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Review Paper

Exploring *L*-Fuzzy Bi-ideals in LA-Rings: A Lattice-Theoretic Approach

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Abstract: *The conception of fuzzy subsets was established by Zadeh in 1965 and the fuzzy hypothesis has developed in many guidelines and created applications in aextensivediversity of fields. The conception of lattice plays a worth mentioning role in the field of mathematics.In this manuscript, we describeL-fuzzy bi-ideals in* $\mathcal{L}$*A-rings and deliver some characterization of diverse classes of* $\mathcal{L}$*A-ring in terms ofL-fuzzy left (resp. right, bi-, generalized bi-, (1,2)-) ideals.*

Keywords: $\mathcal{L}$***A****-ring, L-fuzzy* $\mathcal{L}$*A-subrings, L-fuzzy bi-ideal.*

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I. Introduction

The notion of fuzzy subsets was introduced by zadeh in 1965[16] and the fuzzy theory has developed in many directions and found applications in a wide variety of fields. The concept of lattice plays a significant role in mathematics. The perception of lattice was first defined by Dedekind in 1897, and then developed by Birkhoff [3] in 1933. In 1990, Yuan and Wu [15] applied the concept of fuzzy sets in lattice theory.J.AGoguen [4] replaced the valuation set [0,1] by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy set. Liu[10], introduced the concept of fuzzy subrings. and fuzzy ideals of a ring. Many authors have explored the theory of fuzzy rings. Kuroki[8], characterized regular rings in terms of fuzzy left(right, quasi,bi-)ideals.Ideals in $\mathcal{L}$A-semigroup have been investigated by [12] Kamran [5] extended the notion of $\mathcal{L}$A-semigroup to the left almost group ($\mathcal{L}$A-group).Shah et.al [13], initiated the concept of left almost ring [abbreviated as $\mathcal{L}$A-ring] of finitely non-zero functions, which is a generalization of a commutative semigroupring.A detail work about bi-ideal and fuzzy bi-ideals in a ring can be found by S.K Datta[2]. Further, NasreenKawsar [11] introduced the concept of fuzzy $\mathcal{L}$A-subring and fuzzy bi-ideals in $\mathcal{L}$A-rings.

In this paper, we define L-fuzzy bi-ideals in $\mathcal{L}$A-rings and give some characterization of different classes of $\mathcal{L}$A-ring in terms of L- fuzzy left (right, bi-, generalized bi-, (1,2)-) ideals.

II. Preliminaries

In this section, we give some concepts and results which will be helpful in this paper.

A fuzzy subset $\mu_{\mathcal{A}}$of an $\mathcal{L}$A-ring $\mathcal{R}$, we mean a function $\mu_{\mathcal{A}}:\mathcal{R}\rightarrow[0,1]$ and the complement of $\mu_{\mathcal{A}}$is denoted by$\mu_{\mathcal{A}}{}^{C}$, is also a fuzzy subset of $\mathcal{R}$ defined by$\mu_{\mathcal{A}}{}^{C}(x)=1-\mu_{\mathcal{A}}(x)$for all $x\in\mathcal{R}$.

A fuzzy subset $\mu_{\mathcal{A}}$ of an $\mathcal{L}$A-ring $\mathcal{R}$ is a fuzzy $\mathcal{L}$A-subring of $\mathcal{R}$, if

$$\mu_{\mathcal{A}}(x-y) \geq \min\{\mu_{\mathcal{A}}(x), \mu_{\mathcal{A}}(y)\}$$

$\mu_{\mathcal{A}}(xy) \geq \min\{\mu_{\mathcal{A}}(x), \mu_{\mathcal{A}}(y)\} \, \forall \, x, y \in \mathcal{R}$.

A fuzzy subset $\mu_{\mathcal{A}}$is a fuzzy left (resp.right) ideal of $\mathcal{R}$, if

$$\mu_{\mathcal{A}}(x-y) \geq \min\{\mu_{\mathcal{A}}(x), \mu_{\mathcal{A}}(y)\}$$

$\mu_{\mathcal{A}}(xy) \geq \mu_{\mathcal{A}}(y)(resp. \mu_{\mathcal{A}}(xy) \geq \mu_{\mathcal{A}}(x)) \forall \, x, y \in \mathcal{R}$.

A fuzzy $\mathcal{L}$A-subring $\mu_{\mathcal{A}}$ of an $\mathcal{L}$A-ring $\mathcal{R}$is a fuzzy bi-ideal of $\mathcal{R}$if,

$$\mu_{\mathcal{A}}((xa)y) \geq \min\{\mu_{\mathcal{A}}(x), \mu_{\mathcal{A}}(y)\} \, \forall \, a, x, y \in \mathcal{R}.$$

A fuzzy subset $\mu_{\mathcal{A}}$ of an $\mathcal{L}$A-ring $\mathcal{R}$ is a fuzzy generalized bi-ideal of $\mathcal{R}$ if,

$$\mu_{\mathcal{A}}(x-y) \geq \min\{\mu_{\mathcal{A}}(x), \mu_{\mathcal{A}}(y)\}$$

$$\mu_{\mathcal{A}}((xa)y) \geq \min\{\mu_{\mathcal{A}}(x), \mu_{\mathcal{A}}(y)\} \, \forall \, a, x, y \in \mathcal{R}.$$

A fuzzy LA-subring $\mu_{\mathcal{A}}$ of an $\mathcal{L}$A-ring $\mathcal{R}$is called a fuzzy (1,2)-ideal of $\mathcal{R}$if,

$$\mu_{\mathcal{A}}((xa)(yz)) \geq \min\{\mu_{\mathcal{A}}(x), \mu_{\mathcal{A}}(y), \mu_{\mathcal{A}}(z)\} \, \forall \, a, x, y, z \in \mathcal{R}.$$

III. L- fuzzy $\mathcal{L}$A-subring &L- fuzzy bi-ideals in$\mathcal{L}$A-rings

Definition:3.1
A fuzzy subset $\mu_{\mathcal{A}}$of an $\mathcal{L}$**A**-ring $\mathcal{R}$ is a L- fuzzy $\mathcal{L}$**A**-subring of $\mathcal{R}$,if
$\mu_{\mathcal{A}}(x-y) \geq \mu_{\mathcal{A}}(x) \wedge \mu_{\mathcal{A}}(y)$ and

$\mu_{\mathcal{A}}(xy) \geq \mu_{\mathcal{A}}(x) \wedge \mu_{\mathcal{A}}(y) \forall \, x, y \in \mathcal{R}$.

Definition:3.2
A fuzzy $\mathcal{L}$**A**-subring $\mu_{\mathcal{A}}$ of an $\mathcal{L}$**A**-ring $\mathcal{R}$ is a L- fuzzy bi-ideal of $\mathcal{R}$ if,

$$\mu_{\mathcal{A}}((xa)y) \geq \mu_{\mathcal{A}}(x) \wedge \mu_{\mathcal{A}}(y) \text{ for all } x, y, a \in \mathcal{R}.$$

Example:3.3
Let $\mathcal{R} = \{0,1,2,3,4\}$Define + and $\cdot$ in $\mathcal{R}$ as follows:

+	0	1	2	3	4
0	0	1	2	3	4
1	4	0	1	2	3
2	3	4	0	1	2
3	2	3	4	0	1
4	1	2	3	4	0

.	0	1	2	3	4
0	0	0	0	0	0
1	4	0	1	2	3

2	3	4	0	1	2
3	2	3	4	0	1
4	1	2	3	4	0

Then $\mathcal{R}$ is an $\boldsymbol{\mathcal{L}}$**A**-ring and $\mu_{\mathcal{A}}$ be a L- fuzzy subset of $\mathcal{R}$. we define

$\mu_{\mathcal{A}}(0)= \mu_{\mathcal{A}}(2)=0.7$, $\mu_{\mathcal{A}}(1)= \mu_{\mathcal{A}}(3)= \mu_{\mathcal{A}}(4)=0.6$

Then $\mu_{\mathcal{A}}$is a L- fuzzy bi-ideal of $\mathcal{R}$.

Lemma:3.4

Every L- fuzzy left (resp.right,two sided) ideal of an $\boldsymbol{\mathcal{L}}$**A**-ring $\mathcal{R}$ is a L- fuzzy bi-ideal of $\mathcal{R}$.But the converse is not true in general.

Proof:

Direct .

Proposition: 3.5

Let $\mathcal{R}$ be an $\boldsymbol{\mathcal{L}}$**A**-ring satisfying the property $a = a^2$ for every$a \in \mathcal{R}$. Then every L- fuzzy (1,2) ideal of $\mathcal{R}$ is a L- fuzzy bi-ideal of $\mathcal{R}$.

Proof:

Suppose that, $\mu_{\mathcal{A}}$ is a L-fuzzy (1,2)-ideal of $\mathcal{R}$ and$a, x, y \in \mathcal{R}$. Thus,

$$\begin{aligned} \mu_{\mathcal{A}}\big((xa)y\big) &= \mu_{\mathcal{A}}\big((xa)(yy)\big) \\ &\geq \mu_{\mathcal{A}}(x) \wedge \mu_{\mathcal{A}}(y) \wedge \mu_{\mathcal{A}}(y) \\ &\geq \mu_{\mathcal{A}}(x) \wedge \mu_{\mathcal{A}}(y) \end{aligned}$$

Therefore, $\mu_{\mathcal{A}}$ is a L- fuzzy bi-ideal of $\mathcal{R}$.

Lemma:3.6

Every L- fuzzy bi-ideal of an $\boldsymbol{\mathcal{L}}$**A**-ring $\mathcal{R}$ is a L- fuzzy (1,2) of $\mathcal{R}$.But the converse is not true in general.

Proof:

Straight forward.

IV. Characterization of $\boldsymbol{\mathcal{L}}$A-rings:

An $\boldsymbol{\mathcal{L}}$**A**-ring $\mathcal{R}$ is a left(resp. right) regular, if for every element$x \in \mathcal{R}$, there exists an element $a \in \mathcal{R}$ such that$x = ax^2 (resp.\, x^2 a)$.

An $\boldsymbol{\mathcal{L}}$**A**-ring $\mathcal{R}$ is completely regular if it is regular, left regular and right regular.

An $\boldsymbol{\mathcal{L}}$**A**-ring $\mathcal{R}$ is a (2,2) regular if for every element$x \in \mathcal{R}$, there exists an element $a \in \mathcal{R}$such that$x = (x^2 a)x^2$.

An $\boldsymbol{\mathcal{L}}$**A**-ring $\mathcal{R}$ is a locally associative $\boldsymbol{\mathcal{L}}$**A**-ring if $(a.a).a = a.(a.a)$ for all $a \in \mathcal{R}$.

Proposition:4.1

Let $\mathcal{R}$ be a regular $\boldsymbol{\mathcal{L}}$**A**-ring having the property$a = a^2$ for every$a \in \mathcal{R}$, with left identity e. Then every L- fuzzy generalized bi-ideal of $\mathcal{R}$is a L- fuzzy bi-ideal of $\mathcal{R}$.

Proof:

Let $\mu_{\mathcal{A}}$ be a L- fuzzy generalized bi-ideal of $\mathcal{R}$ and$x, y \in \mathcal{R}$, this implies that there exists,$a \in \mathcal{R}$ such that $x = (xa)x$.We have to show that, $\mu_{\mathcal{A}}$ is aL- fuzzy $\boldsymbol{\mathcal{L}}$**A**-subring of $\mathcal{R}$.Thus,

$$\mu_{\mathcal{A}}(xy) = \mu_{\mathcal{A}}\left(((xa)x)y\right) = \mu_{\mathcal{A}}\left(((xa)x^2)y\right) = \mu_{\mathcal{A}}(((xa)(xx)y)$$
$$= \mu_{\mathcal{A}}((x((xa)x))y) \geq \mu_{\mathcal{A}}(x) \wedge \mu_{\mathcal{A}}(y)$$

Hence $\mu_{\mathcal{A}}$ is a L- fuzzy $\boldsymbol{\mathcal{L}}$**A**-subring of $\mathcal{R}$.

Proposition:4.2

Let $\mathcal{R}$ be a regular locally associative $\boldsymbol{\mathcal{L}}$**A**-ring having the property$a = a^2$ for every $a \in \mathcal{R}$. The for every L-fuzzy bi-ideal $\mu_{\mathcal{A}}$of $\mathcal{R}$, $\mu_{\mathcal{A}}(a^n) = \mu_{\mathcal{A}}(a^{2n})$for all$a \in \mathcal{R}$,where n is any positive integer.

Proof:

For $n = 1$.Let $a \in \mathcal{R}$, this implies that there exists an element $x \in \mathcal{R}$,such that $a = (ax)a$.

Now, $a = (ax)a = (a^2x)a^2$, because $a = a^2$. Thus

$$\mu_{\mathcal{A}}(a) = \mu_{\mathcal{A}}((a^2x)a^2)$$
$$\geq \mu_{\mathcal{A}}(a^2) \wedge \mu_{\mathcal{A}}(a^2) = \mu_{\mathcal{A}}(a^2)$$
$$=\mu_{\mathcal{A}}(aa) \geq \mu_{\mathcal{A}}(a) \wedge \mu_{\mathcal{A}}(a) = \mu_{\mathcal{A}}(a)$$
$$\Rightarrow \mu_{\mathcal{A}}(a) = \mu_{\mathcal{A}}(a^2).$$

Now, $a^2 = aa = \left((a^2x)a^2\right)\left((a^2x)a^2\right) = (a^4x^2)a^4$, then the result is true for $n = 2$.

Suppose that the result is true for $= k$,

i.e., $\mu_{\mathcal{A}}(a^k) = \mu_{\mathcal{A}}(a^{2k})$.

Now,$a^{k+1} = a^k a = \left((a^{2k}x^k)a^{2k}\right)\left((a^2x)a^2\right) = \left(a^{2(k+1)}x^{k+1}\right)a^{2(k+1)}$.

Thus,

Now, $\mu_{\mathcal{A}}(a^{k+1}) = \mu_{\mathcal{A}}(a^{2(k+1)}x^{k+1})a^{2(k+1).}$

$$\geq \mu_{\mathcal{A}}\left(a^{2(k+1)}\right) \wedge \mu_{\mathcal{A}}\left(a^{2(k+1)}\right)$$
$$= \mu_{\mathcal{A}}\left(a^{2(k+1)}\right)$$
$$=\mu_{\mathcal{A}}(a^{k+1}a^{k+1})$$
$$\geq \mu_{\mathcal{A}}(a^{k+1}) \wedge \mu_{\mathcal{A}}(a^{k+1})$$
$$= \mu_{\mathcal{A}}(a^{k+1})$$
$$\Rightarrow \mu_{\mathcal{A}}(a^{k+1}) = \mu_{\mathcal{A}}\left(a^{2(k+1})\right.$$

Hence by induction method, the result is true for all positive integers.

Proposition:4.3

Every L-fuzzy generalized bi-ideal of (2,2)-regular $\boldsymbol{\mathcal{L}}$**A**-ring $\mathcal{R}$with left identity e, is a L-fuzzy bi-ideal of $\mathcal{R}$.

Proof:

Suppose that$\mu_{\mathcal{A}}$ is a L-fuzzy generalized bi-ideal of $\mathcal{R}$ and $x, y \in \mathcal{R}$,this an means that there exists an element $a \in \mathcal{R}$such that$x = (x^2a)x^2$. We have to show that $\mu_{\mathcal{A}}$is a L-fuzzy LA-subring of $\mathcal{R}$. Thus

$$\mu_{\mathcal{A}}(xy) = \mu_{\mathcal{A}}\left(((x^2a)x^2)y\right) = \mu_{\mathcal{A}}\left(((x^2a)(xx))y\right)$$
$$=\mu_{\mathcal{A}}((x(x^2a)x)y) \geq \mu_{\mathcal{A}}(x) \wedge \mu_{\mathcal{A}}(y)$$

Therefore $\mu_{\mathcal{A}}$is a L-fuzzy $\boldsymbol{\mathcal{L}}$**A**-subring of $\mathcal{R}$.

Proposition:4.4

Let $\mathcal{R}$ be a (2,2)- regular locally associative $\boldsymbol{\mathcal{L}}$**A**-ring. Then for every L-fuzzy bi-ideal $\mu_{\mathcal{A}}$of $\mathcal{R}$, $\mu_{\mathcal{A}}(a^n) = \mu_{\mathcal{A}}(a^{2n})$for all $a \in \mathcal{R}$,where n is any positive integer.

Proof:

Similar to Proposition 4.2

Proposition:4.5

Every L-fuzzy generalized bi-ideal of a right regular $\mathcal{LA}$-ring $\mathcal{R}$with left identity e, is a L-fuzzy bi-ideal of $\mathcal{R}$.

Proof:

Suppose that $\mu_{\mathcal{A}}$is a L-fuzzy generalized bi-ideal of $\mathcal{R}$and$x, y \in \mathcal{R}$,this means that there exists an element$a \in \mathcal{R}$ such that $x = x^2a$. We have to show that $\mu_{\mathcal{A}}$is a L-fuzzy $\mathcal{LA}$-subring of $\mathcal{R}$. Thus

$$\mu_{\mathcal{A}}(xy) = \mu_{\mathcal{A}}((x^2a)y) = \mu_{\mathcal{A}}\left(((xx)(ea))y\right)$$

$$=\mu_{\mathcal{A}}((((ae)(xx))y) \geq \mu_{\mathcal{A}}(x) \wedge \mu_{\mathcal{A}}(y)$$

Therefore $\mu_{\mathcal{A}}$is a L-fuzzy $\mathcal{LA}$-subring of $\mathcal{R}$.

Proposition:4.6

Every L-fuzzy generalized bi-ideal of a left regular $\mathcal{LA}$-ring $\mathcal{R}$ with left identity e, is a L-fuzzy bi-ideal of $\mathcal{R}$.

Proof:

Let$\mu_{\mathcal{A}}$be a L-fuzzy generalized bi-ideal of $\mathcal{R}$and $x, y \in \mathcal{R}$,this implies that there exists $a \in \mathcal{R}$such that$x = ax^2$. We have to show that $\mu_{\mathcal{A}}$is a L-fuzzy $\mathcal{LA}$-subring of $\mathcal{R}$. Thus

$$\mu_{\mathcal{A}}(xy) = \mu_{\mathcal{A}}((ax^2)y) = \mu_{\mathcal{A}}\left((a(xx))y\right)$$

$$=\mu_{\mathcal{A}}((x(ax))y) \geq \mu_{\mathcal{A}}(x) \wedge \mu_{\mathcal{A}}(y)$$

Hence$\mu_{\mathcal{A}}$is a L-fuzzy $\mathcal{LA}$-subring of $\mathcal{R}$.

Proposition:4.7

Let $\mathcal{R}$ be a regular and regular locally associative $\mathcal{LA}$-ring. Then for every L-fuzzy right ideal $\mu_{\mathcal{A}}$of $\mathcal{R}$, $\mu_{\mathcal{A}}(a^n) = \mu_{\mathcal{A}}(a^{3n})$for all $a \in \mathcal{R}$,where n is any positive integer.

Proof:

For $n = 1$. Let $a \in \mathcal{R}$,this implies that there exists an element$x \in \mathcal{R}$such that $a = (ax)a$ and$a = a^2x$.

Now, $a = (ax)a = (ax)(a^2x) = a^3x^2$. Thus

$$\mu_{\mathcal{A}}(a) = \mu_{\mathcal{A}}(a^3x^2) \geq \mu_{\mathcal{A}}(a^3) = \mu_{\mathcal{A}}(aa^2) \geq \mu_{\mathcal{A}}(a) \wedge \mu_{\mathcal{A}}(a^2)$$

$$\geq \mu_{\mathcal{A}}(a) \wedge \mu_{\mathcal{A}}(a) \wedge \mu_{\mathcal{A}}(a)=\mu_{\mathcal{A}}(a)$$

$$\Longrightarrow \mu_{\mathcal{A}}(a) = \mu_{\mathcal{A}}(a^3).$$

Here, $a^2 = aa = (a^3x^2)(a^3x^2) = a^6x^4$, then the result is true for $n = 2$.

Assume that the result is true for $n = k$,

ie, $\mu_{\mathcal{A}}(a^k) = \mu_{\mathcal{A}}(a^{3k})$.

Now, $a^{k+1} = a^ka = (a^{3k}x^{2k})(a^3x^2) = (a^{3(k+1)}x^{2(k+1)}$. Thus

$$\mu_{\mathcal{A}}(a^{k+1}) = \mu_{\mathcal{A}}(a^{3(k+1}x^{2(k+1)}) \geq \mu_{\mathcal{A}}(a^{3(k+1)}) = \mu_{\mathcal{A}}(a^{3k+3}).$$

$$= \mu_{\mathcal{A}}(a^{k+1}a^{2k+2})$$

$$\geq \mu_{\mathcal{A}}(a^{k+1}) \wedge \mu_{\mathcal{A}}(a^{2k+2})$$

$$\geq \mu_{\mathcal{A}}(a^{k+1}) \wedge \mu_{\mathcal{A}}(a^{k+1}) \wedge \mu_{\mathcal{A}}(a^{k+1})$$

$$=\mu_{\mathcal{A}}(a^{k+1})$$

$$\Rightarrow \mu_{\mathcal{A}}(a^{k+1}) = \mu_{\mathcal{A}}(a^{3(k+1)}).$$

Hence by induction method, the result is true for all positive integers.

Author Contribution Statement

R. Sumathi: Conceptualization, Methodology, Formal analysis, Writing—original draft preparation. **Sathappan KE:** Validation, Investigation, Writing—review and editing. **ParveenBanu M**: Literature review, Visualization, Data curation. **Kalaiselvan S**: Supervision, Resources, Project administration, Writing—review and editing. All authors have read and approved the final manuscript.

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Ethics approval and consent to participate

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Consent for publication

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Conflicts of Interest

The authors declare no competing interests**.**

References:

[1] Atanassov K.T, Intuitionistic fuzzy sets,Fuzzy sets and Systems 20,(1986),87-96.
[2] Datta S.K, "On Bi-Ideals and Fuzzy Bi-Ideals of Rings", M.Phil. Thesis, Under The Guidance of Professor Tapandatta, Department of Pure Mathematics, University OfCalcutta (1997).
[3] GarretBirkhof: Lattice Theory, American Mathematical Society Colloquium Publications,Volume XXV.
[4] GoguenJ.A L-Fuzzy Sets J.Math.Anal.Appl.18,145-174 (1967).
[5] Kamran M.S, Conditions For LA-Semigroup To Resemble Associative Structures, Ph.D Thesis, Quaid-I- Azam University, Islamabad 1993.
[6] Kim K.H, Lee J.G "On intuitionistic fuzzy bi-ideals of semigroup"Tark J Math 29(2005),201-210.
[7] Kuroki,N., "On fuzzy ideals and fuzzy bi-ideals in Semigroups",Fuzzy sets and systems 5(1981),203-215.
[8] Kuroki.N, Regular Fuzzy Duo Rings, Inform.Sci., 94 (1996) 119-139.
[9] Lajos.S, Szasz.F, Bi-ideals in associative rings, ActaSci. Math., 32(1971) 185-193.
[10] Liu W.J, Fuzzy Invariant Subgroups and Ideals, Fuzzy Sets and Systems, 8 (1982) 133-139.
[11] NasreenKausar "Fuzzy Bi-Ideals in $\mathcal{LA}$-Rings" Italian Journal of Pure and Applied Mathematics- N. 44-2020 (745-763).
[12] Protic P.V and Stevanovic.N, AG-Test and Some General Properties of Abel Grossmann's Groupoids,Pure Math, Appli., 6 (1995) 371-383.
[13] Shah.TandRehman.I, On $\mathcal{LA}$-Rings Of Finitely Non-Zero Functions, Int.J.Contempt.Math. Sci.,5 (2010) 209-222.
[14] Thomas K.V and LathaNair.S, Intuitionistic fuzzy sublattice and ideals, Fuzzy information and Engg;3(2011)321-331.
[15] Yuan,B. And Wu,W. "Fuzzy Ideals On A Distributive Lattice", Fuzzy Sets And Systems 35(1990) 231-240 .
[16] Zadeh L.A, Fuzzy Sets, Information and Control, 8 (1965), 338–353.