Quest Journals
Journal of Research in Applied Mathematics
Volume 11 ~ Issue 11 (November 2025) pp: 63-66
ISSN (Online): 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

Formation of Cyclic Coding From Uncertainty Collections

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Abstract: *In this paper, the main objective is study the codes arising from uncertainty sets and parameterizations sets. We discuss properties of fuzzy linear coders and fuzzy cyclic codes by means of fuzzy linear space. Finally, the codes developed by using parameterization codes with examples.*
AMS Mathematics subset classification: *03E72, 06072, 11T71*

Keywords: *Fuzzy set, parametric set, linear space, Fuzzy linear space, Fuzzy cyclic codes, Posset, Block-code, Binary number.*

*Received 06 Nov., 2025; **R**evised 17 Nov., 2025; **A**ccepted 19 Nov., 2025 © The author(s) 2025.*
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I. Introduction:

The theory of error-correcting codes was first introduced by Claude Shannon in 1948 and then gradually developed by time to time by different researchers. Messages in the form of bit strings are encoded by translating then into longer bit strings, called code word. A set of code word is called a code. Coding theory is concerned with reliability of communication over noisy channel. Algebraic codes [13] are used for data compressing error correction and for network coding. There are many types of codes which is important to its algebraic structures such as linear block-codes, Hamming codes, BCH codes and so on. The most common type of code is a linear code over the finite field F_q. More literature can be found in [13].Fuzzy sets were introduced by Lofti Zadeh [14] as an extension of crisp set which is called classical set. Fuzzy set theory permits the gradual assessment of the membership of elements in a set which is described in the interval [0, 1]. It can be used in a wide range of domains where the information is partial and vague. Fuzzy set perhaps is the most suitable frame work to model uncertain data. Fuzzy sets have a several applications in the areas such as signal processing, decision making, control theory, pattern recognition, computer version and so on. The complexities of modelling uncertain data are the main problem in engineering, environmental sciences etc. The fuzzy set theory [14], probability theory, rough set theory [12] etc., are well known and useful mathematical tools which describes uncertainty but each of them has its own limitation pointed out by Molodtsov [12]. Therefore, Molodtsov introduced the theory of soft sets [12] to model vague and uncertain information. A soft set is a parametrised collection of subsets of a universe of discourse. A huge amount of literature can be found in [[1],[2],[3], [5], [9],[10],[11]]. In this paper, the main objective is study the codes arising from uncertainty sets and parameterization's sets. We discuss properties of fuzzy linear coders and fuzzy cyclic codes by means of fuzzy linear space. Finally, the codes developed by using parameterization codes with examples. This paper organized as follows. In section2, preliminary concepts of uncertainty sets, parametric collection of objects and inner codes are presented. In section 3, discussed linear algebraic codes, In section 4, codes arising from uncertainty linear space and P-fuzzy sets are discussed. In section 5, codes from soft sets, called soft linear codes are presented along with their fundamental definitions and examples.

II. Preliminaries

In this section, we first recall the basis definitions.

Definition-2. 1 [11]**:** A fuzzy set λ in R (Real line) is defined to be a set ordered pairs, where λ (x) is called the membership function for the first set.

Definition-2.2 [11]**:** The α -cut of $\alpha -$ level set of fuzzy set λ is a set consisting of those elements of the universe 'X' whose membership values exceed the level α that is
$\lambda_\alpha = \{x \in X / \lambda(x) \geq \alpha\}$.

Definition -2.3: Let S be a non-empty set and $(P, \leq)$ a partially ordered set (poset). Any function $\lambda: S \rightarrow P$ is a p-fuzzy set on S. Also, for $p \in P, \lambda: S \rightarrow \{0,1\}$. So that for $x \in S, \lambda_p(x) = 1$, if and only i$f\ \lambda(x) \geq p$. Here $\lambda_p(x)$ is a characteristic functions of a p-level subset (or a p-cut) that
$\lambda_p = \{x \in S/\lambda(x) = 1\}$.
Lemma-2.4: Let $\lambda: S \rightarrow p$ be a fuzzy set. For every $x \in S, if\ \lambda_p(x) = p$, then p is a supremum of the class to which it belongs, that is $p = v[p]\sim$.
Theorem-2.5: (Decomposition of p-fuzzy sets): If $\lambda: S \rightarrow p$ is a p-fuzzy set on S. For$x \in S$,$\lambda(x) = \max\{p \in P/\lambda_p(x) = 1\}$. That is , the supremum on the right exists in $(P, \leq)$. For every
$x \in S$ and is equal to $A(x)$.
Definition-2.6: Let V_n be a n-dimensional linear space on a field F_q, λ a fuzzy subset of V_n, if for any $x, y \in V_n$, $\alpha \in F_q$, we have

(i) $\lambda(x + y) \geq \min\{\lambda(x), \lambda(y)\}$

(ii) $\lambda(\alpha x) \geq \lambda(x)$

Then, A is the fuzzy linear subspace of V_n on F_q.
Definition -2.7: A pair (F, λ) is called a soft set over U_1, where F is a mapping given by $F: A \rightarrow P(U)$. Here U refers to an intial universe and E to be set of parameters. P(U) denote the power set of U and $\lambda \subseteq E$. Usually, (F, λ) is denoted as F_λ.
Definition -2.8: Let f_λ and g_λ be two soft sets over a common universe U, then f_λ is called a soft subset of g_δ denoted by $f_\lambda \subseteq g_\delta\ if$

(i) $\lambda \subseteq \delta$

(ii) $\forall\ e \in \lambda, f(e)\ and\ g(e)$ Are identical approximations.

III. Linear Algebraic codes

Definition 3.1: A code C is any non-empty subset of F_q^n, the code C is called linear if it is am F_q-linear subspace of F_q^n. the number 'n' is the length of the code.
Definition-3.2: The hamming distance'd' on F_q^n is given by $d(x, y) = \{i/1 \leq i \leq n, x_i \neq y_i\}$ where $x = (x_1, x_2, \dots .. x_n)$ and $y = (y_1, y_2 \dots . . y_n)$.
Definition-3.3: The minimum distance of code $C \subseteq F_q^n$ is given by $d(c) = \min\ \{d(x, y)/x, y \in C, x \neq y\}$.
Definition- 3.4: The linear code C is called linear [n, k]-code, if $\dim(c) = k$.
Definition-3.5: Let C be a linear [n, k]-code. Let G be a $k \times n$ matrix whose rows forms a basis of C, then G is called generator matrix of the order C.
Definition-3.6: Let C be an [n, k]- code over F_q, then the dual code of C is denoted as $C^\perp$ and is defined to be $C^\perp = \{y \in F_q^n/x. y = 0, \forall\ x \in C\}$.
Defintion-3.7: A code C is called self-orthogonal code, if $C \subset C^\perp$.

IV. Formation of Coding from Uncertainty sets

This section deals with construction of codes from uncertainty set by means fuzzy linear space. Block-code from fuzzy sets are also discussed.
Definition -4.1: Let F_2 be a binary symmetric channel, then the fuzzy linear subspace λ of V_n is called a fuzzy linear code, where V_n is the n-dimensional linear space over F_2.
Lemma-4.2: λ is the fuzzy linear subspace, if for any $\alpha \in [0,1]$, if $\lambda_\alpha \neq \emptyset$, λ_α is a linear subspace of V_n.
Definition-4.3: λ is the fuzzy linear subspace, if and only if for any $\in [0,1]$, if $\lambda_\alpha \neq \emptyset$, λ_α is a linear code.
Definition-4.4: A fuzzy linear space λ of V_n is called a fuzzy cyclic code, if for any $(\alpha_0, \alpha_1, \dots \alpha_{n-1}) \in V_n$, we have $\lambda(\alpha_{n-1}, \alpha_0, \dots \alpha_{n-1}) \geq \lambda(\alpha_0, \alpha_1, \dots, \alpha_{n-1})$.
Lemma-4.5: A is a fuzzy cyclic code, if and only if $\in [0,1]$, if $\lambda_\alpha \neq \emptyset$, λ_α is a cyclic code.
Lemma-4.6: Let λ_1 and λ_2 be two fuzzy cyclic codes, then (i) $\lambda_1 \cap \lambda_2$ and
(ii) $\lambda_1 + \lambda_2$ are fuzzy cyclic codes. M-fuzzy sets are considered to be mapping from an arbitrary non-empty set S into a partially ordered set M.
Proof: Let $S = \{1,2,3 \dots . n\}$ and let $(M, \leq)$ be a finite partially ordered set. Every M-fuzzy set on S determines a binary block-code V of length n in the following way. For every class $[p]\sim(p \in P)$, there corresponds a code word $V_{[p]} = x_1, x_2, \dots x_n$, such that $x_i = j$ if and only if $\lambda_{ij}(i) = j$, for $i \in S\ and\ j \in [0,1]$.
For any $x, y \in V, x = x_1, x_2 \dots . x_{n_1}, y = y_1, y_2 \dots y_n\ if\ x \leq y$ if and only if $y_1 \leq x_1, \dots y_n \leq x_n \dots \dots (1)$ where $\leq$ is the ordinary ordering relation on the lattice $(\{0,1\}, \leq); 0 < 1$.
Theorem-4.7: Every finite partially ordered set $(p, \leq)$ determines a block-code V, such that $(p, \leq)$ is isomormorphic to $(V, \leq)$.
Proof: Let $p = \{p_1, p_2 \dots p_n\}$ and let $\lambda: p \rightarrow p$ be the mapping as an M-fuzzy set on P.

The decomposition of λ gives a family $\{\lambda_p/p \in P\}$ which is the required code under the above defined ordering relation. Now consider the mapping$h: p \rightarrow \{\lambda_p/p \in P\}$, such that $h(p) = \lambda_p$, by lemma-( ), every $(\sim)$ $-$class contains exactly one element and thus'h' is one to one of $p \leq q$, then $\lambda_q \subseteq \lambda_p$. By (1) follows that $\lambda_p \subseteq \lambda_{q,}$ and 'h' is an isomorphism.

Theorem-4.8: Let$V = \{V_1, V_2, \ldots V_n\} \subseteq \{0,1\}^n$, be a binary block-code, such that for every $i \in \{1,2 \ldots n\}$ at least one code word has a non-zero i^{th}-cordinates. Then there is a M-fuzzy set which corresponds to V if and only if, for every $i \in \{1,2 \ldots n\}$, max $(v \in V/V(i) = 1) \in V$.

The Hamming distance d(x, y) between x and y from $\{0,1\}^n$ is the number of coordinates in which x and y differ that is, $d(x,y) = \left\{\frac{i}{x_i} \neq y_i\right\}, x, y \in \{0,1\}^n$. The code distance of $V \subseteq \{0,1\}^n$ is the minimum Hamming distance between two code word in V defined by $d(v)$ and is defined as $d(V) = \min\{d(x,y); x, y \in \{0,1\}^n, x \neq y\}$.

V. Formation of coding from soft linear code

In this section, the codes evolved from soft sets, called soft linear codes are discussed along with examples.

Defintion-5.1: Let F_q be a finite field and $V = F_q^n$ be a vector space overF_q, where n is a positive integer. Let $p(V)$ be the power set of V and f_λ is called soft linear code over V if and only if, F_e is subspace of V which is a linear code.

Example 5.2: Let $F_q = F_2$ and $V = F_2^2$ is a vector space over F_2 and let f_A be a soft set over $V = K_2^3$. $F_{e1} = \{000,111\}, F_{e2} = \{000,101,011,110,111\}$.

Defintion-5.3: Let f_A be a soft code over $V = F_q^n$. Then D_s is called soft dimensions f_A if $D_s = \{\dim(f_e), \forall\, e \in \lambda\}$.

Example-5.4: Let f_A be a soft code defined as above example. Then the soft dimension is given by $D_s = \{\dim(f_e) = 1,\ \dim(f_e) = 2\} = \{1,2\}$.

Defintion-5.5: A soft linear code f_A over F_q^n of soft dimension D_s is called soft linear $[n, D_s]$ $-$code.

Definition-5.6: Let f_A be a soft code over $V = F_q^n$. Then the soft minimum distance of f_A is denoted by $S_d(f_A)$ and is defined as $S_d(f_A) = d(f_e)/d(f_e)$ is the minimum distance of the code f_e for all $e \in X$.

Example-5.7: Let $F_q = F_2$ be a finite field and $V = F_2^3$ is a vector space over F_2 and f_A be a soft set over $V = F_2^3$. Then clearly f_A is a soft code over $V = F_2^3$, where
$F_{e_1} = \{000,111\}$, $F_{e_2} = \{000,101,110,011\}$. The minimum distance of the code $F_{e_1} = 3$ and $F_{e_2} = 2$. thus the soft minimum distance of the soft code f_A is given as $S_d(f_A) = \{d(F_{e_1}) = 3, d(F_{e_2}) = 2\} = \{3,2\}$.

Definition-5.8: Let f_A be a soft $[n, D_s]$ -code over the field F_q and let the vector space $V = F_q^n$. Then the soft dual code of f_q is defined to be $(f_A)^\perp = (F_e)^\perp$; $(F_e) \perp$ is the dual code of f_e for all $e \in \lambda$.

Example-5.9: Let $F_q = F_2$ be a finite field and $V = F_2^3$ is a vector space F_2 and let f_A be a soft set over $V = F_2^3$. Then clearly f_A is soft code over $V = F_2^3$, wher $F_{e_1} = \{000,111\}$ and $F_{e_2} = \{000,101,110,011\}$. Then the soft dual code of f_A is define as $(f_A)^\perp$ where $(f_A)^\perp = \{000,110,101,011\}$ and $(f_{e_2})^\perp = \{000,111\}$.

Definition-5.10: A soft linear code f_A in $V = F_q$ over the field F_q is called soft self-dual code if $(f_A)^\perp = f_A$.

Defintion-5.11: A soft linear code f_A in $V = F_q^n$ over the field F_q is called complete-soft code if for all $e \in \lambda$, the deal of F_e also exists in f_A.

Example-5.12: Let $F_q = F_2$ be a finite field and $V = F_2^3$ is a vector space over F_2 and let f_A be a soft set over $V = F_2^3$. Then f_A is a soft code over $V = F_2^3$, where $F_{e_1} = \{000,111\}$ and $F_{e_2} = \{000,110,101,011\}$. Then f_A is a complete-soft code. Since the dual of F_{e_1} and F_{e_2} are also the dual of $F_{e_2}\ is\ F_{e_1}$.

Theorem-5.13: All complete-soft codes are trivially soft codes but the convex is not true in general.

Conclusion: we discussed in this article, codes from uncertainty sets and parametric sets. The advantage of soft linear code is that it can send n-message to n-persons.

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