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Review Paper

The Invariant Measures of Stochastic Evolution Equations with Gaussian Noise

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ABSTRACT:
We consider the applicability of the SEE to the solution of optimality problems often appear as heat equation under mild semi linear conditions and we establish the existence of invariant measure in a corrupted Gaussian noise.
KEYWORDS: *Stochastic Evolution equation, Gaussian process, Hilbert space, S-Linear Stochastic evolution equations, Abstract Heat Equation.*

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I. INTRODUCTION

The research for the existence, uniqueness and regularity of invariant measures for stochastic evolution equations (SEEs) in Hilbert spaces has received a robust research attention in recent years. However, there appears to be more work in the stochastic evolution equations in Hilbert spaces in terms of different conditions on the coefficients of the stochastic evolution equations. Our focus is to examine and establish the long-term behaviours of a dynamical of dynamical and lattice systems and derive conditions for solution with Gaussian noise which are applied to the optimality of control problems especially in finance (see [1],[2], [3]), and recast such problems as an abstract heat equation.

There are some methods developed in establishing the invariant measures of SEEs in Hilbert spaces using different conditions. Compactness method and dissipativity method pioneered by Da Prato, Gatarek and Zabczyk (see [4]). The uniqueness of invariant measures using the Feller property and predictability (see [5] and [6]) on one hand, on the other hand is to apply the Lyapunov approach ([7] and [8]). Importantly, Da Prato and Zabczyk also established the regularity of invariant measures in Hilbert space by the application of semi-group theory ([11]).

In the analytical approach, the work of (Shigekawa, I. (see [10]) established the invariant measures for the infinite dimensional stochastic systems by the application of logarithmic Sobolev inequality.

The novelty of our work is establishment of procedures and applicability of our approach pioneered by the work of Da Prato and Zabczyk to the solution of abstract heat equation under mild semi linear SEEs , the investigation of the fundamental relationships and the existence of invariant measure.

The structure of the work is as follows. In section 1, we examined the general structures and relevant works done in SEEs. In section 2, we state the form of stochastic evolution equations in Hilbert space and its existence and uniqueness results. In section 3, we establish the necessary conditions of SEEs under the Gaussian noise. Section 4 is the applicability of Gaussian noise in finance.

II. STOCHASTIC EVOLUTION EQUATIONS IN HILBERT SPACE

Consider the stochastic evolution equation on $\mathcal{H}$ of the form

$$dX_t = AX(t)dt + F\big(X(t)\big)dt + \sigma\big(X(t)\big)dW(t),\ t \geq 0,$$
$$X(0) = x_0 \in \mathcal{H}, t \in [0,T],\ T < \infty$$
(1)

where A is infinitesimal generator of a C_0-semigroup of $\mathcal{H}$. $\Lambda = (\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ is the usual probability space, x_0 is the valued $\mathcal{F}_0$ measurable random variable, σ defined such that

$\mathcal{H} \to \mathbb{R}$ $and\, F$ is self-measurable mapping in $\mathcal{H}, W(t)_{t\geq 0}$ is Q-cylindrical Wiener process defined on a separable Hilbert space $\mathcal{H}$ on Λ

Theorem 2.1: Given a separable Hilbert space with inner product $\langle\cdot,\cdot\rangle_{\mathcal{H}}$ $and\ dual\ \mathcal{H}^*, norm$ $\|\cdot\|$, and a Markov semigroup $\mathcal{P}_t$, $\mathcal{P}_t\Phi(y)\pi_t(x, dy), t \geq 0, \Phi \in \mathcal{C}_b(\mathcal{H})$.
Proof: Assume that for some $x_0 \in \mathcal{H}$, $\exists$ $the\ lim_{t\to\infty}\mathcal{P}_t\Phi(x_0) = \mathcal{F}_{x_0}(\Phi), \forall\Phi \in \mathcal{C}_b(\mathcal{H})$. (2)
Then , we show that $\mathcal{F}_{x_0}$ is a positive functional from $\mathcal{C}_b(\mathcal{H})$ $into$ $\mathbb{R}$, and $\mathcal{F}_{x_0}$ is invariant for $\mathcal{P}_t$, given that $\mathcal{F}_{x_0}(\mathcal{P}_t\Phi) = \mathcal{F}_{x_0}(\Phi)$, $\forall\, \Phi \in \mathcal{C}_b(\mathcal{H})$ $and\ t \geq 0$. (3)
At $\Phi = \mathcal{P}_s(\Phi), \Phi \in \mathcal{C}_b(\mathcal{H}) \Rightarrow \lim_{t\to+\infty}\mathcal{P}_t\mathcal{P}_s\Phi(x_0) = \mathcal{F}_{x_0}(\mathcal{P}_s\Phi)$ (4)
Again, $\lim_{t\to+\infty}\mathcal{P}_t\mathcal{P}_s\Phi(x_0) = \lim_{t\to+\infty}\mathcal{P}_{t+s}\Phi(x_0) = \mathcal{F}_{x_0}(\Phi)$ (5)
And $Let\ \mu T(E) = \frac{1}{T}\int_0^T \pi_t(x_0, E)\, dt$, $E \in \mathcal{B}(\mathcal{H}), T > 0$ (6)
$\Rightarrow$ (6) is valid since $[0, +\infty) \to \mathbb{R}$, $t \to \pi_t(x_0, E) = \mathcal{P}_t 1_t(x_0)$ is a Borel.

Theorem 2.2: $Let\ \mathcal{P}_t$ be a Markov semigroup. Assume that for some $x_0 \in H, the\ \ set\ \ (\mu T)T > 0$, defined by (16) is tight. Then there is an invariant measure.
Proof : By the Prokhorov theorem, there exists a sequence $T_n \uparrow \infty$ and a probability measure $\mu \in \mathcal{P}(H) such\ \ that$ $\lim_{n\to\infty}\int_H \Phi(x)\mu T_n(dx) = \int_H \Phi(x)\mu(dx)$ $\forall\Phi \in \mathcal{C}_b(H)$, by Fubini theorem, we can recast it as $\lim_{n\to\infty}\frac{1}{T_n}\int_0^{T_n}\mathcal{P}_t\Phi(x_0)dt = \int_H \Phi(x)\mu(dx)$ $\forall\Phi \in \mathcal{C}_b(H)$. (17) $Let\ \Phi = \mathcal{P}_s\Phi, for\ s \geq 0$, $\Phi \in \mathcal{C}_b(H)$ since $\mathcal{P}_t$ is Feller.

Again , $\lim_{n\to\infty}\frac{1}{T_n}\int_0^{T_n}\mathcal{P}_{t+s}\,\Phi(x_0)dt = \int_H \mathcal{P}_s\Phi(d\mu)$ $\forall\Phi \in \mathcal{C}_b(H)$. (7)

We need to establish that $\lim_{n\to\infty}\frac{1}{T_n}\int_0^{T_n}\mathcal{P}_{t+s}\,\Phi(x_0)dt = \int_H \Phi(d\mu)$ is invariant. From (17) (8)
$\lim_{n\to\infty}\frac{1}{T_n}\int_0^{T_n}\mathcal{P}_{t+s}\,\Phi(x_0)dt = \frac{1}{T_n}\int_0^{T_n+s}\mathcal{P}_{\mathrm{t}}\,\Phi(x_0)dt$

$= \frac{1}{T_n}\int_0^{T_n}\mathcal{P}_t\Phi(x_0)dt + \frac{1}{T_n}\int_{T_n}^{T_n+s}\mathcal{P}_t\Phi(x_0)dt - \frac{1}{T_n}\int_0^s\mathcal{P}_t\Phi(x_0)dt \ \ \Rightarrow \int_H \Phi(x)\mu(dx)\ as\ n \to \infty$.

Definition 2.1: $Let\ \mathcal{H}$ be a separable Hilbert space and $Q \in \mathcal{L}(\mathcal{H})$ be non-negative such that $\mathcal{T}r(Q) < \infty$. Then there exists a complete orthonormal system $\{e_i\} \in \mathcal{H}$ with a bounded sequence μ_i such that $Qe_i = \mu_i e_i$, $i = 1,2, \ldots$ (9)

Definition 2.2: A $\mathcal{H}$-valued stochastic process $W(t)$, $t \geq 0$ is called Q- Wiener process if
(i) $W(0) = 0$
(ii) W is continuous
(iii) W is independent increments
(iv) $\mathcal{L}\big(W(t) - W(s)\big) = \mathcal{N}(0, (t-s)Q)$, $\forall\, t \geq s \geq 0$
And covariance operator $(t-s)Q$

Proposition 2.1:
$Let\ W\ be\ a\ Q - Wiener\ process\ with\ TrQ < \infty$. Then $W$is a Gaussian process on $\mathcal{H}, \mathbb{E}(W(t)) = 0, Cov\big(W(t)\big) = tQ,\ \ for\ t \geq 0,\ W(t) = \sum_{j=1}^{\infty}\mu_j^{\frac{1}{2}}\beta_j(t)e_j$ (10)
$where\ \beta_j(t) = \ \mu_j^{\frac{1}{2}} < W(t), e_j > \mathcal{H}, j = 1,2, \ldots$ are real valued Brownian motions on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ with a convergent series in $\mathcal{L}^2(\Omega, \mathcal{F}, \mathcal{P})$.

Assumption 2.1: $Let\ \mathcal{U} = (\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ be a filtered probability space and $(W(t))_{t\geq 0}$ is a cylindrical Wiener process defined on $\mathcal{U}$ for each $t \geq 0$ $and\ W(t)$ is a bounded linear operator in $\mathcal{L}^2(\Omega, \mathcal{F}, \mathcal{P})$ such that :
(i) $\forall t \geq 0,\ and\ \psi, \varphi \in \mathcal{H}$, $\mathbb{E}[W(t)\psi W(t)\varphi] = t\langle\psi, \varphi\rangle\mathcal{H}$,
$(ii)\ for\ each\ \psi \in \mathcal{H}, W(t)\psi, t \geq 0,$ is a real valued $\{\mathcal{F}_t\}_{t\geq 0}$ – Wiener process.

Definition 2.3: Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a probability space and $(\mathcal{H}, (\cdot,\cdot)_{\mathcal{H}}, \|\cdot\|_{\mathcal{H}})$ a separable Hilbert space and $Let\ Q \in \mathcal{L}(\mathcal{H})$ be a bounded, self-adjoint, positive semi-definite operator $Q \in \mathcal{L}(\mathcal{H})$.

The trace of $\mathcal{Q}$ is stated by: $Tr(\mathcal{Q}) = \sum_{i\in\mathcal{N}}(\mathcal{Q}e_i, e_i)_{\mathcal{H}}$ $where$ $(e_i)_{i\in\mathcal{N}}$ (11)
is an orthonormal basis of $\mathcal{H}$.
Furthermore, by the spectral theorem for compact operators, yields the existence of an orthonormal basis. Again, any $\mathcal{Q}$- Wiener process $W: [0,T] \times \Omega \rightarrow \mathcal{H}$ is a $\mathcal{Q}$- Wiener process with respect to a normal filteration ([11]).

Proposition 2.2:
Consider a $\mathcal{H}$- valued Gaussian random variable X with mean $m \in \mathcal{H}$ and covariance operator $\mathcal{Q} \in \mathcal{L}(\mathcal{H})$, where $\mathcal{Q}$ is self-adjoint, positive semi-definite and with finite trace such that $\mathcal{P} \cdot X^{-1} = \mathcal{N}(m, \mathcal{Q})$ Then, $\forall\, h \in \mathcal{H}$, $(X,h)_{\mathcal{H}}$ is a real-valued Gaussian random variable with the following conditions:

(i) $\mathbb{E}[(X,h)_{\mathcal{H}}] = (m,h)_{\mathcal{H}}, \forall\, h \in \mathcal{H}$
(ii) $\mathbb{E}[(X-m,h)_{\mathcal{H}}(X-m,\mathcal{N})_{\mathcal{H}}] = (\mathcal{Q}h, v)_{\mathcal{H}},\ \forall\, h, v \in \mathcal{H}$
(iii) $\mathbb{E}[\|X-m\|^2_{\mathcal{H}}] = Tr\,(\mathcal{Q})$

Definition 2.4: A $\mathcal{Q} -$ Wiener process $W: [0,T] \times \Omega \rightarrow \mathcal{H}$ is called a $\mathcal{Q} -$ Wiener process with respect to a filteration $(\mathcal{F}_t)_{t\in[0,T]}$ if
(a) W is adapted to $(\mathcal{F}_t)_{t\in[0,T]}$
(b) $W(t) - W(s)$ is independent on $\mathcal{F}_s$ $\forall\, 0 < s < t < T$

Furthermore, a normal filteration $(\mathcal{F}_t)_{t\in[0,T]}$ such that $\mathcal{F}_0$ contains all sets $\mathcal{A} \in \mathcal{F}$ $with$ $P(\mathcal{A}) = 0$ and
$\mathcal{F}_t = \mathcal{F}_{t+} = \bigcup_{s>t} \mathcal{F}_s,\ \forall t \in [0,T]$

We now extend this to the martingale property of $\mathcal{Q}$- Wiener process (see[4])
Let $\Lambda: [0,T] \times \Omega \rightarrow \mathcal{B}_s$ (a Banach space), then $\left(\mathcal{F}_t\right)_{t\in[0,T]}$ is a Martingale if Λ is adapted with
$\mathbb{E}\left[\|\Lambda\|_{\mathcal{B}_s}\right] < \infty$ $\forall t \in [0,T]$ and $\mathbb{E}[\Lambda(t)|\mathcal{F}_s] = \Lambda(s)\ \forall\, 0 \le s \le t \le T$

Proposition 2.3:
Let $W: [0,T] \times \Omega \rightarrow \mathcal{U}$ be a $\mathcal{Q}$-Wiener process with respect to a filteration $(\mathcal{F}_t)_{t\le T}$ on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge0}, \mathcal{P})$. Then W is a continuous square-integrable $(\mathcal{F})_{t\in[0,T]}$- martingale provided the covariance operator $\mathcal{Q} \in \mathcal{L}(\mathcal{U})$ of the Wiener process has a finite trace.

III. SOLUTIONS TO S-LINEAR STOCHASTIC EVOLUTION EQUATIONS.

We state some properties and present some assumptions.
Let $T > 0$ be a real number and $(\mathcal{H}, (\cdot,\cdot), \|\cdot\|)$ and $(\mathcal{U}, (\cdot,\cdot)_{\mathcal{U}}, \|.\|_{\mathcal{U}})$ be separable Hilbert spaces. Also, Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge0}, \mathcal{P})$ be a complete probability space and a mapping defined by $W: [0,T] \times \Omega \rightarrow \mathcal{U}$ be a cylindrical $\mathcal{Q}$-Wiener process on Hilbert space adapted to filteration$\{\mathcal{F}_t\}_{t\in[0,T]}$. We consider a predictable stochastic process $\Psi: [0,T] \times \Omega \rightarrow \mathcal{H}$ a solution to stochastic evolution equations driven by the $\mathcal{Q}$- Wiener process$W$.
$d\Psi(t) + [A\Psi(t) + f(t, \Psi(t)]dt = g\big(t, \Psi(t)\big)dW(t)$ for $0 \le t \le T$, $(0) = \Psi_0$ (12)

Assumption 3.1:
The mapping $f: [0,T] \times \Omega \times \mathcal{H} \rightarrow \mathcal{H}^{-1}$, $(t, \omega, h) \rightarrow f(t, \omega, h)$ is $\mathcal{P}_T \times \frac{\mathcal{B}_{\mathcal{H}}(\mathcal{H})}{\mathcal{B}(\mathcal{H}^{-1})}$ measurable, $\exists$ a constant $\mathcal{C} > 0$ $such\ that$ $\|f(0,\omega,0)\|_{-1} \le \mathcal{C}\ \forall\ \omega \in \Omega$ and
$\|f(t,\omega,h_1) - f(t,\omega,h_2)\|_{-1} < \mathcal{C}\|h_1 - h_2\|, \forall\, h_1, h_2 \in \mathcal{H}, \omega \in \Omega, t \in [0,T]$ (13)
Again , $\exists$ a constant $\mathcal{C} > 0$ such that
$\|f(t_1,\omega,h) - f(t_2,\omega,h)\|_{-1} \le \mathcal{C}(1 + \|h\|)(t_2 - t_1)^{\frac{1}{2}}$ (14)
$\forall\, h \in \mathcal{H}, 0 \le t_1 \le t_2 \le T, \omega \in \Omega$

Definition 3.1: Let $\alpha \ge 2$, $\mathcal{A}$ predictable stochastic process: $\Psi: [0,T] \times \Omega \rightarrow \mathcal{H}$ is an $\alpha - fold$ integrable mild solution of (12) if $\sup_{t\in[0,T]} \|\Psi(t)\|_{\mathcal{L}^P(\Omega;\mathcal{H})} < \infty$ and $\forall t \in [0,T]$ (15)
$\Rightarrow\ \Psi(t) = \mathbb{E}(t)\Psi_0 - \int_0^t \mathbb{E}(t-\beta)\, f(\beta, \Psi(\beta)d\beta + \int_0^t \mathbb{E}(t-\beta)\, g(\beta, \Psi(\beta))dW(\beta)$ (16)
(16) is the variation of constant formula for stochastic evolution equations.

Lemma 3.1: Given a predictable stochastic process
$Z_\xi: [0,T] \times \Omega \rightarrow \mathcal{H}$ $with$ $\mathcal{P}\big(Z_\xi(t) \in \mathcal{H}^r\big) = 1\ \forall\ t \in [0,T]$ and $\sup_{t\in[0,T]} \big\|Z_\xi(t)\big\|_{\mathcal{L}^{\mathcal{P}}(\Omega;\,\mathcal{H}^r)} < \infty.$
$Then, \forall\ s \in [0, r+1], \exists\ a\ constant\ \mathcal{K} = \mathcal{K}(r,s,\mathcal{A},f)\ such\ that\ \forall\ 0 \le \tau_1 \le \tau_2 \le T$,
$\left\|A^{\frac{s}{2}} \int_{\tau_1}^{\tau_2} E(\tau_2 - \sigma) f\left(\tau_2, Z_\xi(\tau_2)\right) d\sigma\right\|_{\mathcal{L}^{\mathcal{P}}(\Omega;\mathcal{H})}$ (17)

$$\leq \mathcal{K}\left(1+\sup_{t\in[0,T]}\left\|Z_{\xi}(t)\right\|_{\mathcal{L}^{\mathcal{P}}(\Omega;\,\mathcal{H}^{r})}\right)(\tau_2-\tau_1)^{\frac{1+r-s}{2}} \tag{18}$$

Again , for some

$$\delta > \frac{r}{2}\ \exists\ \mathcal{K}_{\delta}\ such\ that\ \left\|Z_{\xi}(t_2)-Z_{\xi}(t_2)\right\|_{\mathcal{L}^{\mathcal{P}}(\Omega;\mathcal{H})} \leq \mathcal{K}_{\delta}|t_2-t_1|^{\delta}\ \forall\ t_1,t_2\in[0,T] \tag{19}$$

It implies that

$$\mathcal{K}=\mathcal{K}(\delta,s,r,f,\mathcal{K}_{\delta})\ \ \left\|A^{\frac{3}{2}}\int_{\tau_1}^{\tau_2}E(\tau_2-\sigma)\Big(f\big(\tau_2,Y(\tau_2)\big)-f\big(\sigma,Y(\sigma)\big)\Big)d\sigma\right\|_{\mathcal{L}^{\mathcal{P}}(\Omega;\mathcal{H})} \tag{20}$$

$Let\ \alpha=(1+2\delta-s)$ such that,

$$\leq\frac{\mathcal{K}}{\alpha}\left(1+\sup_{\sigma\in[0,T]}\|Y(\sigma)\|_{\mathcal{L}^{\mathcal{P}}(\Omega;\mathcal{H})}\right)\leq\mathcal{K}\left(1+\sup_{\sigma\in[0,T]}\|Y(\sigma)\|_{\mathcal{L}^{\mathcal{P}}(\Omega;\mathcal{H})}\right)(\tau_2-\tau_1)^{\alpha} \tag{21}$$

We noted that this technique to stochastic integral first appeared in (see[13])
We consider some examples to emphasis the applied lemmas and propositions

Examples : Consider a stochastic heat equation with additive noise in the unit interval. We are to find a measurable mapping $X:[0,T]\times\Omega\times[0,1]\to\mathbb{R}$ such that

$$dX(t,\tau)=\frac{d^2X}{d\tau^2}(t,\tau)dt+dW(t,\tau),\ \forall\ t\in(0,T],\tau\in[0,1],$$
$$X(t,0)=X(t,1)=0\quad,\quad\forall\ t\in(0,T],$$
$$X(0,\tau)=X_0(\tau)\quad,\quad\forall\ \tau\in(0,1). \tag{22}$$

where the initial condition X_0 is a random variable defined as $X_0:\Omega\times[0,1]\to\mathbb{R}$, such that $\forall\omega\in\Omega, X_0(\omega,\cdot)$ is a smooth function that satisfies the boundary condition.

We are to recast the SPDE of (22) as an abstract SEE in Hilbert space $\mathcal{H}$.
$Let\ \mathcal{H}=\mathcal{L}^2([0,1],\mathcal{B}([0,1]),d\tau;\ \mathbb{R})\ and\ Q$- Wiener process on $\mathcal{H}\ with\ Tr<\infty$
$dX(t)+AX(t)dt=dW(t),\ \forall\ t\in[0,T],X(0)=X_0$ where the mild solution is given by
$X(t)=E(t)X_0+\int_0^t E(t-\tau)dW(\tau),\ \ t\in[0,T]$ with a component of stochastic convolution and stochastic process $X:[0,T]\times\Omega\to\mathcal{H}$ as a solution to (22).

IV CONCLUSION

We considered the invariant measure for s stochastic evolution equation in Hilbert space and applied the conditions to the problem of abstract heat equation for a more corrupted Gaussian noise under a mild solution conditions which easy and straight forward to handle.

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