v_1.

(ii) k ≥ 5, assign the label 1 to the vertex v_1, if k = n, 2n, 3n….. otherwise v_1 is an even number which is less than k.

From all the above two cases we observe that $| t_{mf}(i) - t_{mf}(j) | \leq 1$, for all i, j$\in\{1, 2, 3,\dots,k\}$ and hence f is k-total geometric mean cordial labeling and Y_n – tree is k-total geometric mean cordial graph.

Theorem 2.4.

Any F_n tree, $n \geq 3$ is a k- total geometric mean cordial graph.

Proof :

Let F_n be a graph of n+2 vertices and n+1 edges. Let P_n be a path $u_1\ u_2\ u_3\ \dots u_n$ and v_1, v_2 be the pendant vertices and $n = kt + r, 0 \leq r < n$

Consider the vertices $u_1, u_2, u_3, \dots, u_t$. Assign the label k to the vertices $u_1, u_2, u_3, \dots, u_t$. Next assign the label $k-1$ to the vertices $u_{t+1}, u_{t+2}, u_{t+3}, \dots, u_{2t}$. Now we assign the label $k-2$ to the vertices $u_{2t+1}, u_{2t+2}, u_{2t+3}, \dots, u_{3t}$, and assign the label $k-3$ to the vertices $u_{3t+1}, u_{3t+2}, u_{3t+3}, \dots, u_{4t}$. Proceeding like this assign the label 1 to the vertices $u_{(k-1)t+1}, u_{(k-1)t+2}, u_{(k-1)t+3}, \dots, u_{(k)t}$ and u_{kt+1} .

Now we consider the vertices $u_{kt+2}, u_{kt+3}, u_{kt+4}, \dots$

Case (i) : k is odd

We assign the label to the vertices in the following manner.

If k = n = 3, assign the label 2 to the vertex u_1, label 3 to the vertex u_2 and label 1 to the vertex u_n.

If n > 3, k $\geq$ 5, assign the odd number to the vertices $u_{kt+2,} u_{kt+3}, u_{kt+4} \dots \dots$ with the condition that odd numbers are $\leq$ k.

The labeling pattern of v_1 and v_2 is given below

Subcase:

(i) If $k = n = 3$, assign the odd number to the vertex v_1 and even number to the vertex v_2 which are less than k. Otherwise v_1 and v_2 must be odd which are less than or equal to k.

(ii) If $k \geq 5$, assign the odd number to the vertex v_1 and even number to the vertex v_2 which are less than or equal to k.

Case (ii) : k is even

Assign the odd number to the vertices $u_{kt+2}, u_{kt+3}, u_{kt+4}, ...$ with the condition that odd numbers are < k

Now the labeling pattern of v_1 and v_2 is given below

Subcase:

(i) If $k = 4$, the vertices v_1 and v_2 are odd or even alternatively, which are less than or equal to k.

(ii) If $k \geq 6$, assign the odd number to the vertex v_1 and even number to the vertex v_2 if $k =$ n, 2n, 3n,… otherwise v_1 and v_2 is an even number which are less than or equal to k.

From all the above two cases we observe that $| t_{mf}(i) - t_{mf}(j) | \leq 1$, for all $i, j \in \{1, 2, 3,\ldots, k\}$ and hence f is k-total geometric mean cordial labeling and F_n – tree is k-total geometric mean cordial graph.

Theorem 2.5.

Any Cycle C_n is a 4-total geometric mean cordial graph.

Proof :

Let $u_1 u_2 \ldots u_n u_1$ be the cycle C_n

Case (i) : $n \equiv 1 \pmod 4$

The $\frac{n+3}{4}$ vertices $u_1, u_2, \ldots, u_{\frac{n+3}{4}}$ are labeled with 1, the $\frac{n-1}{4}$ vertices $u_{\frac{n+7}{4}}, u_{\frac{n+11}{4}}, \ldots, u_{\frac{n+1}{2}}$ are labeled with 2, the label 3 to the $\frac{n-1}{4}$ vertices $u_{\frac{n+3}{2}}, u_{\frac{n+5}{2}}, \ldots, u_{\frac{3n+1}{4}}$ are labeled with 3. Finally the $\frac{n-1}{4}$ vertices $u_{\frac{3n+5}{4}}, u_{\frac{3n+9}{4}}, u_{\frac{3n+13}{4}}, \ldots, u_n$ are labeled with 4.

Case (ii) : $n \equiv 2 \pmod 4$

The $\frac{n+2}{4}$ vertices $u_1, u_2, \ldots, u_{\frac{n+2}{4}}$ are labeled with 1, the $\frac{n-2}{4}$ vertices $u_{\frac{n+6}{4}}, u_{\frac{n+10}{4}}, \ldots, u_{\frac{n}{2}}$ are labeled with 3, the $\frac{n-2}{4}$ vertices $u_{\frac{n+2}{2}}, u_{\frac{n+4}{2}}, \ldots, u_{\frac{3n-2}{4}}$ are labeled with 2. Finally the $\frac{n+2}{4}$ vertices $u_{\frac{3n+2}{4}}, u_{\frac{3n+6}{4}}, u_{\frac{3n+10}{4}}, \ldots, u_n$ are labeled with 4.

Case (iii) : $n \equiv 3 \pmod 4$

The $\frac{n+1}{4}$ vertices $u_1, u_2, \ldots, u_{\frac{n+1}{4}}$ are labeled with 1, the $\frac{n+1}{4}$ vertices $u_{\frac{n+5}{4}}, u_{\frac{n+9}{4}}, \ldots, u_{\frac{n+1}{2}}$ are labeled with 4, the $\frac{n+1}{4}$ vertices $u_{\frac{n+3}{2}}, u_{\frac{n+5}{2}}, \ldots, u_{\frac{3n-1}{4}}$ *and* u_n are labeled with 2. Finally the $\frac{n-3}{4}$ vertices $u_{\frac{3n+3}{4}}, u_{\frac{3n+7}{4}}, u_{\frac{3n+11}{4}}, \ldots, u_{n-1}$ are labeled with 3.

Case (iv) : $n \equiv 4 \pmod 4$

It is easy to verify $n \le 8$.

If $n \ge 12$, The $\frac{n+4}{4}$ vertices $u_1, u_2, \ldots, u_{\frac{n}{4}}$ and u_{n-1} are labeled with 1, the $\frac{n-4}{4}$ vertices $u_{\frac{n+4}{4}}, u_{\frac{n+8}{4}}, \ldots, u_{\frac{n-2}{2}}$ are labeled with 3, the $\frac{n-4}{4}$ vertices $u_{\frac{n}{2}}, u_{\frac{n+2}{2}}, \ldots, u_{\frac{3n-12}{4}}$ *and* u_{n-2} are labeled with 2. Finally the $\frac{n+4}{4}$ vertices $u_{\frac{3n-8}{4}}, u_{\frac{3n-4}{4}}, u_{\frac{3n}{4}}, u_{\frac{3n+4}{4}}, u_{\frac{3n+8}{4}}, \ldots, u_{n-3}$ *and* u_n are labeled with 4.

The vertex labeling f is a 4- total geometric mean cordial labeling

Nature of n	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$	$t_{mf}(4)$
$n \equiv 1 \bmod 4$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n-1}{2}$	$\frac{n-1}{2}$
$n \equiv 2 \bmod 4$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$
$n \equiv 3 \bmod 4$	$\frac{n-1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n-1}{2}$
$n \equiv 4 \bmod 4$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$

In all the above four cases we see that $| t_{mf}(i) - t_{mf}(j) | \le 1$, for all $i, j \in \{1, 2, 3, \ldots, k\}$ and hence f is 4-total geometric mean cordial labeling and C_n is 4-total geometric mean cordial graph.

Theorem 2.6.

The Comb $P_n \odot K_1$ is a 4-total geometric mean cordial graph.

Proof :

Let P_n be a path $u_1\, u_2\, u_3\, \ldots u_n$. Let $V(P_n \odot K_1) = V(P_n) \cup \{v_i : 1 \le i \le n\}$ and $E(P_n \odot K_1) = E(P_n) \cup \{u_i v_i : 1 \le i \le n\}$

Case (i) : n is even

The n-1 vertices $v_1, v_2, v_3, \ldots, v_{n-1}$ are labeled with 1, the label 2 to the one vertex v_n, the $\frac{n}{2}$ vertices $u_1, u_2, u_3, \ldots, u_{\frac{n}{2}}$ are labeled with 3. Finally the $\frac{n}{2}$ vertices $u_{\frac{n+2}{2}}, u_{\frac{n+4}{2}}, \ldots, u_n$ are labeled with 4.

Case (ii) : n is odd

The n-1 vertices $v_1, v_2, v_3, \ldots, v_{n-1}$ are labeled with 1, the label 2 to the one vertex v_n, the $\frac{n+1}{2}$ vertices $u_1, u_2, u_3, \ldots, u_{\frac{n-1}{2}}$ and u_n are labeled with 3. Finally the $\frac{n-1}{2}$ vertices $u_{\frac{n+1}{2}}, u_{\frac{n+3}{2}}, \ldots, u_{n-1}$ are labeled with 4. Clearly $t_{mf}(1) = n-1$ and $t_{mf}(2) = t_{mf}(3) = t_{mf}(4) = n$

In the above two cases we see that $| t_{mf}(i) - t_{mf}(j) | \le 1$, for all $i, j \in \{1, 2, 3, \ldots, k\}$ and hence f is 4-total geometric mean cordial labeling and $P_n \odot K_1$ is 4-total geometric mean cordial graph.

Theorem 2.7.

Star $K_{1,n}$, $n \geq 3$ is a 4-total geometric mean cordial graph.

Proof :

Let u be the centre vertex of the star $K_{1,n}$. Let u_i $(1 \leq i \leq n)$ be the pendant vertices adjacent to u. Assign the label 3 to the vertex u.

Case (i) : $n \equiv 1 \pmod 4$

The $\frac{n+1}{2}$ vertices $u_1, u_2, u_3, \ldots, u_{\frac{n+1}{2}}$ are labeled with 1, the $\frac{n-1}{4}$ vertices $u_{\frac{n+3}{2}}, u_{\frac{n+5}{2}}, \ldots, u_{\frac{3n+1}{4}}$ are labeled with 3. Finally the $\frac{n-1}{4}$ vertices $u_{\frac{3n+5}{4}}, u_{\frac{3n+9}{4}}, u_{\frac{3n+13}{4}}, \ldots, u_n$ are labeled with 4.

Case (ii) : $n \equiv 2 \pmod 4$

The $\frac{n}{2}$ vertices $u_1, u_2, \ldots, u_{\frac{n}{2}}$ are labeled with 1, the $\frac{n-2}{4}$ vertices $u_{\frac{n+2}{2}}, u_{\frac{n+4}{2}}, \ldots, u_{\frac{3n-2}{4}}$ are labeled with 3. Finally the $\frac{n+2}{4}$ vertices $u_{\frac{3n+2}{4}}, u_{\frac{3n+6}{4}}, u_{\frac{3n+10}{4}}, \ldots, u_n$ are labeled with 4.

Case (iii) : $n \equiv 3 \pmod 4$

The $\frac{n+1}{2}$ vertices $u_1, u_2, \ldots, u_{\frac{n+1}{2}}$ are labeled with 1, the $\frac{n-3}{4}$ vertices $u_{\frac{n+3}{2}}, u_{\frac{n+5}{2}}, \ldots, u_{\frac{3n-1}{4}}$ are labeled with 3. Finally the $\frac{n+1}{4}$ vertices $u_{\frac{3n+3}{4}}, u_{\frac{3n+7}{4}}, u_{\frac{3n+11}{4}}, \ldots, u_n$ are labeled with 4.

Case (iv) : $n \equiv 4 \pmod 4$

The $\frac{n}{2}$ vertices $u_1, u_2, u_3, \ldots, u_{\frac{n}{2}}$ are labeled with 1, the $\frac{n}{4}$ vertices $u_{\frac{n+2}{2}}, u_{\frac{n+4}{2}}, \ldots, u_{\frac{3n}{4}}$ are labeled with 3. Finally the $\frac{n}{4}$ vertices $u_{\frac{3n+4}{4}}, u_{\frac{3n+8}{4}}, u_{\frac{3n+12}{4}}, \ldots, u_n$ are labeled with 4.

The vertex labeling f is a 4 total geometric mean cordial labeling

Nature of n	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$	$t_{mf}(4)$
n≡ 1 mod 4	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n-1}{2}$
n≡ 2 mod 4	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n+2}{2}$
n≡ 3 mod 4	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n-1}{2}$	$\frac{n+1}{2}$
n≡ 4 mod 4	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n+2}{2}$	$\frac{n}{2}$

From the above four cases we observe that $| t_{mf}(i) - t_{mf}(j) | \leq 1$, for all $i, j \in \{1, 2, 3, \ldots, k\}$ and hence f is 4-total geometric mean cordial labeling and $K_{1,n}$ is 4-total geometric mean cordial graph.

Theorem 2.8.

The graph $P_n \odot 2K_1$ is a 4-total geometric mean cordial for all values of n.

Proof:

Let P_n be a path $u_1, u_2, u_3 \ldots \ldots . u_n$, and v_i , w_i be the pendant vertices adjacent to u_i $(1 \leq i \leq n)$. It is easy to verify that $| V(P_n \odot 2K_1)| + | E(P_n \odot 2K_1)| = 6n-1$.

Case (i): $n \equiv 1 \pmod 4$

The $\frac{3n-1}{2}$ vertices $v_1, v_2, v_3, \ldots, v_n$ *and* $w_1, w_2, w_3, \ldots, w_{\frac{n-1}{2}}$ are labeled with 1,the $\frac{3n+1}{4}$ vertices $u_1, u_2, u_3, \ldots, u_{\frac{3n+1}{4}}$ are labeled with 3.Finally allocate the label 4 to the $\frac{3n+1}{4}$ vertices $u_{\frac{3n+5}{4}}, u_{\frac{3n+9}{4}}, u_{\frac{3n+13}{4}}, \ldots, u_n$ *and* $w_{\frac{n+1}{2}}, w_{\frac{n+3}{2}}, w_{\frac{n+5}{2}}, \ldots, w_n$

Case (ii): $n \equiv 2 \pmod 4$

The $\frac{3n}{2}$ vertices $v_1, v_2, v_3, \ldots, v_n$ *and* $w_1, w_2, w_3, \ldots, w_{\frac{n}{2}}$ are labeled with 1, the $\frac{3n+2}{4}$ vertices $u_1, u_2, u_3, \ldots, u_{\frac{3n-2}{4}}$ and w_n are labeled with 3. Finally the $\frac{3n-2}{4}$ vertices $u_{\frac{3n+2}{4}}, u_{\frac{3n+6}{4}}, \ldots, u_n$ *and* $w_{\frac{n+2}{2}}, w_{\frac{n+4}{2}}, \ldots, w_{n-1}$ are labeled with 4.

Case (iii): n ≡ 3 (mod 4)
The $\frac{3n-1}{2}$ vertices $v_1, v_2, v_3, \ldots, v_n$ *and* $w_1, w_2, w_3, \ldots, w_{\frac{n-1}{2}}$ are labeled with 1, the $\frac{3n+3}{4}$ vertices $u_1, u_2, u_3, \ldots, u_{\frac{3n+3}{4}}$ are labeled with 3, the $\frac{3n-1}{4}$ vertices $u_{\frac{3n+7}{4}}, u_{\frac{3n+11}{4}}, u_{\frac{3n+15}{4}}, \ldots, u_n$ *and* $w_{\frac{n+1}{2}}, w_{\frac{n+3}{2}}, w_{\frac{n+5}{2}}, \ldots, w_n$ are labeled with 4.

Case (iv): n ≡ 4 (mod 4)
The $\frac{3n}{2}$ vertices $v_1, v_2, v_3, \ldots, v_n$ *and* $w_1, w_2, w_3, \ldots, w_{\frac{n}{2}}$ are labeled with 1, the $\frac{3n+4}{4}$ vertices $u_1, u_2, u_3, \ldots, u_{\frac{3n}{4}}$ and w_n are labeled with 3, the $\frac{3n-4}{4}$ vertices $u_{\frac{3n+4}{4}}, u_{\frac{3n+8}{4}}, u_{\frac{3n+12}{4}}, \ldots, u_n$ *and* $w_{\frac{n+2}{2}}, w_{\frac{n+4}{2}}, w_{\frac{n+6}{2}}, \ldots, w_{n-1}$ are labeled with 4.

The vertex labeling f is a 4 total geometric mean cordial labeling

Nature of n	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$	$t_{mf}(4)$
n≡ 1 mod 4	$\frac{3n-1}{2}$	$\frac{3n-1}{2}$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$
n≡ 2 mod 4	$\frac{3n}{2}$	$\frac{3n}{2}$	$\frac{3n-2}{2}$	$\frac{3n}{2}$
n≡ 3 mod 4	$\frac{3n-1}{2}$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$
n≡ 4 mod 4	$\frac{3n}{2}$	$\frac{3n}{2}$	$\frac{3n}{2}$	$\frac{3n-2}{2}$

From the above four cases we observe that $| t_{mf}(i) - t_{mf}(j) | \le 1$, for all i, j∈{1, 2, 3,…,k} and hence f is 4-total geometric mean cordial labeling and $P_n \odot 2K_1$ is 4-total geometric mean cordial graph.

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