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Review Paper

Mathematical Solution of the One-Dimensional Advection-Dispersion Equation with Radioactive Decay Factor Using a Change of Variable and Integral Transform Technique

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Abstract: *Mathematical study of the one-dimensional advection-dispersion equation with radioactive decay factor is derived using a generalized integral transform method to investigate the movement of absorbing solutes in hydraulic homogenous porous formations. The solution is derived under conditions of steady-state flow and arbitrary initial and inlet boundary conditions. The results obtained by this solution agree well with the results obtained by numerically inverting Laplace transform-generated solutions previously published in the literature. For mathematical simplicity it is hypothesized that the sorption processes are based on linear equilibrium isotherms and that the local chemical equilibrium assumption is valid. The result from several simulations, compared with predictions based on the classical advection-dispersion equation with constant coefficients, indicate that at early times, retardation affects the transport behavior of absorbing solutes. The center of mass appears to move more slowly, and solute spreading is enhanced in the radioactive decay case. The mathematical solution presented in this paper provides more flexibility with regard to the inlet conditions.*

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I. Introduction

The impact of contaminant flow and the solutions of advection dispersion equations have been studied by many researchers in the past years and still in progress. Gelhar et. al. (1979), Sudheendra (2010, 2011). Aral et.al (1996) and others, have provided methodologies for improving the description and prediction of non-reacting solute transport in complex structured formations, compared with the prediction based on the classical advection-dispersion equation with constant coefficients. On the other hand, the transport of absorbing solutes in geochemically as well as hydraulically heterogeneous porous media has received little attention.

For the importance case of transport of sorbing solutes in geochemically homogeneous porous media, the effects of sorption are commonly accounted for by a dimensionless retardation factor, which may be defined as the ration of the average interstitial fluid velocity to the propagation velocity of the solute. Excluding the possibilities of mass transport limitations and solute transformation or decay, any observed fluctuations on the retardation factor are attributed solely to the variability of the distribution coefficient, which is an experimentally obtained measure of sorption or solute retention by the solid formation. Sorption processes can be complex and depend on many variables, including temperature, pressure, solution pH, and ionic strength, sorbent surface charge, sorbent sorptive capacity, and the presence of species that complete for sorption sites. Spatial or temporal fluctuations in any of these variables accordingly affect the distribution coefficient and, consequently, the movement of sorbing solutes in subsurface porous media. (Karckhoff (1984) and Sudheendra (2014)). Although such a correlation is not fully reliable for every solute-sorbent system (Curtis and Roberts, 1985), it can explain to some extent the variable decay factor observed in field experiments (Roberts et.al 1986).

Garabedian (1987) &Sudheendra (2012) employed spectral methods to analyze reactive solute macro-dispersion under the assumption that the log-hydraulic conductivity is linearly related to both the porosity and the distribution coefficient. His result indicate that solute spreading is enhanced when there is negative correlation between the log-hydraulic conductivity and the distribution coefficient. The present work is focused on the transport of pollutants but otherwise non-reacting solutes under local equilibrium conditions in a one-dimensional unsaturated porous medium. Mathematical solutions are employed to solve the one-dimensional advection-dispersion equation with uniform, steady fluid flow conditions and radioactive decay factor, for a semi-infinite medium and flux-type inlet boundary condition.

The main objective of the study is to provide mathematical model for better understanding of transport of pollutant through unsaturated porous media. A mathematical model is an important tool and can play a crucial role in understanding the mechanism of groundwater pollution problems. It is a simplified description of physical reality expressed in mathematical terms. Mathematical models that attempt to simulate atmospheric processes involved in groundwater pollution are based, in general, on the equation of mass conservation for individual pollutant species. Such models relate in one equation the effects of all the physical aspects and dynamic processes that influence the mass balance on groundwater which include transport, diffusion, removal of pollutants and loss or transformation through chemical reactions.

II. Mathematical Model

The Advection-Dispersion equation along with initial condition and boundary conditions can be written as

$$\frac{\partial C}{\partial t} + w_1 \frac{\partial C}{\partial z} = D_1 \frac{\partial^2 C}{\partial z^2} - \lambda_1 C \qquad (1)$$

Initially, saturated flow of fluid of concentration, *C = 0*, takes place in the porous media.
Thus, the appropriate boundary conditions for the given model

$$\left.\begin{array}{ll} C(z,\ 0) = 0 & z \geq 0 \\ C(0,\ t) = C_0 e^{-\gamma t} & t \geq 0 \\ C(\infty,\ t) = 0 & t \geq 0 \end{array}\right\}. \qquad (2)$$

To reduce equation (2) to a more familiar form, we take

$$C(z,\ t) = \Gamma(z,\ t)\ Exp\ \left[\frac{w_1 z}{2D_1} - \frac{w_1^2 t}{4D_1} - \lambda_1 t\right] \qquad (3)$$

Substituting equation (3) into equation (1) gives

$$\frac{\partial \Gamma}{\partial t} = D_1 \frac{\partial^2 \Gamma}{\partial z^2} \qquad (4)$$

The initial and boundary conditions (2) transform to

$$\left.\begin{array}{ll} \Gamma(0,t) = C_0\ Exp\ \left[\dfrac{w_1^2 t}{4D_1} + (\lambda_1 - \gamma)t\right] & t \geq 0 \\ \Gamma(z,0) = 0 & z \geq 0 \\ \Gamma(\infty,\ t) = 0 & t \geq 0 \end{array}\right\}$$

Equation (4) can be verified for a time dependent influx of the fluid at $z = 0$. The solution of equation (4) may be obtained readily by use of Duhamel's theorem (Carslaw and Jaeger, 1947).

If $C = F(x, y, z, t)$ is the solution of the diffusion equation for semi-infinite media in which the initial concentration is zero and its surface is maintained at concentration unity, then the solution of the problem in which the surface is maintained at temperature $\varphi(t)$is

$$C = \int_0^t \varphi(\tau)\ \frac{\partial}{\partial t}\ F(x,\ y,\ z,\ t - \tau)\ d\tau$$

The boundary conditions are

$$\left.\begin{array}{ll} \Gamma(0,t) = 0 & t \geq 0 \\ \Gamma(z,0) = 1 & z \geq 0 \\ \Gamma(\infty,t) = 0 & t \geq 0 \end{array}\right\}$$

The Laplace transform of equation (4) is

$$L\ \left[\frac{\partial \Gamma}{\partial t}\right] = D_1 \frac{\partial^2 \Gamma}{\partial z^2}$$

Hence, this reduces to an ordinary differential equation

$$\frac{\partial^2 \bar{\Gamma}}{\partial z^2} = \frac{p}{D_1} \bar{\Gamma} \qquad (5)$$

The solution of the equation is $\bar{\Gamma} = Ae^{-qz} + Be^{qz}$where, $q = \pm\sqrt{\frac{p}{D_1}}$.

The boundary condition as $z \to \infty$ requires that $B = 0$ and boundary condition at $z = 0$ requires that $A = \frac{1}{p}$ thus the particular solution of the Laplace transformed equation is

$$\bar{\Gamma} = \frac{1}{p} e^{-qz}$$

The inversion of the above function is given in any table of Laplace transforms. The result is

$$\Gamma = 1 - erf(\frac{z}{2\sqrt{D_1 t}}) = \frac{2}{\sqrt{\pi}} \int_{\frac{z}{2\sqrt{Dt}}}^{\infty} e^{-\eta^2} d\eta$$

Using Duhamel's theorem, the solution of the problem with initial concentration zero and the time dependent surface condition at $z = 0$ is

$$\Gamma = \int_0^t \varphi(\tau) \; \frac{\partial}{\partial t} \left[\frac{2}{\sqrt{\pi}} \int_{\frac{z}{2\sqrt{D(t-\tau)}}}^{\infty} e^{-\eta^2} d\eta \right] d\tau$$

Since $e^{-\eta^2}$ is a continuous function, it is possible to differentiate under the integral, which gives

$$\frac{2}{\sqrt{\pi}} \frac{\partial}{\partial t} \int_{\frac{z}{2\sqrt{D_1(t-\tau)}}}^{\infty} e^{-\eta^2} d\eta = \frac{z}{2\sqrt{\pi D_1} (t-\tau)^{3/2}} \; Exp\left[\frac{-z^2}{4D_1(t-\tau)}\right]$$

The solution to the problem is

$$\Gamma = \frac{z}{2\sqrt{\pi D_1}} \int_0^t \varphi(\tau) \; Exp\left[\frac{-z^2}{4D_1(t-\tau)}\right] \frac{d\tau}{(t-\tau)^{3/2}} \qquad (6)$$

Putting $\mu = \frac{z}{2\sqrt{D_1(t-\tau)}}$ then the equation (6) can be written as

$$\Gamma = \frac{2}{\sqrt{\pi}} \int_{\frac{z}{2\sqrt{D_1 t}}}^{\infty} \varphi\left(t - \frac{z^2}{4D_1\mu^2}\right) \; e^{-\mu^2} d\mu$$

Since $\varphi(t) = C_0 \; Exp\left(\frac{w_1^2 t}{4D_1} + (\lambda_1 - \gamma) \; t\right)$ the particular solution of the problem be written as

$$\Gamma(z,t) = \frac{2C_0}{\sqrt{\pi}} \; Exp\left(\frac{w_1^2 t}{4D_1} + (\lambda_1 - \gamma) \; t\right) \left\{ \int_0^{\infty} Exp\left(-\mu^2 - \frac{\varepsilon^2}{\mu^2}\right) \; d\mu - \int_0^{\alpha} Exp\left(-\mu^2 - \frac{\varepsilon^2}{\mu^2}\right) \; d\mu \right\} \qquad (7)$$

where, $\alpha = \frac{z}{2\sqrt{D_1 t}}$ and $\varepsilon = \sqrt{\left(\frac{w_1^2}{4D_1} + \lambda_1 - \gamma\right)} \; \left(\frac{z}{2\sqrt{D_1}}\right)$.

III. Evaluation of the integral solution

The integration of the first term of equation (7) gives

$$\int_0^{\infty} Exp\left(-\mu^2 - \frac{\varepsilon^2}{\mu^2}\right) \; d\mu = \frac{\sqrt{\pi}}{2} e^{-2\varepsilon}.$$

Noting that

$$-\mu^2 - \frac{\varepsilon^2}{\mu^2} = -\left(\mu + \frac{\varepsilon}{\mu}\right)^2 + 2\varepsilon = -\left(\mu - \frac{\varepsilon}{\mu}\right)^2 - 2\varepsilon.$$

The second integral of equation (9) may be written as

$$I = \int_0^{\alpha} Exp \; \left(-\mu^2 - \frac{\varepsilon^2}{\mu^2}\right) \; d\mu = \frac{1}{2}\left\{ e^{2\varepsilon} \int_0^{\alpha} Exp \; \left[-\left(\mu + \frac{\varepsilon}{\mu}\right)^2\right] d\mu + e^{-2\varepsilon} \int_0^{\alpha} Exp \; \left[-\left(\mu - \frac{\varepsilon}{\mu}\right)^2\right] d\mu \right\} \qquad (8)$$

Let $a = \varepsilon/\mu$ and the integral may be expressed as

$$I_1 = e^{2\varepsilon} \int_0^{\alpha} Exp \; \left[-\left(\mu + \frac{\varepsilon}{\mu}\right)^2\right] d\mu$$

$$= -e^{2\varepsilon} \int_{\varepsilon/\alpha}^{\infty} \left(1 - \frac{\varepsilon}{a^2}\right) Exp \; \left[-\left(\frac{\varepsilon}{a} + a\right)^2\right] \; da + e^{2\varepsilon} \int_{\varepsilon/\alpha}^{\infty} Exp \; \left[-\left(\frac{\varepsilon}{a} + a\right)^2\right] \; da \quad (9)$$

Further, let, $\beta = \left(\frac{\varepsilon}{a} + a\right)$

in the $\beta = \frac{\varepsilon}{a} + a$ first term of the above equation, then

$$I_1 = -e^{2\varepsilon} \int_{\alpha + \frac{\varepsilon}{\alpha}}^{\infty} e^{-\beta^2} d\beta + e^{2\varepsilon} \int_{\frac{\varepsilon}{\alpha}}^{\infty} Exp \; \left[-\left(\frac{\varepsilon}{a} + a\right)^2\right] \; da. \qquad (10)$$

Similar evaluation of the second integral of equation (11) gives

$$I_2 = e^{-2\varepsilon} \int_{\frac{\varepsilon}{\alpha}}^{\infty} Exp \; \left[-\left(\frac{\varepsilon}{a} - a\right)^2\right] \; da - e^{-2\varepsilon} \int_{\varepsilon/\alpha}^{\infty} Exp \; \left[-\left(\frac{\varepsilon}{a} - a\right)^2\right] \; da$$

Again substituting $-\beta = \frac{\varepsilon}{a} - a$ into the first term, the result is

$I_2 = e^{-2\varepsilon} \int_{\frac{\varepsilon}{\alpha}-\alpha}^{\infty} e^{-\beta^2} d\beta - e^{-2\varepsilon} \int_{\varepsilon/\alpha}^{\infty} Exp \left[-\left(\frac{\varepsilon}{a} - a\right)^2 \right] da.$

Noting that

$$\int_{\varepsilon/\alpha}^{\infty} Exp \left[-\left(a + \frac{\varepsilon}{a}\right)^2 + 2\varepsilon \right] da = \int_{\varepsilon/\alpha}^{\infty} Exp \left[-\left(\frac{\varepsilon}{a} - a\right)^2 - 2\varepsilon \right] da$$

Substitution into equation (8) gives

$$I = \frac{1}{2}\left(e^{-2\varepsilon} \int_{\frac{\varepsilon}{\alpha}-\alpha}^{\infty} e^{-\beta^2} d\beta - e^{2\varepsilon} \int_{\alpha+\frac{\varepsilon}{\alpha}}^{\infty} e^{-\beta^2} d\beta \right). \quad (11)$$

Thus, equation (7) may be expressed as

$$\Gamma(z,\ t) = \frac{2C_0}{\sqrt{\pi}} Exp \left(\frac{w_1^2 t}{4D_1} + (\lambda_1 - \gamma)\ t \right) . \left\{ \frac{\sqrt{\pi}}{2} e^{-2\varepsilon} - \frac{1}{2}\left[e^{-2\varepsilon} \int_{\frac{\varepsilon}{\alpha}-\alpha}^{\infty} e^{-\beta^2} d\beta - e^{2\varepsilon} \int_{\alpha+\frac{\varepsilon}{\alpha}}^{\infty} e^{-\beta^2} d\beta \right] \right\} \quad (12)$$

However, by definition,

$$e^{2\varepsilon} \int_{\alpha+\frac{\varepsilon}{\alpha}}^{\infty} e^{-\beta^2} d\beta = \frac{\sqrt{\pi}}{2} e^{2\varepsilon}\ erfc\ \left(\alpha + \frac{\varepsilon}{\alpha}\right)$$

also, $e^{-2\varepsilon} \int_{\frac{\varepsilon}{\alpha}-\alpha}^{\infty} e^{-\beta^2} d\beta = \frac{\sqrt{\pi}}{2} e^{-2\varepsilon} \left(1 + erf\ \left(\alpha - \frac{\varepsilon}{\alpha}\right)\right).$

Writing equation (12) in terms of error functions, we get

$$\Gamma(z,\ t) = \frac{C_0}{2} Exp\ \left(\frac{w_1^2 t}{4D_1} + (\lambda_1 - \gamma)\ t\right) . \left[e^{2\varepsilon} erfc\ \left(\alpha + \frac{\varepsilon}{\alpha}\right) + e^{-2\varepsilon} erfc\ \left(\alpha - \frac{\varepsilon}{\alpha}\right) \right] \quad (13)$$

Thus, Substitution into equation (3) the solution is

$$\frac{C}{C_0} = \frac{1}{2} Exp\ \left[\frac{w_1 z}{2D_1} - \gamma t\right] . \left[e^{-2\varepsilon} erfc\ \left(\alpha - \frac{\varepsilon}{\alpha}\right) + e^{2\varepsilon} erfc\ \left(\alpha + \frac{\varepsilon}{\alpha}\right) \right]$$

Re-substituting for ε and α gives

$$\frac{C}{C_0} = \frac{1}{2} Exp\left[\frac{w_1 z}{2D_1} - \gamma t\right] \left[Exp\left[\frac{\sqrt{w_1^2 + 4D_1(\lambda_1 - \gamma)}}{2D_1} z \right] erfc\left[\frac{z + \sqrt{w_1^2 + 4D_1(\lambda_1 - \gamma)}}{2\sqrt{D_1 t}} t \right] + Exp\left[-\frac{\sqrt{w_1^2 + 4D_1(\lambda_1 - \gamma)}}{2D_1} z \right] . erfc\left[\frac{z - \sqrt{w_1^2 + 4D_1(\lambda_1 - \gamma)}}{2\sqrt{D_1 t}} t \right] \right] \quad (14)$$

Here the boundaries are non-symmetrical and the solution of this particular problem is obtained by the first term of equation (14). Due to the asymmetric boundary imposed in the general problem which leads to the second term. However, it should be noted also that if a point a great distance away from the source is considered, then it is possible to approximate the boundary condition by$C(-\infty,\ t) = C_0$, which leads to a symmetrical solution.

IV. Results &Conclusions:

The transform method coupled with the generalized integral transform technique is used to obtain the analytical solutions. Solutions are obtained for both first- and third-type inlet boundary conditions. The developed analytical solutions for finite domain are compared with solutions for the semi-infinite domain to clarify how the exit boundary influences the one-dimensional transport in a porous medium system.The properties of the soil in the region considered must be homogeneous in the sub region. The analytical method is somewhat more flexible than the standard form of other methods for one-dimensional transport model. Figures 1 to 4 represents the concentration profiles verses time in the adsorbing media for depth z = 10m and Retardation factor R=1. It is seen that for a fixed velocity w, dispersion coefficient D and distribution coefficient K_d, C/C_0 decreases with depth as porosity n decreases due to the distributive coefficient K_d and if time increases the concentration decreases for different time and decay chain.

Accordingly, the analytical solutions derived for the finite domain will thus be particularly useful for analyzing the one-dimensional transport in unsaturated porous medium with a large dispersion coefficient whereas the analytical solution for semi-infinite domain is recommended to be applied for a medium system with a small dispersion coefficient. Moreover, the developed solution is especially useful for validating numerical model simulated solution because realistic problems generally have a finite domain.

From this paper, we conclude that the mathematical solutions have been developed for predicting the possible concentration of a given dissolved substance in steady unidirectional seepage flows through semi-infinite, homogeneous, and isotropic porous media subject to source concentration that vary with the radioactive decay using a change of variable and integral transform technique. The expressions take into account the contaminants as well as mass transfer from the liquid to the solid phase due to adsorption. For simultaneous dispersion and adsorption of a solute, the dispersion system is considered to be adsorbing at a rate proportional to its concentration.

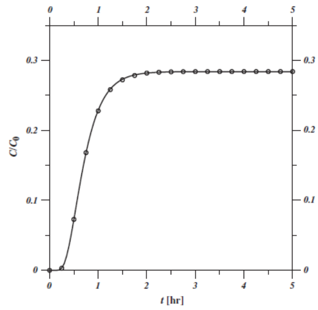

Fig. 2: Break-through-curve for C/C_0 v/s time for z=10m, R=1.0, λ=0.5 &γ = 0

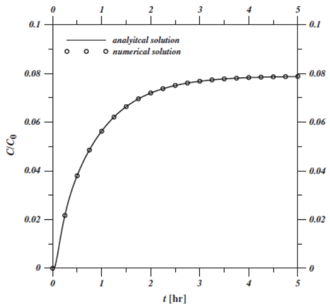

Fig. 3: Break-through-curve for C/C_0 v/s time for z=10m, R=1.0, λ=0.5 &γ = 0.25

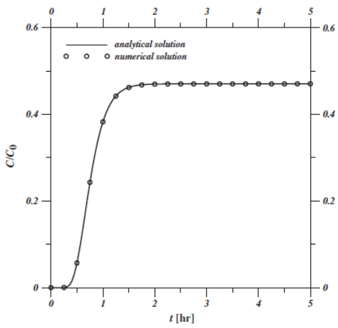

Fig. 3: Break-through-curve for C/C_0 v/s time for z=10m, R=1.0, λ=0.5 &γ = 0.5

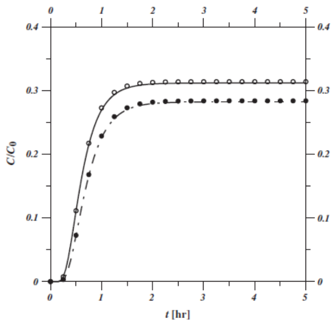

Fig. 4: Break-through-curve for C/C_0 v/s time for z=10m, R=1.0, λ=0.5, γ = 0.75 & 1.0

REFERENCES:

[1]. A. Verruijt (1990): Modelling Groundwater Flow and Pollution. D Reidel Publishing Co., Tokyo.
[2]. Al-Niami A N S and Rushton K R 1977 Analysis of flow against dispersion in porous media; J. Hydrol. 33 87–97.
[3]. Aral M M and Liao B 1996 Analytical solutions for two-dimensional transport equation with time-dependent dispersion coefficients; J. Hydrol. Engg. 1(1) 20–32.
[4]. A. Kumar, D. K. Jaiswal and N. Kumar, "Analytical Solutions to One-Dimensional Advection-Diffusion Equation with Variable Coefficients in Finite Domain," Journal of Earth System Science, Vol. 118, No. 5, 2009, pp. 539-549
[5]. Banks R B and Ali J 1964 Dispersion and adsorption in porous media flow; J. Hydraul. Div. 90 13-31.
[6]. Barry, D.A., Parlange, J.Y., Sander, G.C., and M. Sivaplan (1993): A class of exact solutions for Richards equation. J. of Hydrology. 142: 29-46.
[7]. Basak, P., and V.V.N. Murthy (1981): Groundwater quality improvement through nonlinear diffusion. J. of Hydrology. 53: 151-159.
[8]. Bear, J., and A. Verruijt (1990): Modelling Groundwater Flow and Pollution. D Reidel Publishing Co., Tokyo.
[9]. Branes, C.J (1989): Solute and water movement in unsaturated soils. Water Resour. Res., 25(1): 38-42.
[10]. Bruce, J.C.; and Street, R.L. (1966). Studies of free surface flow and twodimensional dispersion in porous media. Report No. 63, Civil Eng. Dept. Stanford Univivrsity.
[11]. Celia, M.A., E.T. Bouloutas and R.L. Zarba (1990): A general mass conservative numerical solution of the unsaturated flow equation, Water Resour. Res. 26 (7): 1483-1496.
[12]. Crank J 1975 the Mathematics of Diffusion (London: Oxford Univ. Press).
[13]. Chen, J.S., Liu, C.W., Liao, C.M., 2003. Two-dimensional Laplace-transformed power series solution for solute transport in a radially convergent flow field. Advances in Water Resources 26, 1113–1124.
[14]. De Smedt, F. and Wierenga, P.J., 1979b. Mass transfer in porous media with immobile water. J. Hydrol., 41: 59967.

[15]. Domenico, P.A. and G.A. Robbins 1984: A Dispersion Scale Effect in Model calibrations & Field Tracer Expts. J. of Hydrology 70: 123-132.
[16]. Eungyer Park & Hongbin Zhan (2001) : Analytical solutions of contaminant transport from finite one-, two- and three-dimensional sources in a finite-thickness aquifer. J. of Contaminant Hydrology. 53 : 41-61
[17]. Gelhar L W, Welty C and Rehfeldt K R 1992 A critical review of data on field-scale dispersion in aquifers; Water Resour. Res. 28(7) 1955–1974.
[18]. Guerrero, J. S. Pérez., Pontedeiro, E.M., Van Genuchten, M.Th., and Skaggs, T.H., "Analytical solutions of the one, two and three dimensional advection-dispersion solute transport equation subject to time-dependent boundary conditions", Chemical Engineering Journal, Vol. 221, pp. 487-491, 2013.
[19]. Harleman, D.R.F., and R.R., Rumer, Jr (1963): Longitudinal and lateral dispersion in an isotropic porous medium. J. Fluid Mech., 16 : 1-12.
[20]. Hanks, R.J., Klute A., and E.Bresler (1969): A numerical methods for estimating infiltration, Redistribution drainage, and evaporation of water from soil. Water Resour. Res., 5(5) : 1064-1069.
[21]. Jaiswal, D.K., Kumar, A., Kumar, N., Yadav, R.R., 2009. Analytical solutions for temporally and spatially dependent solute dispersion of pulse type input concentration in one-dimensional semi-infinite media. Journal of Hydro environmental Research 2, 254–263.
[22]. Kim Kue-Young, Kim T, Kim Y and Woo Nam-Chil 2007 A semi-analytical solution for ground water responses to stream–stage variations and tidal fluctuations in a coastal aquifers; Hydrol. Process. 21 665–674.
[23]. Kumar, A. and Yadav, R. R., "One-dimensional solute transport for uniform and varying pulse type input point source through heterogeneous medium", Environmental Technology, Vol.36, No.4, pp.487-495, 2015.
[24]. Leij F J, Toride N and van Genutchen M Th 1993 Analytical solutions for non equilibrium solute transport in three-dimensional porous media; J. Hydrol. 151 193-228.
[25]. Liping Pang & Bruce Hunt (2001): Solutions and verification of a scale-dependent dispersion model. J. Contam. Hydrol., in press.
[26]. Oberhettinger and Baddi(1973) tables of Laplace transforms Springer- verlag, Newyork
[27]. Ogata, A. and R.B. Banks (1961): A solution of the differential equation of longitudinal dispersion in porous media, U.S. Geol. Surv. Prof. Pap. 411-A, A1-A9.
[28]. Pang L, Gottz M and Close M 2003 Application of the method of temporal moment to interpret solute transport with sorption and degradation; J. Contam. Hydrol. 60 123–134.
[29]. Paniconi, C., Aldama, A.A and Wood, E.F (1991): Numerical evaluation of iterative and noniterative methods for the solution of nonlinear Richards equation. Water Resour. Res., 27(6): 1147-1163.
[30]. Pullan A.J. (1990): The quasilinear approximation for unsaturated porous media flow. Water Resour. Res. 26(6): 1219-1234.
[31]. Sanskrityayn, A., Bharati , V. K. and Kumar, N., "Analytical solution of advection dispersion equation with spatiotemporal dependence of dispersion coefficient and velocity using green's function method", Journal of Groundwater Research, Vol. 5, No. 1, pp.24-31, 2016.
[32]. Severino, G., Tartakovsky, D., Srinivasan, G., & Viswanathan, H. (2012). Lagrangian models of reactive transport in heterogeneous porous media with uncertain properties. In , Vol. 468. Proceedings of the royal society of london A: Mathematical, physical and engineering sciences (pp. 1154e1174).
[33]. Sirin, H. (2006). Ground water contaminant transport by non-divergence free, unsteady and non-stationary velocity fields. Journal of Hydrology, 330(3-4), 564-572.
[34]. Smedt, F.D. (2007). Analytical solution and analysis of solute transport in rivers affected by diffusive transfer in the hyporheic zone. Journal of Hydrology, 339(1-2), 29-38.
[35]. Sudheendra S.R., 2010 A solution of the differential equation of longitudinal dispersion with variable coefficients in a finite domain, Int. J. of Applied Mathematics & Physics, Vol.2, No. 2, 193-204.
[36]. Sudheendra S.R., 2011. A solution of the differential equation of dependent dispersion along uniform and non-uniform flow with variable coefficients in a finite domain, Int. J. of Mathematical Analysis, Vol.3, No. 2, 89-105.
[37]. Sudheendra S.R. 2012. An analytical solution of one-dimensional advection-diffusion equation in a porous media in presence of radioactive decay, Global Journal of Pure and Applied Mathematics, Vol.8, No. 2, 113-124.
[38]. Sudheendra S.R., Raji J, & Niranjan CM, 2014. Mathematical Solutions of transport of pollutants through unsaturated porous media with adsorption in a finite domain, Int. J. of Combined Research & Development, Vol. 2, No. 2, 32-40.
[39]. Sudheendra S.R., Praveen Kumar M. & Ramesh T. Mathematical Analysis of transport of pollutants through unsaturated porous media with adsorption and radioactive decay, Int. J. of Combined Research & Development, Vol. 2, No. 4, 01-08.(2014)
[40]. van Genuchten M Th and Alves W J 1982 Analytical solutions of the one dimensional convective-dispersive solute transport equation; US Dept. Agriculture Tech. Bull. No. 1661 151p. van Kooten J J A 1996 A method to solve the advection dispersion equation with a kinetic adsorption isotherm; Adv. Water Res. 19 193–206.
[41]. Wang, F.C. and V. Lakshminarayana (1968) : Mathematical simulation of water movement through unsaturated non-homogeneous soils. Soil Science Soc. Am. Proc. 32: 329-334.
[42]. Warrick, A.J., Biggar, J.W., and D.R. Nielsen (1971): Simultaneous solute and water transfer for an unsaturated soil. Water Resour. Res. 7(2); 1216-1225.
[43]. Wortmanna, S.; Vilhena, M.T.; Moreirab, D.M.; and Buske, D. (2005). A new analytical approach to simulate the pollutant dispersion in the PBL. Atmospheric Environment, 39(12), 2171–2178.
[44]. Yates S R 1990 An analytical solution for one-dimensional transport in heterogeneous porous media; Water Resour. Res. 26 2331–2338
[45]. Yates S R 1992 An analytical solution for one-dimensional transport in porous media with an exponential dispersion function; Water Resour. Res. 28 2149–2154.
[46]. Zyvolski, G., Bruch, J.C., and James M. Sloss (1976) : Solution of equation for two dimensional infiltration problems. Soil Science 122 (2) : 65-70.
[47]. Singh, S., Singh, J., Shukla, J.B.,"Modelling and analysis of the effects of density dependent contactrates on the spread of carrier dependent infectious diseases with environmental discharges" in Model. Earth Syst. Environ. 5, 21–32(2019).