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Review Paper

Tarig Transform Analysis of Undamped Mechanical Oscillating System Subjected To a Periodic Force

ONUOHA, N.O.
DEPARTMENT OF MATHEMATICS, IMO STATE UNIVERSITY, OWERRI

ABSTRACT
Theoretical analysis of undamped mechanical oscillating system subjected to periodic force is discussed in this paper using Tarig transform technique. We first applied Tarig transform to elementary first and second order ordinary differential equations to illustrate the approach. Thereafter, it was used to solve the undamped mechanical system equations; system subjected to periodic force and forced free system. Tarig solutions obtained are bounded and oscillatory solutions in the time variable, t. It was observed that periodic force (external force) reduces the rate at which a mechanical system oscillates. Hence, the force reduces the amplitude of oscillation of the mechanical oscillating system.
Keywords: Tarig transform, Oscillating systems, undamped mechanical system, periodic force, Amplitude of oscillation, bounded solution, Differential equations

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I. Introduction

Tarig and Salih [13] derived the Tarig transform of partial derivatives and applied it to initial value problems. Shaheed and Muhammad [9], proposed the new variant, Adomian decomposition Tarig transform method and applied it to linear and nonlinear differential equations. Chitra et al [2], discussed the Tarig transform of higher order homogeneous and non-homogeneous differential equations. Tarig [12] applied Elzaki transform to first and second order differential equations. Onuoha [5], applied Kamal transform technique to coupled systems of linear ordinary differential equations. Shrinath and Bhadane [10] defined new generalized fractional Elzaki-Tarig transform and its relations to other fractional integral transforms. Badriah [1] solved nonlinear differential equations using Tarig transform with differential transform. Neetu and Gupta [4] used Tarig transform and variational iteration method to solve fractional delay differential equations. Gnanavel et al [3], applied Tarig transform to linear volterra integral equations of first kind.

Many others investigated the mechanical oscillating system subjected to various loading systems (external force applied). Mechanical oscillating systems are physical systems that experience variation that repeat itself in a mechanical structure. The mechanical oscillating system in many cases is modeled using mass-spring equation. Studying mechanical oscillating systems, one investigates the behaviour of the system when subjected to different forces. Rahul and Rohit [7], used Gupta integral transform to analyze the response of undamped oscillator subjected to triangular pulse. Sujito et al [11] solved differential equations that modeled oscillating system. Volodymyr and Nataliya [14], studied the dynamical behaviour of a 2-DOF mechanical system subjected to an external harmonic force. Rahul et al [8], investigated the responses of mechanical and electrical oscillators by the application Gupta transform. Onuoha [6] studied the forced Van der Pol oscillator equation by regular parameter perturbation and asymptotic expansion techniques.

1. Definition

Tarig transform of a given function $f(t)$ is defined in [13], as

$$T\{f(t)\} = \frac{1}{u}\int_0^\infty e^{-\frac{t}{u^2}}\,dt = F(u) \qquad t>0, u\neq 0$$

(1)

2. Tarig Transform of some Elementary Functions

(i) Tarig transform of a constant function, p

$$T\{p\} = \frac{p}{u}\int_0^\infty e^{-\frac{t}{u^2}} dt = pu$$

(2)

(ii) Tarig transform of a function, $f(t) = t$

$$T\{t\} = \frac{1}{u}\int_0^\infty e^{-\frac{t}{u^2}} t\, dt = u^3$$

(3)

(iii) Tarig transform of a function , $f(t) = t^2$

$$T\left\{t^2\right\} = \frac{1}{u}\int_0^\infty e^{-\frac{t}{u^2}} t^2\, dt = 2u^5$$

(4)

(iv) Tarig transform of a function, $f(t) = t^n$

$$T\left\{t^n\right\} = \frac{1}{u}\int_0^\infty e^{-\frac{t}{u^2}} t^n\, dt = n!u^{2n+1}$$

(5)

(v) Tarig transform of an exponential function, $e^{\pm at}$

$$T\left\{e^{\pm at}\right\} = \frac{1}{u}\int_0^\infty e^{-\frac{t}{u^2}} e^{\pm at}\, dt = \frac{u}{1 \mp au^2}$$

(6)

(vi) Tarig transform of trigonometric functions, $\cos at$ and $\sin at$

$$T\{\cos at\} = \frac{1}{u}\int_0^\infty e^{-\frac{t}{u^2}} \cos at\, dt = \frac{u}{1 + a^2u^4}$$

(7)

$$T\{\sin at\} = \frac{1}{u}\int_0^\infty e^{-\frac{t}{u^2}} \sin at\, dt = \frac{au^3}{1 + a^2u^4}$$

(8)

(vii) Tarig transform of derivatives of a given function, $f(t) = x(t)$

Let $T\{x(t)\} = \frac{1}{u}\int_0^\infty e^{-\frac{t}{u^2}} x(t) dt = X(u)$

(9)

Then,

$$T\left\{\frac{dx}{dt}\right\} = \frac{1}{u}\int_0^\infty e^{-\frac{t}{u^2}} \frac{dx}{dt} dt = \frac{1}{u^2} X(u) - \frac{1}{u} x(0)$$

(10)

$$T\left\{\frac{d^2x}{dt^2}\right\}=\frac{1}{u}\int_0^\infty e^{-\frac{t}{u^2}}\frac{d^2x}{dt^2}dt=\frac{1}{u^4}X(u)-\frac{1}{u^3}x(0)-\frac{1}{u}x'(0)$$

(11)

3. Tarig Transform of some Elementary Ordinary Differential Equations

3.1. First order ordinary differential equations

Example 1

Consider the equation

$$\frac{dx}{dt}+x=0$$

(12a)

$$x(0)=1$$

(12b)

Taking the Tarig transform of equation (12a), we get

$$\frac{1}{u^2}X(u)-\frac{1}{u}x(0)+X(u)=0$$

(13)

Applying the initial condition, equation (12b) on equation (13), we get

$$X(u)=\frac{u}{1+u^2}$$

(14)

Taking the inverse Tarig transform of equation (14), we get

$$x(t)=e^{-t}$$

(15)

Example 2

Given the differential equation

$$\frac{dx}{dt}+2x=t$$

(16a)

$$x(0)=1$$

(16b)

Taking the Tarig transform of equation (16), we get

$$\frac{1}{u^2}X(u)-\frac{1}{u}x(0)+2X(u)=u^3$$

(17)

Applying the initial condition, equation (16b) on equation (17), we get

$$X(u)=\frac{u\left(u^4+1\right)}{1+2u^2}$$

(18)

We first simplify the right hand side of equation (18) and we get

$$X(u)=\frac{1}{2}u^3-\frac{1}{4}u+\frac{5u}{4\left(1+2u^2\right)}$$

(19)

Taking the inverse Tarig transform of equation (19), we get

$$x(t)=\frac{1}{2}t-\frac{1}{4}+\frac{5}{4}e^{-2t}$$

(20)

3.2. Second order ordinary differential equations

Example 3

Consider the equation

$$\frac{d^2x}{dt^2}-3\frac{dx}{dt}+2x=0$$

(21a)

$$x(0)=1$$

$$x'(0)=4$$

(21b)

Taking the Tarig transform of equation (21a), we get

$$\frac{1}{u^4}X(u)-\frac{1}{u^3}x(0)-\frac{1}{u}x'(0)-3\left\{\frac{1}{u^2}X(u)-\frac{1}{u}x(0)\right\}+2X(u)=0$$

(22)

Applying the initial conditions, equation (21b) on equation (22), we get

$$X(u)=\frac{u\left(1+u^2\right)}{\left(1-u^2\right)\left(1-2u^2\right)}$$

(23)

Expanding the right hand side of equation (23) into sum of its partial fractions, we get

$$X(u)=\frac{3u}{1-2u^2}-\frac{2u}{1-u^2}$$

(24)

Taking the inverse Tarig transform of equation (24), we get

$$x(t)=3e^{2t}-2e^{t}$$

(25)

Example 4

Given the second order nonhomogeneous ordinary differential equation

$$\frac{d^2x}{dt^2}-3\frac{dx}{dt}+2x=4e^{3t}$$

(26a)

$$x(0)=-3$$

$$x'(0)=5$$

(26b)

Taking the Tarig transform of equation (26a), we get

$$\frac{1}{u^4}X(u)-\frac{1}{u^3}x(0)-\frac{1}{u}x'(0)-3\left\{\frac{1}{u^2}X(u)-\frac{1}{u}x(0)\right\}+2X(u)=\frac{4u}{1-3u^2}$$

(27)

Applying the initial conditions, equation (26b) on equation (27), we get

$$X(u)=\frac{-38u^5+23u^3-3u}{\left(1-3u^2\right)\left(1-u^2\right)\left(1-2u^2\right)}$$

(28)

We write the right hand side of equation (28) as sum of partial fractions

$$X(u)=u\left\{\frac{2}{1-3u^2}-\frac{9}{1-u^2}+\frac{4}{1-2u^2}\right\}$$

(29)

Taking inverse Tarig transform of equation (29), we get

$$x(t)=2e^{3t}-9e^{t}+4e^{2t}$$

(30)

Example 5

$$\frac{d^2x}{dt^2}+4x=9t$$
(31a)
$$x(0)=0$$
$$x'(0)=7$$
(31b)
Taking Tarig transform of equation (31a), we get
$$\frac{1}{u^4}X(u)-\frac{1}{u^3}x(0)-\frac{1}{u}x'(0)+4X(u)=9u^3$$
(32)
Applying the initial conditions, equation (31b) on equation (32), we get
$$X(u)=\frac{9u^7+7u^3}{1+4u^4}$$
(33)
Taking the inverse Tarig transform of equation (33), we get
$$x(t)=\frac{9}{4}t+\frac{19}{8}\sin 2t$$
(34)
4. Undamped Mechanical Oscillating System Subjected to a Periodic Force
Consider the equation of the undamped mechanical system subjected to a periodic force
$$\frac{d^2x}{dt^2}+\alpha^2x=\cos\theta t$$
(35a)
$$x(0)=x'(0)=0$$
(35b)
where $\alpha>0,\ \theta>0$ and $\alpha\neq\theta$
Taking Tarig transform of equation (35a), we get
$$\frac{1}{u^4}X(u)-\frac{1}{u^3}x(0)-\frac{1}{u}x'(0)+\alpha^2X(u)=\frac{u}{1+\theta u^4}$$
(36)
Applying the initial conditions, equation (35b) on equation (36), we get
$$X(u)=\frac{u^5}{\left(1+\theta^2u^4\right)\left(1+\alpha^2u^4\right)}$$
(37)
We then write equation (37) in the form
$$X(u)=\frac{1}{\alpha^2-\theta^2}\left[\frac{u}{\left(1+\theta^2u^4\right)}-\frac{u}{\left(1+\alpha^2u^4\right)}\right]$$
(38)
Taking the inverse Tarig transform of equation (38), we get
$$x(t)=\frac{1}{\alpha^2-\theta^2}\left(\cos\theta t-\cos\alpha t\right)$$
(39)
Equation (39) is the bounded solution of equation (35a) where $x(t)$ is the amplitude of the oscillating system and t is the time variable.
5. Forced Free Undamped Mechanical Oscillating System
Consider the equation of the undamped mechanical system without external force

$$\frac{d^2x}{dt^2}+\alpha^2 x=0$$
(40a)

$$x(0)=0$$
$$x'(0)=1$$
(40b)

where $\alpha>0$

Taking Tarig transform of equation (40a), we get

$$\frac{1}{u^4}X(u)-\frac{1}{u^3}x(0)-\frac{1}{u}x'(0)+\alpha^2 X(u)=0$$
(41)

Applying the initial conditions, equation (40b) on equation (41), we get

$$X(u)=\frac{u^3}{\left(1+\alpha^2u^4\right)}$$
(42)

Taking the inverse Tarig transform of equation (42), we get

$$x(t)=\frac{1}{\alpha}\sin\alpha t$$
(43)

Equation (43) is the bounded solution of equation (40a) where $x(t)$ is the amplitude of the oscillating system and t is the time variable.

6. Analysis

The solutions obtained for the two different mechanical oscillating systems are periodic and bounded solutions in t. The effect of the external force (periodic force) on the amplitude of the mechanical oscillating system at different times shall be observed from the numerical values of $x(t)$ using Excel Microsoft package.

Table I: Computed values of the amplitude, $x(t)$ of the mechanical oscillating system subjected to periodic force at various values of time, t and fixed value of $\alpha\,(\alpha=0.5)$ and $\theta(\theta=0.01)$.

t	$x(t)$
10	2.846507
20	7.279464
30	6.862843
40	2.052737
50	-0.45466
60	2.685411
70	6.676808
80	5.456762
90	0.385306
100	-1.69933
110	1.726568
120	5.261187
130	3.321139
140	-1.85415
150	-3.40542
160	0.324881
170	3.423498
180	0.88384
190	-4.21554
200	-5.11591

Table II: Computed values of the amplitude, $x(t)$ of the forced free mechanical oscillating system at various values of time, t and fixed value of $\alpha\,(\alpha = 0.5)$.

t	$x(t)$
10	9.588511
20	16.82942
30	19.9499
40	18.18595
50	11.96944
60	2.8224
70	-7.01566
80	-15.136
90	-19.5506
100	-19.1785
110	-14.1108
120	-5.58831
130	4.3024
140	13.13973
150	18.76000
160	19.78716
170	15.96974
180	8.24237
190	-1.50302
200	-10.8804

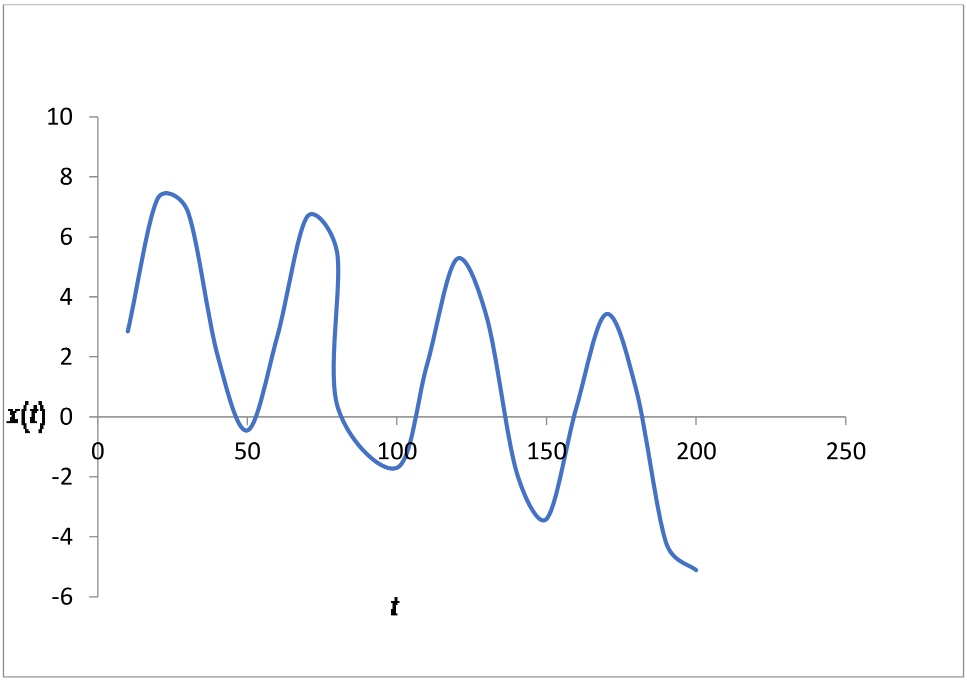

Fig I: Variation of the amplitude, $x(t)$ of the mechanical oscillating system subjected to periodic force with time, t and fixed value of $\alpha(\alpha = 0.05)$ and $\theta(\theta = 0.01)$

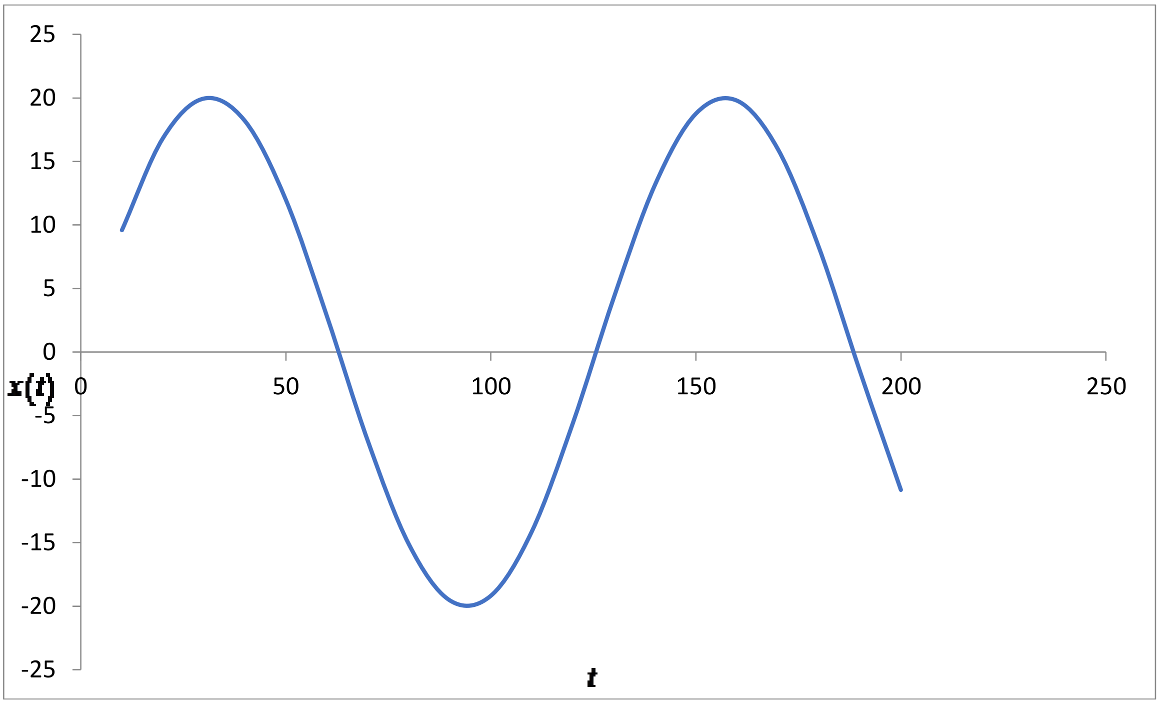

Fig II: Variation of the amplitude, $x(t)$ of the forced free mechanical oscillating system with time, t and fixed value of $\alpha(\alpha = 0.05)$

II. Conclusion

Tarig transform is a half line integral transform. It transforms the ordinary differential equations to algebraic equations. We were able to use it to solve elementary first and second order ordinary differential equations and the process showed that Tarig transform is an effective mathematical technique of solving linear differential equations. The new integral transform, Tarig transfom was also applied to undamped mechanical oscillating system subjected to periodic force and forced free mechanical oscillating system. For the two systems, the solutions obtained are oscillator and bounded solutions in the time variable, t. The results obtained showed that the periodic force applied to the oscillating mechanical system has significant effect on the system. The presence of the periodic force reduces the amplitude of the oscillation of the mechanical oscillating system as illustrated in Table I and Table II. Without the external force, the mechanical oscillating system oscillates very fast thereby increasing its amplitude of oscillation.

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