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Review Paper

Emad–Falih Transform Based Technique for Solving Fourth-Order Odes with Constant Coefficients

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Abstract
In this article, new integral transform-based technique has been evolved for obtaining solutions of fourth order ordinary differential equations with constant coefficients encountered in various applications of sciences and engineering. For this purpose Emad –Falih integral transform has been used. In order to validate accuracy and simplicity of the proposed scheme, few numerical experiments have been demonstrated.

Keywords: *Emad-Falih integral transform; Fourth- Order ODEs with constant coefficients; Numerical Examples.*
MSC Classifications: *35A08, 44A05*

I. Introduction

Owing to its significant and substantial use in different applications of sciences and engineering, third order ordinary differential equations have captured the consideration of scientists. It has been observed in the literature that many semi-analytical, analytical, and also computational techniques have been implemented to solve the third order mathematical models. It has been proved that analytical methods are essential for solving third order ordinary differential equations as it is difficult to solve them numerically. Homotopy Perturbation Method (HPM), Tanh method, Homotopy Analysis Method (HAM), Adomian Decomposition Method (ADM), Tau method, Variational Iteration Method (VIM), etc. are few examples of these techniques. In this research, we have discovered an efficacious classical method to solve third order ordinary differential equations based on the Emad-Falih transform. The standard form of fourth order ordinary differential equation is:

$$ay^{iv} + by''' + cy'' + dy' + ey = f(t)$$

in some continuous domain having initial conditions $y(0) = \alpha,\ y^{'}(0) = \beta,\ y^{''}(0) = \gamma,\ y^{'''}(0) = \delta$. Here, the parameters $a,\ b,\ c,\ d,\ e,\ \alpha,\ \beta,\ \gamma, \delta$ are real numbers. In [1], Emad-Falih integral transform has been discussed for finding soluions of differential equations. Analytical solutions of telegraph equations have been procured with the assistance of Emad-Falih integral transform-based technique in [2]. Elzaki transform has been discussed in [3], for solving differential equations. In [4], Elzaki transform based method has been develoed to solve partial differential equations. Connection between Laplace and Elzaki transforms have been discussed in [5]. Elzaki transform and Sumudu transform have been utilised for solving differential equations in [6]. Properties of Elzaki transform along with its applications have been discussed in [7]. Homotopy perturbation method based on Elzaki transform has been used to solve nonlinear partial differential equations in [8]. Analytical and semi-analytical solutions of Burger's equations are demonstrated in [9], where Elzaki transform and homotopy perturbation method are applied for this purpose. Homotopy perturbation method has been utilised to solve coupled Burger's equations having time fractional derivatives in [10].

This research has been organised as: An Introduction to Emad-Falih integral transform alongwith its properties has been presented in Section 2. In Section 3, few numerical observations have been demonstrated to validate the accuracy and simplicity of the propoundedtechnique for solving third order ordinary differential equations. Conclusion is discussed in Section 4.

II. Emad Falih Integral Transform and its properties

We define Emad Falih integral transform for the exponential order function given below.

$$B = \{f(t): \exists K, m_1, m_2 > 0, |f(t)| < Ke^{m^2 j|t|}\ If\ t \in (-1)^j X[0,\infty)\}$$

where $f(t)$ is a function, K is a finite constant and m_1, m_2 may or may not be finite. The Emad Falih integral transform is defined as:

$$EF\{f(t)\} = \frac{1}{\varphi}\int_0^\infty f(t).\,e^{-\varphi^2 t}dt,$$

where $t \geq 0, m_1 \leq \varphi \leq m_2$ and φ is a variable which is a factor to the variable t used in the function $f(t)$.
Emad Falih integral transform for some of the functions are as:

a. $EF\{k\} = \frac{k}{\varphi^3}$, where k is a constant.
b. $EF\{t^n\} = \frac{n!}{\varphi^{2n+3}}$, where n is an integeral positive number.
c. $EF\{e^{at}\} = \frac{1}{\varphi(\varphi^2-a)}$,
d. $EF\{\sin at\} = \frac{a}{\varphi(\varphi^4+a^2)}$,
e. $EF\{\cos at\} = \frac{\varphi}{(\varphi^4+a^2)}$,
f. $EF\{\sinh at\} = \frac{a}{\varphi(\varphi^4-a^2)}$,
g. $EF\{\cosh at\} = \frac{\varphi}{(\varphi^4-a^2)}$.

Emad Falih integral transform of some partial derivative are given below:

a. $EF\left[\frac{\partial}{\partial t}f(x,t)\right] = -\frac{f(x,0)}{\varphi} + \varphi^2.EF\{f(x,t)\}$,
b. $EF\left[\frac{\partial^2}{\partial t^2}f(x,t)\right] = -\frac{f'(x,0)}{\varphi} + \varphi^2.EF\{f'(x,t)\}$,
c. $EF\left[\frac{\partial^n}{\partial t^n}f(x,t)\right] = -\frac{f^{(n-1)}(x,0)}{\varphi} + \varphi^2.EF\{f^{(n-1)}(x,t)\}$.

III. Numerical Experiments

Let us discuss few numerical experiments to portray the simplicity and precision of the scheme being proposed for solving fourth order ordinary differential equations having constant coefficients.

Example 1: Consider fourth- order ODE in the form:

$$y'^{v}(t) = 2 \qquad (1)$$

having initial conditions as

$$y(0) = y'(0) = y''(0) = y'''(0) = 0$$

and exact solution as:

$$y(t) = \frac{t^4}{12}$$

Applying Emad-Falih transform both sides, we obtain

$$EF\{y^{iv}\} = EF\{2\}$$

This implies

$$-\frac{y'''(0)}{\varphi} - \varphi y''(0) - \varphi^3 y'(0) - \varphi^5 y(0) + \varphi^8 EF(y) = \frac{2}{\varphi^3}$$

Using initial conditions, we obtain

$$\varphi^8 EF(y) = \frac{2}{\varphi^3}$$

Dividing by φ^6, we obtain

$$EF(y) = \frac{2}{\varphi^{11}}$$

Taking inverse Emad-Falih transform, we obtain

$$y(t) = 2.\frac{t^4}{24} = \frac{t^4}{12}$$

which is an exact solution.

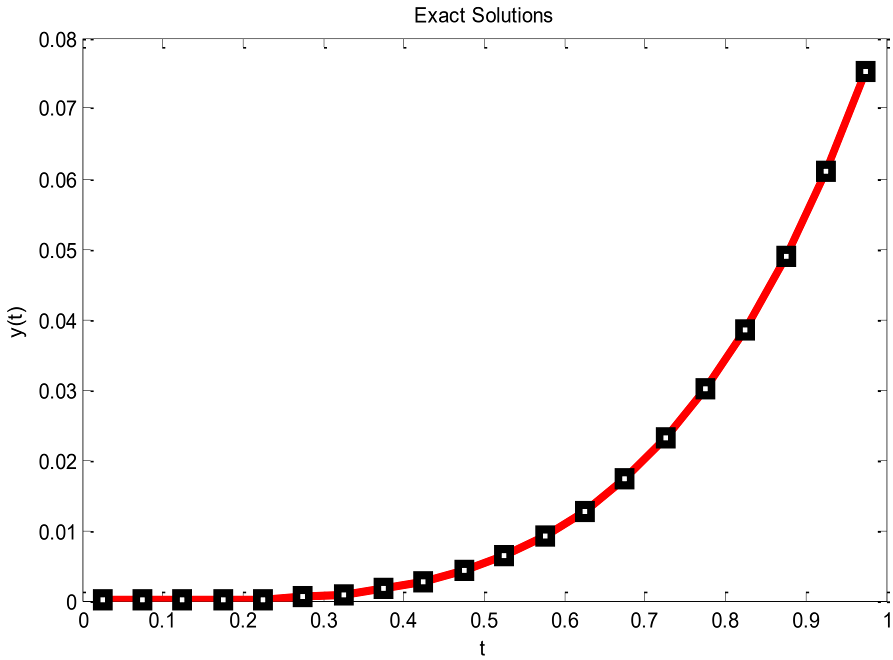

Figure –I: Exact Solutions of Example 1

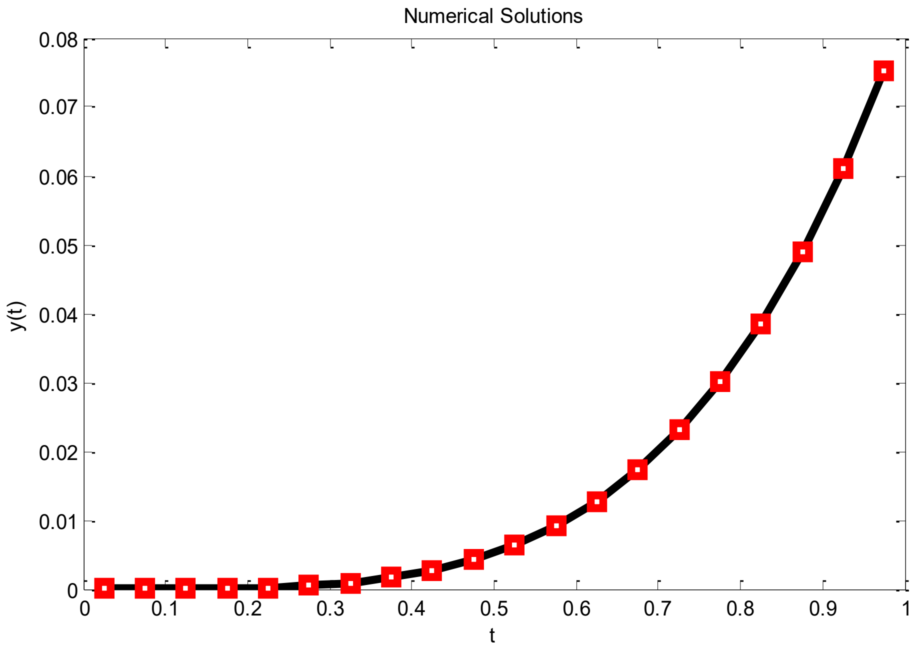

Figure –II: Emad-Falih Transform Solutions of Example 1

Figure I and Figure II depict the exact solution and the solution based on Emad –Falih transform method.
Example 2: Consider the fourth order ODE of the form:

$$y^{iv}(t) = e^{-t} \tag{2}$$

having initial conditions

$$y(0) = y'(0) = y''(0) = y'''(0) = 0$$

and exact solution as:

$$y(t) = -1 + t - \frac{t^2}{2} + \frac{t^3}{6} + e^{-t}$$

Applying Emad-Falih transform both sides of (2), we obtain

$$EF\{y^{iv}\} = EF\{e^{-t}\}$$

This implies

$$-\frac{y'''(0)}{\varphi}-\varphi y''(0)-\varphi^3 y'(0)-\varphi^5 y(0)+\varphi^8 EF(y)=\frac{1}{\varphi(\varphi^2+1)}$$

Using initial conditions, we obtain

$$\varphi^8 EF(y)=\frac{1}{\varphi(\varphi^2+1)}$$

Dividing by φ^8, we obtain

$$EF(y)=\frac{1}{\varphi^9(\varphi^2+1)}$$

After simplifications, we can write it as:

$$EF(y)=\frac{1}{\varphi}-\frac{1}{\varphi^3}+\frac{1}{\varphi^5}-\frac{1}{\varphi^7}+\frac{1}{\varphi^9}-\frac{\varphi}{\varphi^2+1}$$

This implies

$$EF(y)=-\frac{1}{\varphi^3}+\frac{1}{\varphi^5}-\frac{1}{\varphi^7}+\frac{1}{\varphi^9}+\frac{1}{\varphi(\varphi^2+1)}$$

Taking inverse Emad-Falih transform, we obtain

$$y(t)=-1+t-\frac{t^2}{2}+\frac{t^3}{6}+e^{-t}$$

which is an exact solution.

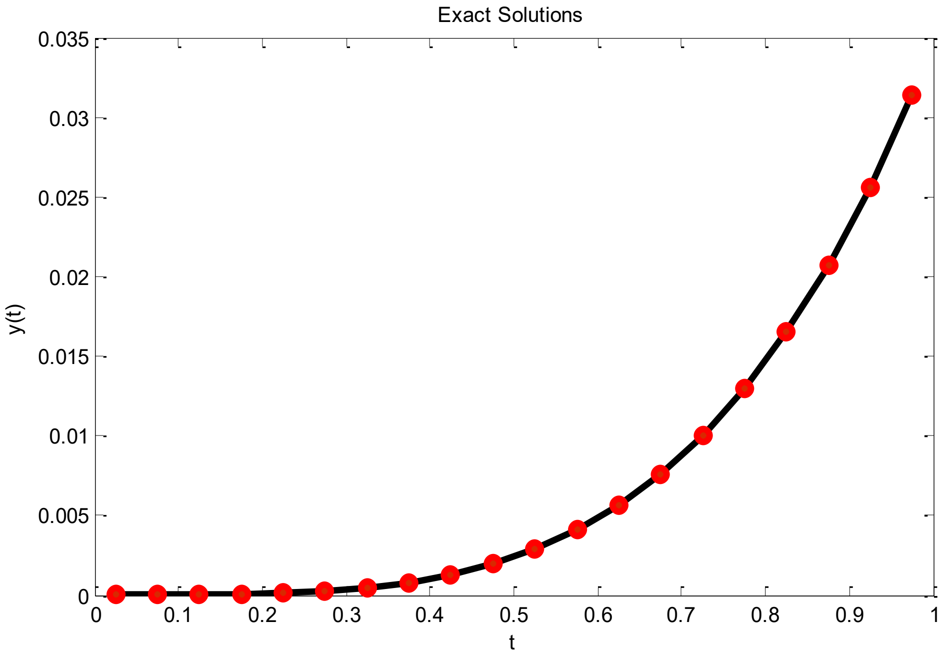

Figure- III: Exact Solutions of Example 2

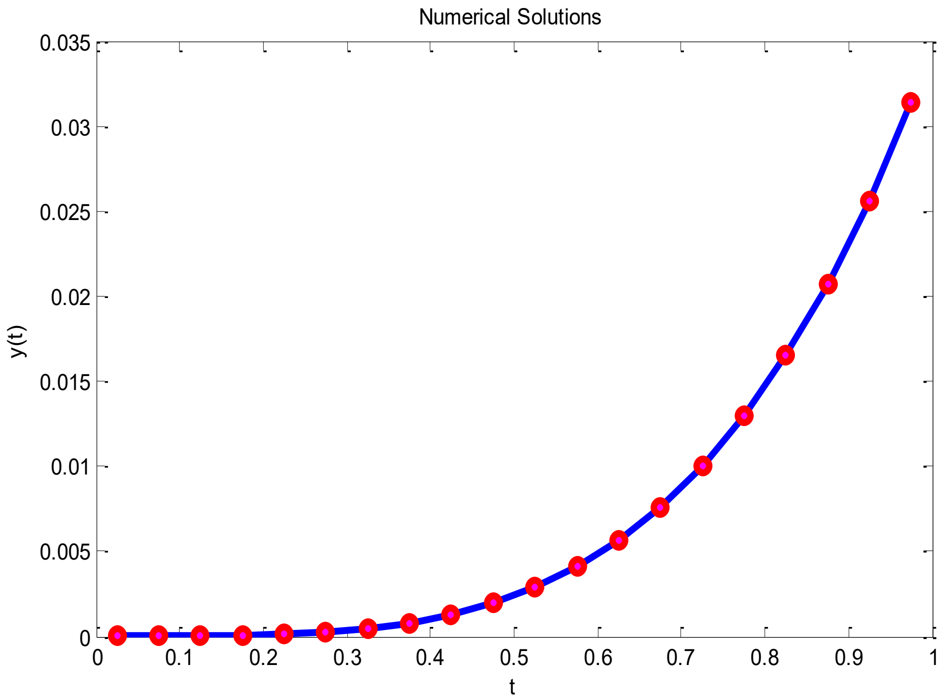

Figure-IV: Emad-Falih Transform based solutions of Example 2

Figure III and Figure IV show exact solution and solution obtained by Emad-Falih transform based numerical technique.

IV. Conclusion

The above graphical and numerical results depict that the Emad-Falih integral transform-based method is a powerful technique to obtain solutions of fourth order ordinary differential equations having constant coefficients. For future scope, this technique will be helpful to solve two- and three- dimensional partial differential equations. The suggested method can also be useful in case of system of ordinary and partial differential equations.

References

[1]. Emad Kuffi, and Sara Falih Maktoof, "Emad-Falih Transform", a new integral transform, Journal of Interdisciplinary Mathematics, Vol. 24, N. 8 (2021), pp. 2381-2390.

[2]. D. P. Patil, Sonal Borse and Darshana Kapadi, Applications of Emad-Falih transform for general solution of telegraph equation, International Journal of Advanced Research in Science, Engineering and Technology, Vol. 9, Issue 6, June 2022, pp. 19450-19454.

[3]. Tarig M. Elzaki (2011), The new integral transform "Elzaki Transform" Global Journal of Pure and Applied Mathematics, 1, pp. 57-64.

[4]. Tarig M. Elzaki & Salih M. Elzaki (2011), Application of new transform "Elzaki Transform" to partial differential equations, Global Journal of Pure and Applied Mathematics, 1, pp. 65-70.

[5]. Tarig M. Elzaki & Salih M. Elzaki (2011), On the Connections between Laplace and Elzaki transforms, Advances in Theoretical and Applied Mathematics, 6(1), pp. 1-11.

[6]. Tarig M. Elzaki, Salih M. Elzaki, and Eman M. A. Hilal, (2012). Elzaki and Sumudu Transforms for solving some differential equations, Global Journal of Pure and Applied Mathematics, ISSN 0973-1768, Volume 8,Number 2, pp. 167-173.

[7]. Tarig M. Elzaki, Salih M. Elzaki and Elsayed A. Elnour (2012), On the New Integral Transform "ELzaki Transform" Fundamental Properties Investigations and Applications, Global Journal of Mathematical Sciences: Theory and Practical,Vol.4, No.1, pp1-13.

[8]. Tarig M. Elzaki and Eman M. A. Hilal (2012), Homotopy Perturbation and ELzaki Transform for solving Nonlinear Partial Differential equations, Mathematical Theory and Modeling, Vol.2,No.3, pp33-42.

[9]. A.T. Salman, H.K. Jassim and N. J. Hassan, An application of the Elzaki homotopy perturbation method for solving fractional Burger's equations, Int. J. Nonlinear Anal. Appl., 13(2), 21-30 (2022).

[10]. A. Yildirim and A. Kelleci, Homotopy perturbation method for numerical solutions of coupled Burgers equations with time-space fractional derivatives, Int. J. Numerical Meth. Heat Fluid Flow 20 (2010), 897–909.