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Review Paper

Non-Unique Fixed Point Results for CIRIC Type Mapping Of Quasi Partial Metric Space

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Abstract: *In the present paper inuence by the work of Ciric,"the authors prove the nonunique xed point using Ciric type (I) contraction for order of self mapping and given dierent type of operator in quasi-partial metric space. Our outcomes are extended, generalize and unite dif- ferent type of known results of various authors. Here we establish the existence of new xed point theorems of certain mappings generaliz- ing the releted work of Erdal Karapinar for non unique xed point. In support of our newly obtained result,we provide one example .*
Keywords: *xed point theory, quasi partial metric space, Ciri ́c type (I) mapping ,Orbital continuity.*
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I. INTRODUCTION

In fixed point theory, the Banach contraction principle is the remarkable result that was introduced by S.Banach [3] in 1922. Over the years, this theory was generalized by various researchers on different metric spaces. And they used the contraction principle for references to their theorems. Karpinar et. al.[14] established quasi-partial-metric space in 2012 and obtained a new non-unique fixed point theorem. He proved the existence of fixed points for self-mapping in quasi-partial metric space. By a generalization of the Banach contraction principle on a complete metric space, Ciric [4] (1974) proved a non-unique fixed point theorem on some maps. In 1976 [1] and 1978 [2], J.Achari, obtained some results on Ciri ́c's non-unique fixed points. In generating spaces, Seong-Hoon Cho[5] (2019) established Fixed point theorems for Ciric type Z-contractions in quasi-family. K. P. Chi et.al. [6] (2012) satisfying generalized a contraction principle in partial metric spaces. S. Gupta and B. Ram [7] (1998) obtained the fixed point theorems with a nonunique fixed point. V. Gupta (2020) obtained quasi-partial metrics aggregation on Ψ-projective expansion with an application. E. Karapnar, I. M. Erhan [8] (2011) established fixed point results for operators on partial metric spaces. E. Karapnar [9,10] (2011 2012) generalized Caristi Kirk's type fixed point in partial metric spaces and proved a new Ciric types non-unique fixed point theorem. H.P.A. Kunzi et.al [11] (2006) introduced Partial quasi-metric spaces. Z. Liu et. al [12] (2006) introduced Ciri ́c type mappings with non-unique fixed and periodic points.S. G. Matthews [13] (1992) obtained Partial metric topology. Pachpatte, B. G [14] (1979) obtained Ciric-type maps with a nonunique fixed point.

By generalizing the Banach contraction principle, Ciric demonstrates an outcome on a non-unique fixed point known as

Ciri´c type contraction [5] (1974).ie.

Let (X,d) be a metric space and $T:(X,d)\to(X,d)$ is a self map, satisfies the following condition

$$min\{d(T\xi,T\eta),d(\xi,T\xi),d(\eta,T\eta)\}- \\ min\{d(\xi,T\eta),d(\eta,T\xi)\}\leq cd(\xi,\eta)$$

where all $\xi,\eta\in X$; $c\in(0,1)$ then there is a fixed point in T whenever X is a T - orbitally complete.

The scope of the present article is to find some fixed points that are non-unique inside the frame of quasi-partial metric space.

Now, we take a look at a few lemmas and definitions pertinent to our primary findings. Here q_l is quasi partial metric space, N is natural quantity and $\Re^+$ is represent all real and positive quantities.

Definition 1.1:(E. Karapınar, 2012) [11]: A function $q_l:P\times P\to\Re^+$ satisfing the condition

(i) $0\leq q_l(\xi,\eta)=q_l(\eta,\eta)=q_l(\xi,\eta)\Longleftrightarrow\xi=\eta$

(ii) $q_l(\xi,\xi)\leq q_l(\xi,\eta)$

(iii) $q_l(\xi,\xi)\leq q_l(\eta,\xi)$

(iv) $q_l(\xi,\eta)+q_l(u,u)\leq q_l(\xi,u)+q_l(u,\eta)$ for all $\xi,\eta,u\in P$

Then (P,q_l) known as quasi-partial metric space (QPMS) and P is a non void set.

Let (P,q_l) is a QPMS, then a function $d_{q_l}:P\times P\to\Re^+$ is a metric on P defined by $d_{q_l}(\xi,\eta)=q_l(\xi,\eta)+q_l(\eta,\xi))-q_l(\xi,\xi)-q_l(\eta,\eta)$

Here we note that every quasi partial metric space q_l on P creates a T_0 topology τ_{q_l}on P, whose base is a open q_l- ball family $\{\mathscr{B}_{q_l}(\xi,\epsilon):\xi\in P,\epsilon\in P\}$ and

$\mathscr{B}_{q_l}(\xi,\epsilon)=\{\eta\in P:q_l(\xi,\eta)\leq q_l(\xi,\xi)+\epsilon\}$ for all $\epsilon>0$ and $\xi\in P$.

Now we defind some principles like continuity, completeness,Cauchy sequemce and Convergence in that manner :

Definition 1.2: Seong-Hoon Cho[5]

a) In a QPMS (P, q_l), $\{\xi_i\}$ is a sequence converges to the limit ξ $\Leftrightarrow q_l(\xi,\xi) = lim_{i\to\infty} q_l(\xi,\xi_i)$.

b) In a QPMS (P, q_l), $\{\xi_i\}$ is known as a Cauchy sequence if $lim_{i,j\to\infty} q_l(\xi_i,\xi_j)$ is exists and finite.

c) In P, if every Cauchy sequence $\{\xi_i\}$ converge with respect to τ_{q_l} to a point $\xi \in P$ analogous $q_l(\xi,\xi) = lim_{i,j\to\infty} q_l(\xi_i,\xi_j)$ then QPMS (P, q_l) known as complete.

d) At $\xi_0 \in P$, a mapping $f : P \to P$ is continuious if for every $\epsilon > 0, \exists\ \lambda > 0$ such that $F(\mathscr{B}_{q_l}(\xi_0,\lambda) \subseteq \mathscr{B}_{q_l}(F\xi_0,\epsilon)$.

Lemma 1.2.1: (E. Karapinar, 2012)[10]

1) In a QPMS (P, q_l), $\{\xi_i\}$ be a Cauchy sequence $\Longleftrightarrow \{\xi_i\}$ is Cauchy sequence in partial metric space.

2) A QPMS (P, q_l) is complete $\Longleftrightarrow$ partial metric space (P, q_p) is also complete.

$$lim_{i\to\infty} q_p(\xi,\xi_i) = 0 \Leftrightarrow q_l(\xi,\xi) = lim_{i\to\infty} q_l(\xi,\xi_i) = lim_{i,j\to\infty} q_l(\xi_i,\xi_j) \quad (1)$$

Lemma 1.2.2:(E. Karapinar, 2012)[10]

In a QPMS (P, q_l), suppose if $\xi_i \to \mathsf{z}_1$as $i \to \infty$ in such a way that $q_l(\mathsf{z}_1, \mathsf{z}_1) = 0$
and $lim_{i\to\infty} q_l(\xi_i,\eta) = q_l(\mathsf{z}_1,\eta)$ for every $\eta \in P$.

Lemma 1.2.3:(E. Karapinar, 2012)[10]

Suppose (P, q_l) is a QPMS, then

(i) If $q_l(\xi,\eta) = 0$ then $\xi = \eta$.

(ii) If $\xi \neq \eta$ then $q_l(\xi,\eta) > 0$.

In a QPMS (P, q_l) let B be a self map. For every $\eta \in P$ and $Y \subseteq P$

1. $\lambda(Y) = sup\{q_l(\xi,\eta) : \xi,\eta \in Y$
2. $O(\xi,i) = \{\xi, B\xi, B^2\xi, B^3\xi.......B^i\xi\}$ where $i \in N$
3. $O(\xi,\infty) = \{\xi, B\xi, B^2\xi, B^3\xi..........\}$.

Definition 1.3:(E. Karapınar, 2012)[10]:

a) A map $B:(P,q_l)\to(P,q_l)$ is called orbitally continuious if

$lim_{n,m\to\infty} q_l(B^{i_n}\xi, B^{i_m}\xi) = lim_{n\to\infty} q_l(B^{i_n}\xi, z_1) = q_l(z_1, z_1)$

which shows that.

$lim_{n,m\to\infty} q_l(BB^{i_n}\xi, BB^{i_m}\xi) = lim_{n\to\infty} q_l(BB^{i_n}\xi, Bz_1) = q_l(Bz_1, Bz_1)$.

b) A QPMS is known as orbitally complete if every Cauchy sequence $\{B^{i_n}\xi,\}_{i=1}^{\infty}$ converge in (P,q_l) if

$$lim_{n,m\to\infty} q_l(B^{i_n}\xi, B^{i_m}\xi) = lim_{n\to\infty} q_l(B^{i_n}\xi, z_1) = q_l(z_1, z_1)$$

Remarks 1.4: For any $j\in N$, the orbital continuity of B implicit orbital continuity of B^j.

2. Aggregation of Quasi Partial Metric and its Projective expansion :

Definition 2.1 (V.Gupta et.al.,2020): Let $(Y_i,q_i)_{i=1}^m$ be a family of quasi-partial metric spaces and $Y=\prod_{i=1}^m Y_i$. Let $\Psi: R_+^m\to R_+$ be a quasi partial metric aggregation function. Then the mapping $D:Y\to Y$ is called a projective $\Psi-$expansion from (Y,Q_Ψ) into itself, if there exist n constants $\lambda_1,\ldots,\lambda_m>1$ such that $q_i(D_i(y),D_i(z))\geq$ $\lambda_i\,\Psi(q_1(y_1,z_1),\ldots q_m(y_m z_m))$ for all $y,z\in X$, where Q_Ψ is the quasi partial metric induced by aggregation of the collection of quasi partial metric spaces $(Y_i,q_i)_{i=1}^m$ through aggregation function Ψ.

Remark 2.1.1 (V.Gupta et.al.,2020): Let collection of nonempty sets represent by$\{Y_i\}_{i=1}^m$ and $Y=\prod_{i=1}^m X_i$. Let D be a self-mapping defined on Y with coordinate functions $D_i:Y\to Y_i, i=1,\ldots m$ such that $D(y)=(D_1(y),D_2(y),\ldots,D_m(y))$ for all $y\in Y$.

Corollary 2.1.2 (V.Gupta et.al.,2020): Let a family of quasi partial metric spaces be $(Y_i,q_i)_{i=1}^m$ with complete metrics $q_i; i=1,\ldots,m$ and $Y=\prod_{i=1}^m X_i$. Let a homogeneous quasi partial metric aggregation function be Ψ such that $\Psi(1,\ldots,1)=\Psi(1i)=1; i=1,\ldots,m$ and an onto projective Ψ-expansion is D . Then D has a unique fixed point $y*$.

Here we introduce some theorems of non-unique fixed points in the structure of QPMS.

3. Result of nonunique fixed point theorem in Ciric type contractive mapping:

Here we introduce some theorems of nonunique fixed point in the structure of QPMS.

Theorem 3.1: Let an orbitally continuous self-map be $B:(P,q_l)\to(P,q_l)$ on P, where (P,q_l) is a quasi-partial-metric space. B has a fixed point whenever (P,q_l) is B- orbitally complete. And For all $\xi,\eta\in P$ and some constant $c\in(0,1)$, if B satisfies the condition

$$min\{q_l(B\xi, B\eta), q_l(\xi, B\xi), q_l(\eta, B\eta)\} - min\{q_l(\xi, B\eta), q_l(\eta, B\xi)\} \leq cq_l(\xi, \eta) \quad (2)$$

Then For every $\xi \in P, \{B^i\xi\}$ converges to a fixed point of B.

Proof:

Let $\xi_0 \in P$ be an arbitrary point and the sequence defined for $i = 0, 1, 2, 3....$ is

$$\xi_{i+1} = B\xi_i \quad (3)$$

It is already proven previously that if there exists a non-negative value i such that $\xi_{i+1} = \xi_i$ then B has a fixed point ξ_i .

Let for every $i = 0, 1, 2.....; \xi_i \neq \xi_{i+1}$. Substitute $\xi = \xi_i$ and $\eta = \xi_{i+1}$ in (2). Then we get

$min\{q_l(B\xi_i, B\xi_{i+1}), q_l(\xi_i, B\xi_i), q_l(\xi_{i+1}, B\xi_{i+1})\} - min\{q_l(\xi_i, B\xi_{i+1}), q_l(\xi_{i+1}, B\xi_i)\} \leq cq_l(\xi_i, \xi_{i+1})$

which implies $min\{q_l(\xi_{i+1}, \xi_{i+2}), q_l(\xi_i, \xi_{i+1}), q_l(\xi_{i+1}, \xi_{i+2})\} - min\{q_l(\xi_i, \xi_{i+2}), q_l(\xi_{i+1}, \xi_{i+1})\} \leq cq_l(\xi_i, \xi_{i+1})$

Therefore

$$min\{q_l(\xi_{i+1}, \xi_{i+2})\} - q_l(\xi_{i+1}, \xi_{i+1})\} \leq cq_l(\xi_i, \xi_{i+1}) \quad (4)$$

Suppoae that $c \in [0, 1)$ and for every $i = 0, 1, 2.....;$ the equation (4) shows that $q_l(\xi_{i+1}, \xi_{i+2}) \leq cq_l(\xi_i, \xi_{i+1})$

Thus we get,

$$q_l(\xi_{i+1}, \xi_{i+2}) \leq cq_l(\xi_i, \xi_{i+1}) \leq c^2 q_l(\xi_{i-1}, \xi_i) \text{ .... } \leq c^{i+1} q_l(\xi_0, \xi_1) \quad (5)$$

Now, we prove $\{\xi_i\}$is a Cauchy Sequence.

Let $i > j$, Then by using equation (5) and the triangle inequality of (iv) of definition 1.1

$$0 \leq q_l(\xi_i, \xi_j) \leq q_l(\xi_i, \xi_{i-1}) + q_l(\xi_{i-1}, \xi_{i-2}) + ... + q_l(\xi_{j+1}, \xi_j) - [q_l(\xi_{i-1}, \xi_{i-1}) + q_l(\xi_{i-2}, \xi_{i-2}) + ... + q_l(\xi_{j+1}, \xi_{j+1})]$$

$$\leq q_l(\xi_i, \xi_{i-1}) + q_l(\xi_{i-1}, \xi_{i-2}) + ... + q_l(\xi_{j+1}, \xi_j)$$

$$\leq [k^{i-1} + k^{i-2} + \cdots k^j] q_l(\xi_0, \xi_1)$$

By G.P. $k^j \frac{1-k^{i-j}}{1-k} q_l(\xi_0, \xi_1)$. Taking limit as $i, j \to \infty$ $q_l(\xi_i, \xi_j) = 0$

Hence, $\{\xi_i\}$is a Cauchy Sequence in (P, q_l)

From lemma 1.3, $\{\xi_i\}$ is also Cauchy in PMS (P, p) as well as in metric space (P, d).

Since (P, q_l) is complete then PMS (P, p) as well as metric space (P, d) are also complete.

Hence $\exists$ a point $z_1 \in$P in such a way that $\xi_i \to z_1$ in (P, d) , from Lemma 1.3

$$q_l(z_1, z_1) = lim_{i\to\infty} q_l(z_1, \xi_i) = lim_{i,j\to\infty} q_l(\xi_i, \xi_j) \qquad (6)$$

which shows that

$$lim_{i\to\infty} q_l(z_1, \xi_i) = 0 \qquad (7)$$

So $q^1(z_1, z_1) = 0$ from equation (6)

Its conclude that z_1 is the fixed point of S. Now substitute $\xi = \xi_i$ and $\eta = z_1$ in equation (2)

Then we obtain,

$min\{q_l(B\xi_i, Bz_1), q_l(\xi_i, B\xi_i), q_l(z_1, Bz_1)\} - min\{q_l(\xi_i, Bz_1), q_l(z_1, B\xi_i)\} \leq cq_l(\xi_i, z_1)$

which shows

$$min\{q_l(B\xi_{i+1}, Bz_1), q_l(\xi_i, \xi_{i+1}), q_l(z_1, Bz_1)\} - min\{q_l(\xi_i, Bz_1), q_l(z_1, B\xi_{i+1})\} \leq cq_l(\xi_i, z_1) \qquad (8)$$

Taking limit as $i \to \infty$, $\quad p(z_1, Bz_1) \leq 0$

from equation (6) and Lemma (1.2.2); $q_l(z_1, Bz_1) = 0$

from equation (1.1) we conclude that,

$$0 \leq p(z_1, Bz_1) = 2q_l(z_1, Bz_1) - q_l(z_1, z_1) - q_l(Bz_1, Bz_1) = -q_l(Bz_1, Bz_1) \leq 0$$

Hence $p(z_1, Bz_1) = 0$ so finally we obtain $z_1 = Bz_1$

For supporting our result one example is given.

Example:

Let $q_l(\xi, \eta) = max\,(\xi, \eta)$ For all $\xi, \eta \in P$,

and $P \in \Re^+$ then (P, q_l) is a QPMS. Let $\xi = 2 \geq \eta = 1$.

then $B\xi = \frac{\xi^3}{1+4\xi^2} = .4706$ (approx.) and $B\eta = \frac{\eta^3}{1+4\eta^2} =\cdot\ 0.2$

Suppose $B : P \to P$ such that $B\xi = \frac{\xi^3}{1+4\xi^2}$for all $\xi \in P$,Then

$q_l(\xi,\eta) = max(\xi,\eta) = max(2,1) = 2 = \xi$

$q_l(\xi,B\xi) = max(\xi, \frac{\xi^3}{1+4\xi^2}) = max(2, 0.4706) = 2 = \xi$

$q_l(\eta,B\eta) = max(\eta, \frac{\eta^3}{1+4\eta^2}) = max(1, 0.2) = 1 = \eta$

$q_l(\xi,B\eta) = max(\xi, \frac{\eta^3}{1+4\eta^2}) = max(2, 0.2) = 2 = \xi$

$q_l(\eta,B\xi) = max(\eta, \frac{\xi^3}{1+4\xi^2}) = max(1, 0.4706) = 1 = \eta$

$q_l(B\xi,B\eta) = max(\frac{\xi^3}{1+4\xi^2}, \frac{\eta^3}{1+4\eta^2}) = max(0.4706, 0.2) = 0.4706 = B\xi$

from (1)
$min\{q_l(B\xi,B\eta), q_l(\xi,B\xi), q_l(\eta,B\eta)\} - min\{q_l(\xi,B\eta), q_l(\eta,B\xi)\} \leq cq_l(\xi,\eta)$

$$min\{\frac{\xi^3}{1+4\xi^2},\xi,\eta\} - min\{\xi,\eta\} = min\{\frac{\xi^3}{1+4\xi^2},\eta\} \qquad (9)$$

In equation (2) , we take c = 3/4; If

$$min\{\frac{\xi^3}{1+4\xi^2},\eta\} = \frac{\xi^3}{1+4\xi^2} \leq \frac{3}{4}\xi \qquad (10)$$

If

$$min\{\frac{\xi^3}{1+4\xi^2},\eta\} = \eta then \eta \leq \frac{\xi^3}{1+4\xi^2} \qquad (11)$$

From equation (10) and (11), we can prove that $\eta \leq \frac{\xi^3}{1+4\xi^2} \leq \frac{3}{4}\xi$.
Hence, $\xi = 0$ is the fixed point of B.

Theorem 3.2: Suppose $B : (P, q_l) \to (P, q_l)$ a orbitally continuous self map, mapped on B- orbitally complete QPMS with $\epsilon > 0$. Let there exist a point $\eta_0 \in P$ in such a way that for few $i \in N, q_l(\xi_0, B^i(\xi_0)) < \epsilon$ and B satisfy the coditions for all $\xi, \eta \in P$ and some constant $c < 1$

$$0 < q_l(\xi, \eta) < \epsilon \Rightarrow min\{q_l(\xi, B(\xi)), q_l(B(\xi), B(\eta)), q_l(B(\eta), \eta\} -$$

$$min\{q_l(\xi, B(\eta)), q_l(\eta, B(\xi)\} \leq cq_l(\xi, \eta). \qquad (12)$$

Then B has a periodic point.

Proof: Here we take set $D = \{i \in N : q_l(\xi, B^i(\xi)) < \epsilon$, for $\xi \in P$. Let $D \neq 0$,

$j = \min D$ and $\xi \in P$ such that $q_l(\xi, B^j(\xi)) < \epsilon$ .Now consider two cases.

Case(1) When j = 1 then $q_l(\xi, B(\xi)) < \epsilon$. In equation (12) put $\eta = B(\xi)$

$$min\{q_l(\xi, B(\xi)), q_l(B(\xi), B(B(\xi))), q_l(B(B(\xi)), B(\xi))\} -$$

$$min\{q_l(\xi, B(B(\xi))), q_l(B(\xi), B(\xi)\} \leq cq_l(\xi, B(\xi))$$

$$min\{q_l(\xi, B(\xi)), q_l(B(\xi), B^2(\xi)), q_l(B(B(\xi)), B(\xi)\} -$$

$$min\{q_l(\xi, B^2(\xi))\} \leq cq_l(\xi, B(\xi))$$

since $c < 1$, so $\{q_l(\xi, B(\xi)), q_l(\xi, B^2(\xi))\} \leq cq_l(\xi, B(\xi))$
give a contradiction.

Thus $q_l(B(\xi), B^2(\xi)) \leq cq_l(\xi, B(\xi))$

Again in theorem 1, we can suppose an arbitrary point $\xi = \xi_0$ and take the iterative sequence $B\xi_i = \xi_{i+1}$ and prove that $z_1 = Bz_1$ a fixed point of B.

Case(2): Suppose $j \geq 2$, for every $y \in P$ if

$$q_l(B(\eta), \eta) \geq \epsilon \qquad (13)$$

Then from equation (12) and the condition $q_l(\xi, B^j(\xi)) < \epsilon$

$$0 < q_l(\xi, \eta) < \epsilon \Rightarrow min\{q_l(\xi, B(\xi)), q_l(B(\xi), B(\eta)), q_l(B(\eta), \eta\} - min\{q_l(\xi, B(\eta)), q_l(\eta, B(\xi)\} \leq cq_l(\xi, B^j(\xi))$$

it shows that, $min\{q_l(\xi, B(\xi)), q_l(B(\xi), B(B^j(\xi)), q_l(B(B^j(\xi)), B^j(\xi)\} - min\{q_l(\xi, B(B^j(\xi)), q_l(B^j(\xi), B(\xi))\} \leq cq_l(\xi, B^j(\xi))$

$$min\{q_l(B(\xi), B^{j+1}(\xi)), q_l(B^{j+1}(\xi), B^j(\xi))\} - min\{q_l(\xi, B^{j+1}(\xi)), q_l(B^j(\xi), B(\xi)\} \leq cq_l(\xi, B^j(\xi))$$

from equation (13) we obtain $q_l(B^{j+1}(\xi), B^j(\xi)), q_l(B^j(\xi), B(\xi)) > \epsilon$

Therefore $min\{q_l(B(\xi), B^{j+1}(\xi))\} - min\{q_l(\xi, B^{j+1}(\xi))\} \leq cq_l(\xi, B^j(\xi))$
$\Rightarrow q_l(B(\xi), B^{j+1}(\xi)) \leq cq_l(\xi, B^j(\xi))$

Similarly we can say that $q_l(B^2\xi, B^{j+2}(\xi)) \leq cq_l(B(\xi), B^{j+1}(\xi)) \leq c^2 q_l(\xi, B^j(\xi))$

by ongoing same process, for each $t \in N$,we obtain,

$$q_l(B^t(\xi), B^{j+t}(\xi)) \leq q_l(B^{t-1}(\xi), B^{j+t-1}(\xi)) \leq .... \leq c^t q_l(\xi, B^j(\xi))$$

hus for any repetitive sequence $\xi_{i+1} = B^J(\xi_i)$ where $\xi_0 = \xi$

$$q_l(\xi_i, \xi_{i+1}) = q_l(B^{ij}(\xi_0), B^{(i+1)j}(\xi_0)) \leq c^{ij}\, q_l(\xi_0, B^j(\xi_0))$$

using (iv) of def. 1.1 and for each $t \in N$,
$q_l(\xi_i, \xi_{i+t}) \leq q_l(\xi_i, \xi_{i+1}), q_l(\xi_{i+1}, \xi_{i+2}) + ... + q_l(\xi_{i+t-1}, \xi_{i+t})$

$$q_l(\xi_i, \xi_{i+t}) \leq c^{ij}[1 + c^j + ... + c^{(t-1)j}](\xi_0, B^j(\xi_0))$$

$$q_l(\xi_i, \xi_{i+t}) \leq \frac{c^{ij}}{1-c^j}(\xi_0, B^j(\xi_0))$$

Thus $lim_{i\to\infty} q_l(\xi_i, \xi_{i+t}) = 0$

So, in set P, $\{\xi_i\}$ is a Cauchy sequence. For some $\mathsf{z_1} \in P$ where P is B-orbitally complete. Such that

$$lim_{i\to\infty} q_l(B^{ji}(\xi_0), \mathsf{z_1}) = lim_{i\to\infty} q_l(\xi_i, \mathsf{z_1}) = q_l(\mathsf{z_1}, \mathsf{z_1}) = 0 \qquad (14)$$

from remark (1.4) the orbital continuity of S indicate that

$$\begin{aligned} q_l(B^j(\mathsf{z_1}), B^j(\mathsf{z_1})) &= lim_{i\to\infty} q_l(B^j(B^{ij}(\xi_0)), B^j(\mathsf{z_1})) = lim_{i\to\infty} q_l(B^j(B^{ij}(\xi_0)), B^j(B^{ij}(\xi_0))) \\ &= lim_{i\to\infty} q_l(B^{(i+1)j}(\xi_0), B^j(\mathsf{z_1})) = lim_{i\to\infty} q_l(B^{(i+1)j}(\xi_0), B^{(i+1)j}(\xi_0)) \\ &= lim_{i\to\infty} q_l(\xi_{i+1}, B^j(\mathsf{z_1})) = lim_{g\to\infty} q_l(\xi_{i+1}, \xi_{i+1}) \\ &= q_l(\mathsf{z_1}, B^j\mathsf{z_1}) = q_l(\mathsf{z_1}, \mathsf{z_1}) \end{aligned}$$

Thus $q_l(B^j(\mathsf{z_1}), B^j(\mathsf{z_1})) = q_l(\mathsf{z_1}, B^j\mathsf{z_1}) = q_l(\mathsf{z_1}, \mathsf{z_1})$.

Hence from (i) of def. 1.1 , Periodic point of B is $\mathsf{z_1}$

Theorem 3.3: Let P be a non-empty set and provide two quasi-partial-metric spaces

q and δ. B be a self-map. If we assume

a) with respect to q, P is an orbitally complete and orbitally continuous space.

b) $q_l(\xi, \eta) \leq \delta(\xi, \eta)$ for all $\xi, \eta \in P$

c) B satisfy the condition $min\{[\delta(B(\xi), B(\eta))]^2, \delta(\xi, \eta), \delta(B(\xi), B(\eta)), [\delta(\eta, B(\eta))]^2\} - min\{[\delta(B(\eta), B(\xi))]^2,)\delta(\xi, B(\eta)), \delta(\eta, B(\xi)), [\delta(\eta, B(\eta))]^2\} \leq$

$$c\,[\delta(\xi, B(\xi)), \delta(\eta, B(\eta))] \qquad (15)$$

For all $\xi, \eta \in P$ and $c < 1$. Then B has a fixed point in P.

Proof: Let take a point $\xi_0 \in P$ and the sequence ξ_0 defined for $i \geq 1$,

$\xi_1 = B(\xi_0)$ and $\xi_{i+1} = B(\xi_i) = B^{i+1}(\xi_0)$.

In equation (15),Replacing ξ, η with ξ_{i-1}, ξ_i

$$
\begin{aligned}
&min\{[\delta(B(\xi_{i-1}), B(\xi_i))]^2, \delta(\xi_{i-1}, \xi_i), \delta(B(\xi_{i-1}), B(\xi_i)), [\delta(\xi_i, B(\xi_i))]^2\} - \\
&\qquad min\{[\delta(B(\xi_i), B(\xi_{i-1}))]^2, \delta((\xi_{i-1}), B(\xi_i)), \delta(\xi_i, B(\xi_{i-1})), [\delta(\xi_i, B(\xi_i))]^2\} \\
&\qquad \leq c\,[\delta(\xi_{i-1}, B(\xi_{i-1})), \delta(\xi_i, B(\xi_i))]
\end{aligned}
$$

$$
\begin{aligned}
&min\{[\delta(\xi_i, \xi_{i+1})], \delta(\xi_{i-1}, \xi_i), \delta(\xi_i, \xi_{i+1}), [\delta(\xi_i, \xi_{i+1})]^2\} - \\
&\qquad\qquad min\{[\delta(\xi_{i+1}, \xi_i)]^2, \delta(\xi_i, \xi_{i+1}), \delta(\xi_i, \xi_i), [\delta(\xi_i, \xi_{i+1})\}
\end{aligned}
$$

$$\leq c\,[\delta(\xi_{i-1}, \xi_i), \delta(\xi_i, \xi_{i+1})] \tag{16}$$

$\delta(\xi_{i-1}, \xi_i), \delta(\xi_i, \xi_{i+1}) \leq c\,[\delta(\xi_{i-1}, \xi_i), \delta(\xi_i, \xi_{i+1})]$ gives a contradiction

Therefore $\delta(\xi_i, \xi_{i+1}) \leq c\,[\delta(\xi_{i-1}, \xi_i) \leq c\,[\delta(\xi_{i-2}, \xi_{i-1})) \leq \ldots\ldots \leq c^i\,\delta(\xi_0, \xi_1)$

So, for any $t \in N$;

$$\delta(\xi_i, \xi_{i+t}) \leq \frac{c^i}{1-c}\,\delta(\xi_0, \xi_1) \tag{17}$$

and

$$\delta(\xi_i, \xi_{i+q}) \leq \frac{c^i}{1-c}\,\delta(\xi_0, \xi_1) \tag{18}$$

Thus with respect to q, ξ_i is a Cauchy sequence. Later P be a B-orbitally complete then $\exists\ z_1 \in P$.

Such that $lim_{i\to\infty} B^i(\xi_i) = z_1$,from the orbital continuity of B,

$Bz_1 = lim_{i \to \infty} B(B^i(\xi_i) = z_1$.

4. Application:

The goal of complexity analysis in computer science is to determine which algorithm is best, or in another way, the algorithm that uses the least amount of space and time even when dealing with large amounts of inputs and other adequate resources. Asymptotic analysis typically uses for this, with the running time of algorithm A given by the mapping $T_Q : N \to (0, \infty)$. The amount of time or space needed by an algorithm to tackle the task at hand is indicated by the symbol $T_Q(m)$, where $m \in N$ denotes the volume of input data that has to be processed. Let $G(T_Q)$ stand for the collection of all functions between N and $(0, \infty)$.

When evaluating the complexity of algorithm analysis, asymptotic complexity analysis is used rather than exact analysis. Then research an algorithm that, even with huge inputs and other acceptable resources, uses "approximately" the least amount of space and the least amount of running time.

Let $h \in G(T_Q)$ represent the running time or space an algorithm uses to run. Following that, we may establish an asymptotic upper bound for h as follows:

If $m_0 \in N, g \in R^+$, and $k \in G(T_Q)$ are there, then $h(m) \leq kg(m)$ for all $m \in N$ such that $m_0 \leq m$. Then,k represents "approximate" knowledge about the procedure and provides an asymptotic upper bound on h. It had expressed as $h \in U(k)$. A similar asymptotic lower bound for the algorithm can also be defined. The notation $h \in L(k)$ indicates that for all $m \in N$ such that $m_0 \in m$, there exist $m_0 \in N, g \in R^+$, and a function $k \in G(T_Q)$ such that $kg(m) \leq h(m)$. The ideal situation is one in which we can identify a function h that satisfies the criteria $h \in \Upsilon(k)$. where, $\Upsilon(k) = U(k) \cap L(k)$. In this instance, the function h reflects a "tight" asymptotic bound of the algorithm, it represents all asymptotic data about the resources that are best suited to solve the problem.

Let the pair (C^*, d^c) represents the complexity space,

where $C^* = \left\{ h \in G(T_Q) : \sum_{m=1}^{\infty} 2^{-m} \frac{1}{h(m)} < \infty \right\}$ and d^c is the complete quasi partial metric on C defined by

$d^c(h,k) = \sum_{n=1}^{\infty} 2^{-m} max \left\{ \frac{1}{h(m)} - \frac{1}{k(m)}, \frac{1}{k(m)} - \frac{1}{h(m))} \right\}$

The members of C^* are called complexity functions and $d^c(h,k)$ represents the complexity distance from h to gk. Then $d^c(f,g) = 0$ means 'h is as efficient as k'.

We shall resolve the issue by applying the Divide and Conquer technique cited in [16]. In this process, we shall bifurcate the problem into minor problems (based on various resources) and solve them individually using the identical algorithm to identify an appropriate solution. Upon attaining solutions to the insignificant problems, we shall integrate them to obtain an overall solution to the original problem that will signify an algorithm with almost all the appropriate resources.

Proposition 4.1 (V.Gupta et.al.,2020): Let D be an onto self mapping defined on C^* with coordinate functions $D_i : C^* \to C_i^*, i = 1, ...m$ such that

$D(h)(m) = (D_1(h)(m), ..., D_m(h)(m))$ for each $h \in C^*$ and $m \in N$.

satisfying the expansion inequality

$d_i^c(D_i(h)(m), D_i(k)(m)) \geq \lambda i\, \Psi(d_1^c(h_1, k_1), ...d_m^c(h_n, k_n))$

for all $h_i, k_i \in C_i^*, i = 1, ...m$ and $\lambda_1, ..., \lambda_n > 1$. Then $D \in \Upsilon(k)$.

We create the elements $C_i^*, i = 1, ...m; m \in N$ of complexity category C^* by utilizing diverse resources like time, space, data, etc., and $C^* = \prod_{i=1}^m C_i^*$. It's evident that (C_i^*, d_i^c) is a set of complete quasi-partial metric spaces. A function $\Psi : R_+^n \to R_+$ is used to aggregate these elements, and it's ensured that $\Psi(1, 1, ..., 1) \geq 1$. D is an onto self-mapping defined on C with coordinate functions $D_i : C^* \to C_i^*, i = 1, ...m$ in such a way that

$D(h)(m) = (D_1(h)(m), ..., D_m(h)(m))$ for each $h \in C^*$ and $m \in N$.

satisfying the expansion inequality

$d_i^c(D_i(h)(m), D_i(k)(m)) \geq \lambda i\, \Psi(d_1^c(h_1, k_1), ...d_m^c(h_n, k_n))$

for all $h_i, k_i \in C_i^*, i = 1, ...m$ and $\lambda_1, ..., \lambda_n > 1$.

Thus, all the conditions are satisfied and therefore, D has a fixed point h^* i.e. $D \in L(k) \bigcap U(k)$. It follows that $D \in \Upsilon(k)$.

5. Conclusion:

In the progress of functional analysis, all types of metrics have a significant role. Many research has been done in various fields like metric, partial metric, and quasi-partial metric spaces. The conception of quasi-partial metric space is generalized in these forms to some extent. Here we introduce the basic concepts of Ciric type (I) orbitally continuous self-maps in the above-defined space. we know that there may be some possibility of successful research in this space.

Motivated by the work of Erdal Karapinar [11] in partial metric space, we put forward Ciric (I) type contractive mapping in quasi-partial metric space. And proved the fixed point is periodic point as well as orbitally continuous with respect of space. Our outcomes can be extended to the case of coupled fixed points in qpms. The results of this paper are theoretical and analytical. Attempt to design innovative fixed-point solutions in this research. And to extend these results by involving Ciric type (I) contraction mapping using the frame of quasi-partial metric space.

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