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Review Paper

Existence Result for Random Differential Inclusions in Hilbert Spaces

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Abstract: *In this paper, sufficient conditions are given for the existence of solutions for a class of second order stochastic differential inclusions in Hilbert sport through Leray-Schauder Nonlinear Alternative.*
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I. Introduction

The theory for differential and integral inclusions in deterministic cases may be found in ([2, 10, 13, 23]) Random differential and integral inclusions play an important role in characterizing many social, physical, biological and engineering problems. Stochastic differential inclusions are important from the viewpoint of applications, since they incorporate randomness into the mathematical description of the phenomena, and therefore provide a more accurate description of them.

In many cases itis advantageous to treat the second order abstract differential equations directly rather than to convert them to first order systems. For example, Fitzgibbon [9] used the second order abstract differential equations for establishing the boundedness of solutions of the equations governing the transverse motion of an extensible beam. Second order equations have been examined in [6, 29]. The deterministic version of second order systems have been thoroughly investigated by several authors ([7, 21, 22, 28, 29]) while the stochastic version has not yet been treated satisfactorily. In fact abstact second order stochastic evolution equations and inclusions have only recently been studied in [5,17,18] and [19].

Ahmed [1] obtained the existence of solutions of nonlinear stochastic differential inclusions by using the semigroup approach and Banach fixed point theorem. Balasubramaniam [3] studied the existence of solutions of the following functional different all inclusions via integral inclusions.

$$dx(t) \in f(t,x_t)dt + G(t,x_1)d\omega(t), \qquad \text{a.e. } t \in J.$$

$$x(t)=\Phi(t),\ t \in [-r,0]$$

Using Kakutani's fixed point theorem in which G is the set valued map, $\{B(t)\}_{t\geq 0}$ is a Brownian motion or Wiener process and $\phi(t)$ is a suitable initial random variable independent of $\omega(t)$. Kree [15] showed the existence of solution of (1) for G(t,x(t)) using a fixed point argument. Recently, the global existence of solutions for nonlinear stochastic evolution inclusions has been studied in [4] by using the fixed point approach. The results presented in the paper constitute a continuation and generalization of the existence, uniquencess results from [11,4,14,15,25,26,27] to the second order semilinear stochastic evolution inclusions in Hilbert spaces settings.

In this paper, we are interested in the existence results of the following nonlinear second order stochastic differential inclusions.

We investigate the existence results of the following nonlinear second order random differentialinclusions,

$$x''(t,\omega) \in [Ax(t,\omega) + f(t,x(t,\omega),\omega)]dt + G(t,x(t,\omega),\omega)d(t) \quad \text{a.e. } t \in J = [0,b], \omega \in \Omega$$
$$x(0,\omega) = x_{(0,\omega)}, x'(0,\omega) = x_{(1,\omega)} \tag{1}$$

Where A is the infinitesimal generator of a strongly continuous cosine family C(t),t∈J, on a separable Hibert space H with the inner product $(\square,\square)$ and norm $\|\square\|$. Let K be another separable Hilbert space with the inner product $(\square,\square)k$ and norm $\|\square\|k$ Suppose $\{\omega(t)\}_{t\geq 0}$ is a given K-valued Brownian motion or Wiener process with a finite trace nuclear covariance operator Q≥0. We are also employing the same notation $\|\square\|$ for the norm BL(K,H), where BL(K,H) denotes the space of all bounded linear operators from K into H. Further, $f: J \times H \to H$ is a measurable mapping in H-norm and $G: J \times H \to P(L_Q(K,H))(P(L_Q(K,H))$ is the family of all nonempty subsets of $L_Q(K,H)$, a multivalued measurable mapping in $L_Q(K,H)$-norm. Here $L_Q(K,H)$ denotes the space of all Q-Hilbert-Schmidt operators from K into H which is going to be defined below.

II. Preliminaries

Let (Ω, $\bar{s}$ P) be a complete probability space furnished with a complete family of right continuous increasing sub σ-algebras {$\bar{s}_t$ t∈J} satisfying $\bar{s}_1 \subset \bar{s}$. An H-valued random variable is an $\bar{s}$- measurable function x(t): Ω→H and a collection of random variables $S = \{x(t,\omega): \Omega \to H \,|\, t \in J\}$ is called a stochastic process. Usually, we suppress the dependence on ω∈Ω and write x(t) instead of x(t,ω) and x(t): J→H in the space of S. Let $\beta_n(t)$ (n=1, 2, ……) be a sequence of real-valued one-dimensional standard Brownian motions mutually independent over (Ω, $\bar{s}$,P) . For more details of this section the reader may refer [8], Set

$$\omega(t) = \sum_{n=1}^{(x)} \sqrt{\lambda_n \beta_n}(t)\zeta_n, t \geq 0$$

Where $\lambda_n \geq 0, (n = 1, 2....)$ are nonnegative real numbers and $\{\xi_n\}$ (n=1,2,......) is complete orthonormal basis in K. Let $Q \in L(K,K)$ be an operator defined by $Q\zeta_n = \lambda_n \zeta_n$ with finite Tr Q=$\sum_{n=1}^{\infty} \lambda_n < \infty$, (Tr denotes the trace of the operator). Then the above K-valued stochastic process ω(t) is called a Q-Wiener process. We assume that $\bar{s}_t$=σ(ω(s):0≤s≤t) is the σ-algebra generated by ω and $\bar{s}_b = \bar{s}$. Let φ∈L(K,H) and define

$$\|\varphi\|_Q^2 = Tr(\varphi Q \varphi^*) = \sum_{n=1}^{\infty} \left\| \sqrt{\lambda_n \varphi \zeta_n} \right\|^2$$

If $\|\varphi\| Q < \infty$, then φ is called a Q-Hilbert-Schemidt operator. Let $L_Q(K,H)$ denote the space of all Q-Hilbert-Schmidt operators φ: K→H. The completion $L_Q(F,H)$ of L(K,H) with respect to the topology induced by the norm $\|\square\| Q$ where $\|\varphi\|_Q^2 = \langle\langle \varphi, \varphi \rangle\rangle$ is a Hilbert space with the above norm topology.

The collection of all strongly-measurable, square-integrable H-valued random variables, denotes by $L_2(\Omega, \xi, P; H) \equiv L_2(\Omega; H)$ is a Banach space equipped with the norm $\|x(\square)\| L_2 = \left(E\|\square;\omega)\|_H^2\right)^{1/2}$, where the expectation E is defined by $E(h) = \int_\Omega h(\omega)dP.$ Let Z=C(J,L_2(Ω;H)) be the Banach space of all continuous maps from J into L_2(Ω;H) satisfying the condition $E\|x(t)\|^2 < \infty$ and let $\|\square\|_z$ be a norm in Z defined by

$$\|x\|_z = \left(\sup_{t \in J} \|x(t)\|^2\right)^{1/2}$$

It is easy to verify that Z, furnished with the norm topology as defined above is a Banach space.

In a Hilbert space H, a multivalaued map M:H→℘(H) is convex (closed) valued, if M(x) is convex (closed) for all x∈H. M is bounded on bounded sets if $M(V) = \bigcup_{x \in V} M(x)$ is bounded in H, for any bounded set V of H $M(N) \subseteq V$..

M is called upper semicontinous (u.s.c.) on H, if for each $x_* \in H$, the set $M(x_*)$ is a nonempty, closed subset of H, and if for each open set V of H containing $M(x_*)$ there exists an open neighborhood N of x_* such that $M(N) \subseteq V.$

M is said to be completely continuou if M(V) is relatively compact, for every bounded subset V⊆H.

If the multivalued map M is completely continuous with nonempty compact values, then M is u.s.c. if and only if M has a closed graph $(\text{i.e.} x_n \to x_*, y_n \to y_*, y_n \in Mx_n \text{ imply } y_* \in Mx_*)$. M has a fixed point if there is x∈H such that x∈Mx.*

In the following, $\wp_{b,cl,cv}(H)$ denotes the set of all nonempty bounded, closed and convex subsets of H.

A multivalued map M:J→ $\wp_{b,cl,cv}(H)$ is said to be measurable if for each x∈H the mean-square distance between x and M(t) is a measurable function on J. For more details on multivalued maps ([10, 12]).

For each x∈$L_2(L_Q(K,H)$ define the set of selections of G by

$$g \in N_{Gx} = \{g \in L_2(L_Q(K,H)) : g(t) \in G(t, x(t)) \text{ for a.e. } t \in J\}.$$

The following basic result concerning the strongly continuous cosine families is needed of [28, 29] to prove our main results.

Definition 2.1.

(i) A one parameter family {C(t),t∈J} of bounded linear operators in the Hilbert space H is called a strongly continuous cosine family if and only if

(a) C(s + t) +C(s-t) = 2C(s)C(t) for all s, t∈J;

(b) C(0) = I;

(c) C(t)x is continuous in t on J for each fixed x∈H.

(ii) The corresponding strongly continuous sine family {S(t),t∈J} of bounded linear operators in the Hilbert space H is defined by

$$S(t)_x = \int_0^t C(\text{s})\,\text{xds}. \qquad \text{for all } x \in H. \qquad \text{for all t} \in \text{J}.$$

(iii) The infinitesimal generator of a strongly continuous cosine family {C(t), t∈J}, is the operator A:H→H defined by

$$Ax = \frac{d^2}{dt^2} C(t)x\Big|_{t=0,} \qquad x \in D(A),$$

WhereD(A) ={x∈H:C(t)x is twice continuously differentiable in t}.

Lemma 2.2. Let A generate a strongly continuous cosine family C(t),t∈J, of bounded linear operators. Then the following hold:

(i) There exist constants $\bar{M} \geq 1$ and $\omega \geq 0$ such that $\|C(t)\| \leq \bar{M}e^{\omega|t|}$ and hence $\|S(t)\| \leq \bar{M}e^{\omega|t|}$,

(ii) $A\int_t^{t^*} S(\tau)x d\tau = [C(t^*) - \text{C(t)}]x, \text{ for all } 0 \leq \text{t} \leq t^* < \infty$

(iii) There exists $\bar{N} \geq 1$ such that $\|S(t) - S(t^*)\| \leq \bar{N}\left|\int_t^{t^*} e^{\omega|s|ds}\right| \; for 0 \leq t \leq t^* < \infty$

The Uniform Boundedness Principle, together with (i) above, imply that both {C(t),t∈J} and {S(t),t∈J} are uniformly bounded by $M = \bar{M}e^{\omega|b|}$

In addition to the familiar Young, Holder and Minkowskii inequalities, the inequality of the form $\left(\sum_{t=1}^n a_i\right)^m \leq n^{m-1}\sum_{i=1}^n a_i^m$, follows from the convexity of x^m,m≥1, and is helpful to establish various estimates, where a_i are nonnegative constants (i=1, 2, …….n) and $n \in$ □

The considerations in this paper are based on the following alternative ([11).

Theorem 2.3 (Nonlinear alternative for Kakutani maps). Let Y be a Hilbert space, C a closed convex subset of Y. □ an open subset of C and $0 \in$ □ . Suppose that $F : \bar{□} \to \rho_{c,cv}(C)$ is an upper semicontinuous compact map; here $\rho_{c,cv}(C)$ denotes the family of nonempty, compact convex subsets of C. The either

(i) F has a fixed point in $\bar{\Omega}$, or
(ii) There is a $av \in \partial\Omega$ and $\lambda \in (0,1)$ with $v \in \lambda F(v)$

Definition 2.4. The multivalued map $F : J \times H \to \rho(H)$ is said to be L_2-Caratheodory if:

(i) $t \to F(t,y)$ is measurable for each $y \in H$;
(ii) $y \to F(t,y)$ is upper semicontinuous for almost all $t \in J$;
(iii) For each q>0, there exists $h_q \in L_1(J, \mathbb{R}_+)$ such that

$$\|F(t,y)\|^2 := \sup\{E\|g\|^2 : g \in F(t,y)\} \le h_q(t) \text{ for all}$$
$$\|y\|^2 \le q \text{ and for a.e. } t \in J.$$

The following lemma is applicable in the proof of our main result.

Lemma 2.5 [16]. Let I be a compact interval and Y be a separable Hilbert space. Let G be and L_2-Carathodory multivalued map with $N_{G,x} \neq \Phi$ and let Γbe a linear continous mapping $L_2(I,Y)$ to $C(I,Y)$.
Then the operator

$$\Gamma \circ N_G : C(I,Y) \to \rho_{b,cl,cv}(C(I,Y)), x \to (\Gamma \circ N_G, x) = \Gamma(N_{G,} \text{ x})$$

Is a closed graph operator in $C(I,Y) \times C(I,Y)$.

III. Existence Result

We give the definition of the mild solution.

Definition 3.1. An $\bar{s}_t$-adapted stochastic process $x(t);J \to H$ is a mild solution of the abstract Cauchy problem (2) if there exists a function $g \in L_2^{\bar{s}}(L_Q(K,H)$, a selection of $G(t,x(t))$, such that for a.e. $t \in J$, the following integral equation is satisfied.

(3) $x(t) = C(t)_{x0} + S(t)_{x1} + \int_0^t S(t-s) f(s,x(s))ds + \int_0^t S(t-s)g(s)d\omega(s).$

Theorem 3.2 Assume that:

(H1) A is the infinitesimal generator of a given strongly continuous bounded cosine family $\{C(t):t \in J\}$, and there exists a constant $M \geq 1$ such that

$$\|C(t)\| \le M \text{ and } \|S(t)\| \le M \text{ for all } t \ge 0$$

(H2) (i) the function $f : J \times H \to H$ is completely continuous;
(ii) There exist constants $c_1>0$, $c_2 \geq 0$ such that

$$E\|f(t,y)\|^2 \le c_1 E\|y\|^2 + c_2, \text{ for every } t \in J, \text{ and } y \in H;$$

(H3) G: $J \times H \to P(L_Q(K,H)$ is an L_2-Caratheodory function:

(H4) There exist a continuous nondecreasing function $\Psi : \mathbb{R}_+ \to (0,\infty), P \in L_1(J, \mathbb{R}_+)$, and nonnegative number $M_*>0$ such that

$$E\|G(t,y)\|_Q^2 = \sup\{E\|g\|_Q^2 : g \in G(t,y)\} \le P(t)\Psi(E\|y\|^2$$

For almost $t \in J$ and $y \in H$, and

$$\frac{(1-4_{c1}M^2b^2)M_*}{(2M)^2 P\{\|x_0\|_Z^2 + \|x_1\|_Z^2 + c_2b^2 + \Psi(M_*)TrQ\int_0^t P(s)ds\}} > 1,$$

(H5) for each bounded set $\beta \subseteq Z$, and $t \in J$ the set

$$\left\{C(t)x_0 + S(t)x_1 + \int_0^t S(t-s)f(s,x(s))ds \right.$$
$$\left. + \int_0^t S(t-s)g(s)d\omega(s), g \in N_G\beta\right\}$$

Is relatively compact in H, where $x \in \beta$ and $N_G\beta = \cup\{N_{G,x} : x \in \beta\}$ Then there exists at least one mild solution for the system (2) on J.

Proof. Consider the multivalued map$\Phi : Z \to P(Z)$ defined by

$$\Phi x=\left\{h\in Z: h(t)=C(t)x_0+S(t)x_1+\int_0^t S(t-s)f(s,x(s))ds\right.$$
$$\left.+\int_0^t S(t-s)g(s)d\omega(s), a.e. t\in J\right\}$$

where $g\in N_{G,x}$. We shall show that the operator Φ has a fixed point, which then is a solution of the system (2) We divide the proof into several steps.

Step I. Φx is convex for each $x\in Z$.

In fact, if h_1, h_2 belong to Φx, then there exist $g_1, g_2\in N_{G,x}$ such that

$$h_i(t)=C(t)x_0+S(t)x_1+\int_0^t S(t-s)f(s,x(s))ds$$
$$+\int_0^t S(t-s)g_i(s)d\omega(s), i=1,2 \quad t\in j$$

Let $0\in\rho\leq 1$. The for each $t\in J$, we have

$$(\rho h_1+(1-\rho)h_2(t)=C(t)x_0+S(t)x_1+\int_0^t S(t-s)f(s,x(s))ds$$
$$+\int_0^t S(t-s)\rho g_1(s)+(1-\rho)g_2(s)d\omega(s).$$

Since $N_{G,x}$ is convex (because G has convex values), then

$$\rho h_1+(1-\rho)h_2\in\Phi x$$

which complets the proof of Step I.

Step II. Φ maps bounded sets into bounded sets in Z.

Indeed, it is enough to show that there exists a positive constant ℓ such that for each $h\in\Phi x$, $x\in Bq=\{x\in Z:\|x\|_Z^2\leq q, q\in \square\}$ one has $\|h\|_Z^2\leq\ell$. If $h\in\Phi x$, then there exists $g\in N_{G,x}$, such that, for each $t\in J$,

(4) $h(t)=C(t)x_0+S(t)x_1+\int_0^t S(t-s)f(s,x(s)ds+\int_0^t S(t-s)g(d\omega)(s).$

Then for each $h\in\Phi(B_q)$ we have

$$E\|h(t)\|^2\leq 4\left\{E\|C(t)x_0\|^2+E\|S(t)x_1\|^2\right.$$
$$+b\int_0^t\|S(t-s)\|^2 E\|S(t)x(s))\|^2 ds$$
$$\left.+TrQ\int_0^t\|S(t-s)\|^2 E\|g(s)\|_Q^2 ds\right\}$$
$$\leq(2M)^2\left\{E\|x_0\|^2+E\|x_1\|^2+b\int_0^t[c_1E\|x(s)\|^2+c_2]ds\right.$$
$$\left.+TrQ\int_0^t E\|g(s)\|_Q^2 ds\right\}$$

Thus,

$$\|h(t)\|_Z^2\leq(2M)^2\{\|x_0\|_Z^2+\|x_1\|_Z^2+b^2[c_1q+c_2]+TrQ\|h_q\|_{L_1}\}:=\ell$$

Step III. Φ maps bounded sets into equicontinuous sets of Z.

Let $0<t_1<t_2\leq b$. For each $x\in B_q$ and $h\in\Phi x$, there exists $g\in N_{G,x}$ such that (4) holds. Thus, using Lemma 2.2 we have

$$E\|h(t_2)-\mathrm{h}(\mathrm{t}_1\|^2\leq 6\left\{\|[C(t_1)-C(t_2)x_0\|^2+\|[S(t_1)-S(t2)]x_1\|^2\right.$$
$$+b\int_0^{t_1}\|S(t_1-s)-S(t_2-s)\|^2 E\|f(s,x(s))\|^2 ds$$

$$+\int_{t_1}^{t_2}\|S(t_2-s)\|^2 E\|f(s,x(s))\|^2 ds$$

$$+TrQ\int_{t_1}^{t_1}\|S(t_2-s)\|^2 E\|g(s)\|_Q^2 ds$$

$$+TrQ\int_{t_1}^{t_2}\|S(t_1-s)-S(t_2-s)\| E\|g(s)\|_Q^2 ds$$

$$+TrQ\int_{t_1}^{t_2}\|S(t_2-s)\|^2 E\|g(s)\|_Q^2 ds\Big\}$$

Hence,

$$\|h(t_2)-h(t_1)\|_Z^2 \le 6\{\|[C(t_1)-C(t_2)]x_0\|^2+\|[S(t_1)-S(t_2)x_1\|^2$$

$$+b\int_0^{t_1}\|S(t_1-s)-S(t_2-s)\|^2\{c_1 q+c_2\}ds$$

$$+M^2(t_2-t_1)^2\{c_1 q+c_2\}$$

$$+e^{\omega b}(t_2-t_1)TrQ\int_0^{t_1}h_q(s)ds$$

$$+M^2 TrQ\int_{t_1}^{t_2}h_q(s)ds$$

$$++M^2 TrQ\int_{t_1}^{t_2}h_q(s)ds\Big\}.$$

As a consequence of Steps 2.3, (H5) and the Arzela-Ascoli theorem we can conclude that Φ is completely continuous

Step IV. Φ has a closed graph

Let $x_n\to x_*$, $h_n\in\Phi x_n$ and $h_n\to h_*$. We shall prove that $h_*\in\Phi x_*$. Now $h_n\in\Phi x_n$ means that there exists $g_n\in N_{G,xn}$ such that

$$h_n(t)=C(t)x_0+S(t)x_1+\int_0^t S(t-s)f(s,x_n(s)ds$$

$$+\int_0^t S(t-s)g_n(s)d\omega(s), t\in J.$$

We must prove that there exists $g_*\in N_{G,x_*}$ such that

$$h_*(t)=C(t)x_0+S(t)x_1+\int_0^t S(t-s)f(s,x_*(s)ds$$

$$+\int_0^t S(t-s)g_*(s)d\omega(s), t\in J.$$

Since f is continuous then

$$\Big\|\Big(h_n(t)=C(t)x_0+S(t)x_1+\int_0^t S(t-s)f(s,x_n(s)ds\Big)$$

$$-\Big(h_*(t)=C(t)x_0+S(t)x_1+\int_0^t S(t-s)f(s,x_*(s)ds\Big)\Big\|^2\to 0$$

as $n\to\infty$. Consider the linear continuous operator

$$\Gamma: L_2^{\bar{s}}(L_Q(K,H))\to Z$$

$$g\to\Gamma(g)(t)=\int_0^t S(t-s)g(s)d\omega(s)$$

Clearly Γ is linear and continuous. Indeed, one has

$$\|\Gamma g\|^2\le bM^2 TrQ\|g\|_Q^2$$

From Lemma 2.5, it follows that $\Gamma\circ S_G$ is a closed graph operator. Morever,

$$\left(h_n(t) - C(t)x_0 - S(t)x_1 - \int_0^t S(t-s)f(s, x_n(s)ds\right) \in \Gamma(N_{G,x_n})$$

Since $x_n \to x_*$, for some $g_* \in N_{G,x^*}$, it follows from lemma 2.5 that

$$h_*(t) - C(t)x_0 - S(t)x_1 - \int_0^t S(t-s)f(s, x_n(s)ds = \int_0^t S(\text{t}-\text{s})\,\text{g}_*(s)d\omega(s).$$

Therefore Φ is a completely continuous multivalued map, u.s.c. with convex closed values.
In order to prove that Φ has a fixed point by Theorem 2.3, we need one more step.
Step V. We show there exists an open set $U' \subseteq Z$ with $x \notin \lambda\Phi(x)$ for $\lambda \in (0,1)$ and $x \in \vartheta\, U'$.

Let $x \in U'$. Then $x \in \lambda\Phi x$ for some $\lambda > 1$, and there exists $g \in N_{G,x}$ such that

$$x(t) = \lambda^{-1}C(t)x_0$$

$$+S(t)x_1 + \lambda^{-1}\int_0^t S(t-s)f(s, x(s))ds + \lambda^{-1}\int_0^t S(t-s)g(s)\,\text{d}\,\omega(\text{s}), \text{t} \in \text{J}$$

By hypotheses (H1)-(H4), we have for each $t \in J$

$$E\|x(t)\|^2 \leq 4\left\{E\|C(t)x_0\|^2 + E\|[S(t)x_1\|^2\right.$$

$$+b\int_0^t \|S(t-s)\|^2 E\|f(s, x(s))\|^2 ds$$

$$+TrQ\int_0^t \|S(t-s)\| E\|g(s)\|_Q^2 ds$$

$$\left\{\leq (2M)^2\left\{E\|x_0\|^2 + E\|x_1\|^2 + b\int_0^t [c_1 E\|x(s)\|^2 + c_2]ds\right.\right.$$

$$\left.+TrQ\int_{t_1}^{t_2} \|P(s)\|\Psi E\|x(s)\|^2 ds\right\}.$$

Consider the function μ defined by

$$\mu(t) = \sup\{E\|x(s)\|^2 : 0 \leq s \leq t\}, \qquad 0 \leq \text{t} \leq \text{b}$$

From the previous inequality we have

$$\mu(t) \leq (2M)^2\left\{\|x_0\|_Z^2 + \|x_1\|_Z^2 + b^2[c_1\mu(t) + c_2] + TrQ\int_0^t P(s)\Psi(\mu(s))ds\right\}$$

Therefore

$$\left(1 - 4c_1M^2b^2\right)\mu\left(t\right)$$

$$\leq (2M)^2\left\{\|x_0\|_Z^2 + \|x_1\|_Z^2 + c_2b^2 + TrQ\int_0^t P(s)\Psi(\mu(s))ds\right\}$$

Consequently,

$$\frac{\left(1 - 4c_1M^2b^2\right)\|x\|_Z^2}{(2M)^2\left\{\|x_0\|_Z^2 + \|x_1\|_Z^2 + c_2b^2 + \Psi(\|x\|_Z^2)TrQ\int_0^t P(s)ds\right\}} \leq 1$$

The by (H4), there exists M* such that

$$\|x\|_Z^2 \neq M_*$$

$$Set$$

$$U^1 = \{x \in Z : \|x\|_Z^2 < M_* + 1\}.$$

From the choice of U′, there is no $x \in \partial U'$ such that $\lambda x = \lambda\Phi$ for some $\lambda > 1$. As a consequence of the nonlinear alternative of leray-Schauder type [11], we deduce that Φ has a fixed point in U′which is a mild solution of the problem (2).

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