Quest Journals
Journal of Research in Applied Mathematics
Volume 10 ~ Issue 4 (2024) pp: 39-42
ISSN (Online): 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Review Paper

Numerical Discretization and Simulation of Two-Phase Flow in a Porous Reservoir

Zuonaki Ongodiebi. [1*] and Josephine Joseph Etuk[2]
[*1,] *Department of Mathematics, Niger Delta University, P.M.B 071 Amassoma, Bayelsa State.*
[2] *Department of Mathematics, Michael Okpara University of Agriculture, Umudike, Abia State*

Abstract
The problem of the mixed oil-water flow in a porous reservoir is investigated. The governing equations for each phase are derived, leading to coupled equations for the total pressure, velocity sum and water saturation. An explicit saturation implicit pressure scheme is considered. The results are presented and discussed for two-dimensional reservoir which is permeable only at its centre and is initially filled with oil. The results show that the water infiltrates the reservoir only through the permeable region, which is expected. It is also observed that the fluid velocity increases with increasing reservoir permeability. These results are obviously physically realistic, hence validate the model.
Keywords*: Two-phase flowPorous media, Modelling, Saturation, Porousity and Permeability*

R*eceived 26 Apr., 2024;* ***R****evised 30 Apr., 2024;* ***A****ccepted 02 May., 2024 © The author(s) 2024.*
P*ublished with open access at* ***www.questjournals.org***

I. INTRODUCTION

All reservoir oil fields has a life cycle which spans from discovery to abandonment. Therefore, it is very important for reservoir operators to be able to predict when to leave the oil field or when to apply secondary or tertiary recovery method to increase the volume of the recovered oil. This is crucial for the survival of oil industries. Such prediction is usually possible using mathematical tools and reservoir simulations, which consist of elaborate numerical models that take into account the inherent physical processes associated with flows in porous mediaKomal and Khadija (2023).Two phase porous media flow is a complex non-linear process due to the relations between the capillary pressure, phase saturations and conductivities. Some typical examples of two-phase flows include oil recovery by water-flooding process which is often adopted in the petroleum industries. Another example is the flow of water and air in an unsaturated zone of soil or groundwater contamination by non-aqueous phase liquids (NAPL); Knut-Andreas Lie (2015) and Zuonakiand Orukari (2021).Different types of mathematical formulation is used to describe these processes, depending on the driving force of the flow (viscous, capillary and gravitational) as well as the compressibility of the two fluid phases. (Chavent and Jaffre (1987), Helmig (1997) and Corey (1994)). Numerical methods are the required tools in two-phase flow modeling. The development and improvement of the numerical schemes is still a subject of intensive research Arezou*et.al*. (2019). In this research, we investigated the problem of mixed oil-water flow in a porous reservoir. The governing equations for each phase are derived, leading to coupled equations for the total pressure, velocity sum and water saturation. An explicit saturation, implicit pressure scheme is considered

II.

III. MATHEMATICAL FORMULATION

2.1 Two -phase immiscible flow equation

Here,we considered two-phase immiscible fluids with one phase considered to be wetting and the other, nonwettingIn a water – oil system, water is considered the wetting phase while oil is regarded as the non-wetting phase while in an oil – gas system, oil is referred to as the wetting phase while the gas is the non-wetting phase. We refer to the wetting phase by the subscript w and to the non-wetting phase by the subscript n. Thus we have

$$s_w + s_n = 1 \qquad (1)$$

where s_w, s_n are the saturations of the wetting and non-wetting phase respectively. Also, due to the curvature and surface tensionof the interface between the two phases, thepressure in the wetting fluid is less than that in the non-wetting fluid as mention by Held and Celia (2001). The pressure difference is given by the capillary pressure. As an empirical fact, the capillary pressure is a function of the saturation and the wetting phase Mohammad and Pramod (2015), Starikovicuis (2003) and is defined by

$$p_{cnw}(\mathrm{s_w}) = p_n - p_w \tag{2}$$

At this point, we extend Darcy's law from single phase flow to two-phase flow by assuming that the phase pressure forces for each phase to flow. Thus we have:

$$\mathbf{u}_n = -\frac{Kk_{rn}}{\mu_n}(\nabla P_n - \rho_n G) \tag{3}$$

$$\mathbf{u}_w = -\frac{Kk_{rw}}{\mu_w}(\nabla P_w - \rho_w G) \tag{4}$$

Where α represents the phase (wetting and non-wetting), $Kk_{r\alpha}, P_\alpha, \mu_\alpha$ are the phase permeability, phase pressure and phase viscosity respectively and $G = g\nabla D$.

Coupling equations (3) and (4) via IMPES formulation and defining phase mobility λ_α refer to Zuonaki and Adokiye (2023) for details, we have

$$u = -K\left(\lambda\nabla P - \left(\lambda_o\rho_o + \lambda_w\rho_w\right)G\right) \tag{5}$$

With the continuity equation (6) and saturation equation (7);

$$\nabla \cdot u = q \tag{6}$$

$$\frac{\partial s_w}{\partial t} + u \cdot \nabla F(s_w) = 0 \tag{7}$$

where $q = \frac{q_w}{\rho_w} + \frac{q_o}{\rho_o}$, $u = u_w + u_o$, $\lambda_\alpha = \frac{k_{r\alpha}(s_w)}{\mu_\alpha}$, $\lambda = \lambda_w + \lambda_o$ and $F(s_w) = \frac{\lambda_w(s_w)}{\lambda_w(s_w) + \lambda_o(s_o)}$ (8)

Equations (5), (6) and (7) are for the basis for our numerical scheme.

2.2 Numerical Scheme

Let $S_N = \{0, 1, 2, 3, \ldots\}$ and Δt be given. Define $t^{n+1} = t^n + \Delta t \ \forall\ n \in S_N$. Denote by $\delta^n\psi$ *;* the spatial discretization of the quantity ψ at time level t^n and also define $\psi^n = \psi(\vec{x}) \approx \psi(\vec{x}, t^n)$. Then, we present the following numerical scheme for the models (5-7)

$$\left.\begin{aligned} & u^{n+1} + K\lambda(s_w^n)\delta^{n+1}(\nabla P) - (\lambda_o\rho_o + \lambda_w\rho_w)^{n+1}G = 0 \\ & \delta^{n+1}(\nabla \cdot u) = q^{n+1} \\ & \frac{s_w^{n+1} - s_w^n}{\Delta t} + u^{n+1} \cdot \delta^n\left(\nabla F(s_w^n)\right) = 0 \quad \forall\ n \in S_N \end{aligned}\right\} \tag{9}$$

We used the following data: the reservoir is initially filled with purely oil (no water), the left boundary is maintained at water saturation of one and the right maintained at zero. The pressure at the boundary is given by $P = 1 - x$; while

$F(s) = \frac{s^2}{\mu(1-s)^2 + s^2}$, $G = 0$ and $K = K(\vec{x}) = \gamma \max\left\{\frac{1}{100}, \frac{1}{e^{r(\vec{x})}}\right\}$ where

$r(\vec{x}) = 100\left(y - \left(0.5 + 0.1\sin(10x)\right)\right)^2$ and γ is the permeability parameter. Increasing γ will allow us to study the effect of permeability on the flow.

IV. Numerical Results

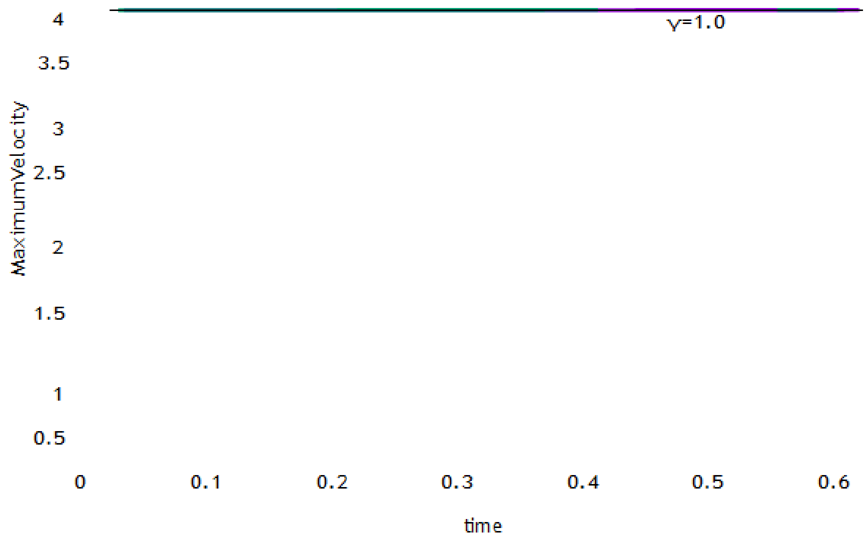
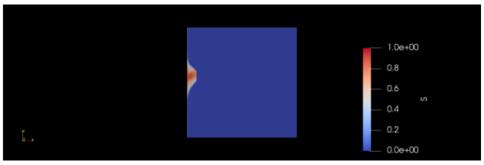


Figure1:Effectofpermeability,γontheflowmaximumvelocity

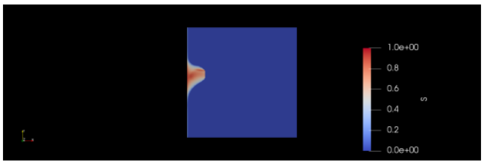
Figure 1, shows the maximum phase velocity as time evolves. As time increases, the flow velocity also increases at different rates of the permeability parameter γ. The greater the permeability implies greater connectivity of the porous medium and hence improved flow.

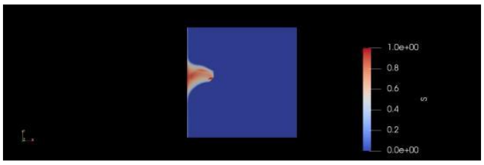

(a)Afterfirsttimestep

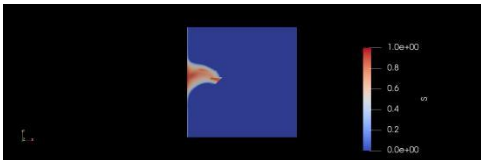

(b)Afterfivetimesteps


(c)Aftertentime steps


(d)Afterfifteentimesteps

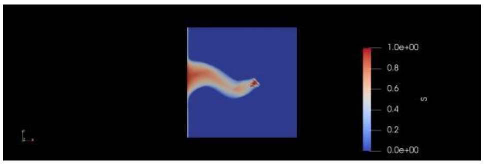

(e)Aftertwentytimesteps


(f)Afterthirtytimesteps

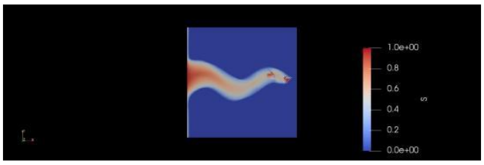

(g)Afterfortytimesteps


(h) Afterseventytimesteps

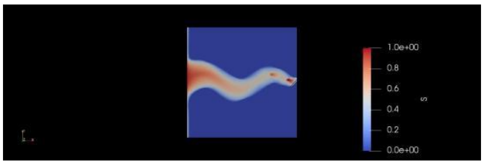
*Corresponding Author: Zuonaki Ongodiebi



(i)Afteronehundredtimesteps



(j)Afteronehundredandfiftytimesteps



(k) After one hundred and sixty time steps

Figure2:Timeevolutionofwatersaturationinthereservoir

Figure 2 consists of the images of time evolution of water saturation in the porous reservoir. Different time steps were considered from one up to one hundred and sixty. Observation from images (a) to (k) show that as the time steps progresses, greater region of the reservoir is saturated with water, creating room for improved mobility of oil within the reservoir

V. Conclusion

The problem of the mixed oil-water flow in a porous reservoir is investigated. The governing equations for each phase are derived, leading to coupled equations for the total pressure, velocity sum and water saturation. An explicit saturation, implicit pressure scheme is considered. The results are presented and discussed for two-dimensional reservoir which is permeable only at its centre and is initially filled with oil. The results show that the water infiltrates the reservoir only through the permeable region, which is expected. It is also observed that the fluid velocity increases with increasing reservoir permeability. These results are obviously physically realistic, hence validate the model.

References

[1]. Arezou, J., Mohammadreza, H., Mostafa, H., and Reza G. (2019). Application of CFD technique to simulate enhanced oil recovery processes: current status and future opportunities. Springer 1-23

[2]. Chavent G., Jaffre J. (1987) Mathematical Models and Finite Elements for Reservoir Simulation,North-Holland, Amsterdam.

[3]. Corey, A.T. (1994). Mechanics of Immiscible Fluids in Porous Media. Water Resources Publications 3rd Edition, U.S.A.

[4]. Held, R. J. and Celia, M. A. (2001). Modeling support of functional relationships between capillary pressure, saturation, interfacial areas and common lines. Advances in Water Resources, 24:325–343.

[5]. Helmig, R. (1997). Multiphase flow and transport processes in the subsurface: A Contribution to the Modeling of Hydrosystems. Springer-Verlag, Berlin.

[6]. Knut-Andreas Lie (2015). An introduction to reservoir simulation using MATLAB. User Guide for the Matlab Reservoir Simulation Toolbox (MRST)

[7]. Komal, M. Sana U. and Khadija T. K. (2023). Mathematical modeling of fluid flow and pollutant transport in ahomogeneous porous medium in the presence of plate stacks. Heliyon https://doi.org/10.1016/j.heliyon.2023.e14329 vol 9, 1-24

[8]. Mohammed, M. and Pramod, K. P. (2015). Flow and diffusion equations for fluid flow in porous rocks for the multiphase flow phenomena. American Journal of Engineering Research, 4(7)68, 139-148

[9]. Nagi, A. A. (2009). Flow and Transport Problems in Porous Media Using CFD M.Sc thesis, Alexandria University.

[10]. Starikovicius, V. (2003). The multiphase flow and heat transfer in porous media. Berichte des Fraunhofer ITWM, Nr. 55,1-30

[11]. Zhangxin, C. Reservoir Simulation Mathematical Techniques in Oil Recovery (2007), Society for Industrial and Applied Mathematics, Philadelphia.

[12]. Zhangxin, C., Huan, G. and Yuanle, M. (2006). Computational Methods for Multiphase Flows in Porous Media. Society for Industrial and Applied Mathematics, Philadelphia.

[13]. Zuonaki Ongodiebi and Adokiye Omoghoyan:Modelling Of Multiphase Porous Media Flow Equations. Global Scientific JournalVol 11, Issue 12, Pp 714-730, 2023

[14]. Zuonaki Ongodiebiand Orukari M. A.: Analytical Solution of Two-Phase Incompressible Flow Equation in a Homogeneous Porous Medium. International Journal of Mathematics and Computer Research Vol.9, Issue 7, Pp:2344-2347, 2021