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Review Paper

Thermodynamics analysis of generalized ghost dark energy model in Brans-Dicke theory

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Abstract In this paper, we study the thermodynamics analysis of generalized ghost dark energy (GGDE) model in Brans-Dicke theory within the framework of flat Friedmann-Lemaitre-Robertson-Walker metric. We assume the well motivated logarithmic form of Brans-Dicke scalar field in terms of the scale factor to discuss the thermodynamics analysis of the model. Generalized second law of thermodynamics is satisfied with in the model if $\dot{S}_{tot} \geq 0$ otherwise model will violate it. It is observed that the generalized second law of thermodynamics is not satisfied with in the model. However for some suitable values of model parameter, the trajectories trying to satisfy GSL.

Keywords: Generalized ghost dark energy,flat,logarithmic,thermodynamics analysis.

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I. INTRODUCTION

The universe is in accelerating phase at present is confirmed by various cosmological observations. There must exist some kind of energy having negative pressure known as dark energy' (DE). The cosmological observation of high redshift surveys of supernovae [1, 2, 3], Wilkinson Microwave Anisotropy Probe (WMAP) [4, 5], Sloan Digital Sky Survey (SDSS) [6, 7] and Planck data [8, 9] confirmed the accelerated expansion of the universe. These observations suggests that the universe contains approximate 27% of dust matter, 68% DE and rest other forms of matter. The most accepted and observationally verified model of DE is cosmological constant model known as ΛCDM model. However, it faces the problem of fine-tuning and cosmic coincidence problems [10]. Recently, some recent research work [11, 12] shows that the ΛCDM model has some major tension with recent observations.Therefore it is not the best description of our Universe.

To solve the problems associated with ΛCDM model, various DE models namely quintessence [13, 14], K-essence [15, 16], tachyon[17, 18], phantom [19, 20, 21], ghost condensate [22, 23], quintom [24, 25, 26], holographic dark energy [27, 28], agegraphic dark energy [29, 30] and many more have been proposed in the literature. In recent years, a new class of DE model so-called ghost dark energy (GDE) has been proposed which has attracted a lot of interest. The origin of GDE comes from Veneziano ghosts in quantum chromodynamics (QCD) theory [31, 32, 33, 34]. It is assumed that the contribution of the ghost field to the vacuum energy can be treated as a possible candidate of DE. The Veneziano ghost field was introduced to solve problems in QCD. The positive and negative norms of QCD ghost field cancel each other and leaves no trace in the physical subspace. It was contended that they have small role to vacuum energy in time-dependent or curved space background [35]. The magnitude of this vacuum energy is of order $\Lambda^3_{QCD}H$, where Λ^3_{QCD} is QCD mass scale [35] and H is known as Hubble's parameter. Using the fact that $\Lambda QCD \sim 100M_eV$ and $H \sim 10^{-33}eV$ for the current time provide right order of magnitude $\rho_D \sim (3 \times 10^{-3}eV)^4$ for ghost energy density [35]. In this way GDE model provides a required amount of DE and solve fine-tuning problem associated with ΛCDM model. The energy density of GDE is given by [35, 36, 37] $\rho_d = n_1 H$, where H is the Hubble's parameter and n_1 is a constant.

The author[38] observed that the role of the Veneziano QCD ghost field to vacuum energy is not exactly of order H and the sub-leading term H^2 seems due to the fact that vacuum expectation value of the energy momentum tensor is conserved in isolation [39]. It was contended that vacuum energy of the ghost field can be written as $H + O(H^2)$. It has been expected that the second term H^2 plays a crucial role in early evolution of the Universe which may act as the early DE [40]. It has been observed that including the second term with original GDE density it has shown better agreement [40, 41] with the observational data as compared to usual GDE. In this generalized model, the energy density is defined as $\rho_g = n_1 H + n_2 H^2$, where n_2 is a constant. We call this form of model the GGDE model.

Brans-Dicke (BD) theory, which was proposed by Brans and Dicke [42] in 1961 is a natural extension of Einstein's GR. In this theory the dynamics of gravity were represented by a scalar field and dynamics of space-time were represented by the metric tensor. In BD theory, Newton's gravitational constant G is not presumed to be constant but is proportional to the inverse of the scalar field ϕ, which can vary from place to place and with time. This theory is dynamical in nature as compare to GR, therefore, it seems more suitable to discuss the dynamical DE models like GGDE models. Therefore, we have considered BD theory to discuss the GGDE model in the paper.

There is lot of literature available on GDE and GGDE to discuss the evolution of the universe [43, 44, 45]. It was observed that features of GDE in BD theory differ from GR. It was shown that the original GDE is not stable for all ranges of parameter spaces in standard cosmology [46] whereas it approaches to a stable phase in BD theory [47]. In Ref. [48], the authors have taken GGDE in BD theory with power-law form of BD scalar field with sub-leading term H^2. However, the power-law relation between scalar field and the scale factor leads to a constant deceleration parameter which does not show the transition phase. To resolve such problem, Kumar and Singh [50] have proposed a well motivated logarithmic relation between scalar field and the scale factor which describes the phase transition. A number of authors [51, 52, 53, 54, 55] have studied the cosmological models in BD theory with logarithmic form and found some interesting results.

Here, we study GGDE model in a dynamical framework such as BD theory using logarithmic form of BD scalar field instead of power-law form. We will discuss the thermodynamics analysis of the GGDE model.

This paper is organized as follows. In Sect. 2, the GGDE model is discussed in BD theory. Sect. 3 explore the thermodynamics behavior of GGDE model. We finish with conclusion in Sect. 4.

2 Model and Field Equations

The action of BD theory in canonical form can be written as [49]

$$S = \int d^4x\sqrt{g}\left(-\frac{1}{8\omega}\phi^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\ \partial_\nu\phi) + \mathcal{L}_m\right), \tag{1}$$

where R is the scalar curvature and ϕ is the BD scalar field. Here, ω is the dimensionless BD coupling parameter between scalar field and gravity. There are no constraints on the coupling parameter ω as far as the theory is concerned whereas there are observational constraints which vary based on the scale of observations and the method being used. The observations of Cassini solar mission put bounds $\omega > 40000$ [59] whereas cosmological scale bounds from various cosmological observations are very low [60, 61, 62].

We assume a homogeneous and isotropic flat FRW space-time to discuss the evolution of the universe which is given by

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-r^2} + r^2(d\theta^2 + \sin^2\theta\ d\phi^2)\right], \tag{2}$$

where a is the cosmic scale factor. We assume that the Universe is filled with pressureless dark matter (DM) and GGDE. We exclude baryonic matter and radiation due to their negligible contribution to the total energy budget during late time evolution. Varying the action (1) with respect the metric tensor $g^{\mu\nu}$ and the BD scalar field ϕ for the FLRW line element in equation (2), the field equations can be obtained as follows

$$\frac{3}{4\omega}H^2\phi^2 - \frac{1}{2}\dot{\phi}^{\,2} + \frac{3}{2\omega}H\phi\dot{\phi} = \rho_m + \rho_g \tag{3}$$

$$\frac{-1}{4\omega}\phi^2\Big(2\frac{\ddot{a}}{a} + H^2\Big) - \frac{1}{\omega}H\phi\dot{\phi} - \frac{1}{2\omega}\phi\ddot{\phi} - \frac{1}{2}\Big(1 + \frac{1}{\omega}\Big)\dot{\phi}^2 = p_g, \tag{4}$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{3}{2\omega}\Big(\frac{\ddot{a}}{a} + H^2\Big)\phi = 0, \tag{5}$$

where ρ_m is energy density of pressureless DM, ρ_g and p_g are respectively the energy density and pressure of GGDE, and $H = \dot{a}/a$ is the Hubble parameter. The GGDE is given by [40]

$$\rho_g = n_1 H + n_2 H^2, \tag{6}$$

where n_1 is a constant with dimension $[energy]^3$, roughly of order Λ^3_{QCD} and n_2 is another constant with dimension $[energy]^2$.

In the framework of BD cosmology, many authors [63, 64, 65, 66] have assumed BD scalar field ϕ as a power-law form of the scale factor, i.e., $\phi \propto a^m$, where m is a constant, to solve the BD field equations. Usually, this form of power-law gives the constant deceleration parameter. Therefore, Kumar and Singh [50] have proposed a logarithmic relation between BD scalar field and scale factor which is able to solve the cosmic coincidence problem and the problem of constant deceleration parameter. In this paper, we assume such logarithmic form of BD scalar field which is given by

$$\phi = \phi_0 \ln(\alpha + \beta a), \tag{7}$$

where ϕ_0, $\alpha > 1$ and $\beta > 0$. Taking time derivative of equation(7), we get

$$\frac{\dot{\phi}}{\phi} = \frac{\beta a H}{(\alpha + \beta a)\ln(\alpha + \beta a)}\ . \tag{8}$$

Using relation in equation (7) and (8), the equation (3) can be written as

$$H^2\Big(1 - \frac{2\omega\beta^2 a^2}{3(\alpha + \beta a)^2[\ln(\alpha + \beta a)]^2} + \frac{2\beta a}{(\alpha + \beta a)\ln(\alpha + \beta a)}\Big) = \frac{4\omega}{3\phi^2}(\rho_m + \rho_g)\ . \tag{9}$$

As usual, let us define the fractional energy densities corresponding to each energy part as

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{4\omega\rho_m}{3\phi^2 H^2} \tag{10}$$

$$\Omega_g = \frac{\rho_g}{\rho_{cr}} = \frac{4\omega\rho_g}{3\phi^2 H^2}, \tag{11}$$

where $\rho_{cr} = 3\phi^2 H^2/4\omega$ is the critical density in BD theory. Using GGDE density equation (6), one can rewrite equation(11)as

$$\Omega_g = \frac{4\omega(n_1 + n_2 H)}{3\phi^2 H} \tag{12}$$

We consider the case where DM and GGDE evolves independent to each other and hence they satisfy the following conservation equations

$$\dot{\rho}_m + 3H\rho_m = 0, \tag{13}$$

$$\dot{\rho}_g + 3H(1 + w_g)\rho_g = 0, \tag{14}$$

where $w_g = p_g/\rho_g$ is the EoS parameter of GGDE. Using equation (10) and (11), Eq.(9) can be written as

$$\Omega_m + \Omega_g = \gamma \tag{15}$$

where

$$\gamma = 1 - \frac{2\omega\beta^2 a^2}{3(\alpha + \beta a)^2[\ln(\alpha + \beta a)]^2} + \frac{2\beta a}{(\alpha + \beta a)\ln(\alpha + \beta a)} \tag{16}$$

It is noted that for $\beta = 0$, we have $\gamma = 1$. In this case, the BD scalar field becomes constant and Einstein gravity is restored. Taking time derivative of(9) and using continuity equations, we get

$$2\frac{\dot{H}}{H^2}\gamma + \frac{1}{H}\dot{\gamma} = -3\Omega_g(1 + w_g + u) - 2\Omega_g(1 + u)\frac{\beta a}{(\alpha + \beta a)\ln(\alpha + \beta a)} \tag{17}$$

. The universe decelerating or accelerating expansion can be discussed with the help of a cosmological parameter known as deceleration parameter(DP). It is defined as $q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}$. For GGDE model in BD theory, we find the value of deceleration parameter as

$$q = \frac{1}{((2\gamma - \Omega_g)n_1 + (\gamma - \Omega_g)2n_2H)}\Big[n_1(\gamma - 2\Omega_g) + n_2H(\gamma - \Omega_g) + \frac{2(\gamma + 1)(n_1 + n_2H)\beta a}{(\alpha + \beta a)\ln(\alpha + \beta a)} - \frac{2(2\omega + 3)(n_1 + n_2H)\beta^2 a^2}{3(\alpha + \beta a)^2[\ln(\alpha + \beta a)]^2} + \frac{2\omega(n_1 + n_2H)\beta^3 a^3}{3(\alpha + \beta a)^3[\ln(\alpha + \beta a)]^2} + \frac{2\omega(n_1 + n_2H)\beta^3 a^3}{3(\alpha + \beta a)^3[\ln(\alpha + \beta a)]^3} - \frac{2(n_1 + n_2H)\beta^2 a^2}{(\alpha + \beta a)^2\ln(\alpha + \beta a)}\Big]$$

3 Thermodynamic Analysis

We will examine whether the Generalized second law of thermodynamics suits our model. Padmanabhan [68] and Jacob [69] derived the first law of thermodynamics starting from the Einstein's field equations for a static and spherically symmetric space-time. The field equations of f(R) theory using only thermodynamical considerations are obtained [70]. Many aspects of thermodynamics have been studied in gravity theories one of which is generalized second law of thermodynamics (GSL). According to GSL, total entropy of the universe (horizon entropy and entropy of the matter inside the horizon) is an increasing function of the time. The GSL has been discussed in both Einstein's theory and modified theories of gravity [71, 72, 73].

Recently, GSL has been investigated in original BD theory and authors have obtained expression of rate of change of total entropy of the flat FLRW universe [75, 74]. In the present paper, we have also considered original BD theory in a flat FLRW universe. Therefore, we have considered the same expression of rate of change of total entropy of the flat FRW universe . Here, in the present work we have taken GGDE in our model while they have not considered any DE component. The total entropy of the universe is given by

$$S_{tot} = S_h + S_{in}, \tag{19}$$

where, S_{tot} denotes total entropy, S_h denotes horizon entropy and, S_{in} denotes entropy of total fluid inside the horizon. The rate of change of total entropy S_{tot} can be obtained as

$$\dot{S}_{tot} = \dot{S}_h + \dot{S}_{in}, \tag{20}$$

where dot represents the time derivative. Authors [75] have considered the entropy of dynamical apparent horizon rather than teleological event horizon which seems more relevant to the study. The entropy of the apparent horizon is given by the relation $S_h = 2\pi A$, where $A = 4\pi R_h^2$ denotes the area of the apparent horizon. Here, R_h denotes radius of the apparent horizon which is related to the Hubble parameter in the flat FRW universe as $R_h = \frac{1}{H}$. Thus, the entropy of the apparent horizon takes the form as $S_h = \frac{8\pi^2}{H^2}$ and it's rate of change is given by

$$\dot{S_h} = -16\pi^2\frac{\dot{H}}{H^3}. \tag{21}$$

The Gibbs law of thermodynamics for fluid inside the horizon gives rise to

$$T_{in}dS_{in} = dE_{in} + p_t dV_h, \tag{22}$$

where the subscript t denotes the total quantity and the volume $V_h = \frac{4}{3}\pi R_h^3$. Now, we can obtain the rate of change in entropy of the fluid inside the horizon as

$$\dot{S_{in}} = \frac{(\rho_t + p_t)\dot{V_h} + \dot{\rho_t}V_h}{T_{in}}. \tag{23}$$

If we consider that the fluid is in thermal equilibrium with the horizon then temperature of fluid inside horizon (T_{in}) and temperature of dynamical apparent horizon (T_h) are same (see [75] and references therein). It is known as Hayward-Kodama temperature which is given by

$$T_h = \frac{2H^2 + \dot{H}}{4\pi H}. \tag{24}$$

It may be observed that this temperature reduces to the Hawking temperature $T_{Hawking} = \frac{H}{2\pi}$ [?] in de Sitter space where $\dot{H} = 0$. Now, the rate of change of entropy of fluid inside the horizon can be obtained using Eqs. (30) and (31) as

$$\dot{S_{in}} = 16\pi^2\frac{\dot{H}}{H^3}\Big(1 + \frac{\dot{H}}{2H^2 + \dot{H}}\Big). \tag{25}$$

Now using eqs. (20), (21) and (25), one can get the rate of change of total entropy as [75]

$$\dot{S}_{tot} = \frac{\left(\frac{4\pi\dot{H}}{H^2}\right)^2}{H\left(\frac{\dot{H}}{H^2}+2\right)}. \quad (26)$$

We must observe $\dot{S}_{tot} \geq 0$ to satisfy GSL of thermodynamics, otherwise model will violate GSL. Using the value of DP in equation (26), we get the expression for $\dot{S}_{tot}$, which is complicated one. Therefore, we will discuss the thermodynamics analysis with the help of the plot. We have plotted the graph of $\dot{S}_{tot}$ against redshift parameter z for various values of the model parameter. Fig.1 is plotted for fixed values of $n_1 = 400$, $n_2 = 2.1$, $\Omega_g = 0.70$, $H = 70$, $\omega = 17$, $\beta = 4.1$ and various values of α. It is observed that $\dot{S}_{tot} < 0$ in past, present and future and the model violate GSL of thermodynamics. Fig.2 has been plotted for fixed values of $n_1 = 400$, $n_2 = 2.1$, $\Omega_g = 0.70$,$H = 70$, $\omega = 17$, $\alpha = 4.1$ and various values of β. It is observed that all the trajectories shows same behavior for all values of β. and the model violate GSL of thermodynamics. However, all the trajectories are trying to satisfy GSL of thermodynamics in future.

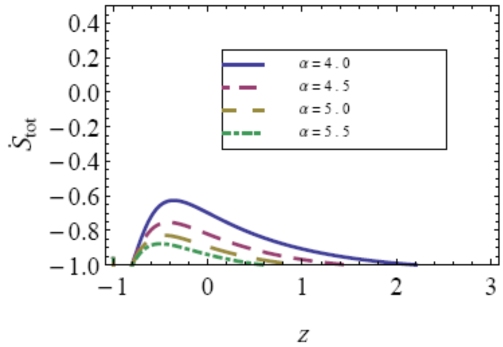

Figure 1: To plot the $\dot{S}_{tot}$ against redshift z, we have taken fixed values of $n_1 = 400$, $n_2 = 2.1$, $\Omega_g = 0.70$,$H = 70$, $\omega = 17$, $\beta = 4.1$ and various values of α

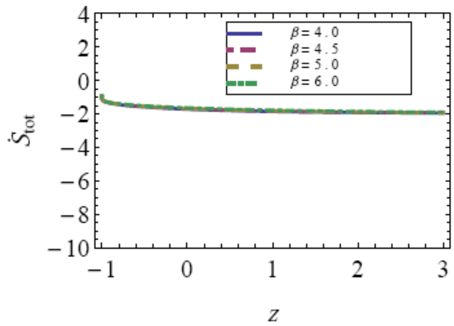

Figure 2: To plot the $\dot{S}_{tot}$ against redshift z, we have taken fixed values of $n_1 = 400$, $n_2 = 2.1$, $\Omega_g = 0.70$,$H = 70$, $\omega = 17$, $\alpha = 4.1$ and various values of β

4 Conclusion

To address one of the main problem associated with present day cosmology of accelerating universe, the concept of GGDE whose energy density takes the form $\rho_D = n_1 H + n_2 H^2$ have been proposed. Veneziano ghosts field in QCD theory is the origin of DE in this model. It was observed that the difference between the vacuum energy of quantum field in Minkowaski space-time and in FLRW Universe can play the role of observed dark energy. In this paper, we have investigated GGDE in BD theory with logarithmic scalar field for flat FLRW Universe. The cosmological implications of GGDE model have been studied to observe the evolution of the Universe. It is observed that the model does not satisfy GSL of thermodynamics. However, for some values of model parameter, the trajectories are trying to satisfy GSL of thermodynamics.

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