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Review Paper

Digit position value and scale: A teaching approach for 4th and 6th grade of elementary school

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ABSTRACT: *This paper presents two teaching approaches, those of digit place value and scale, for 4th and 6th grades of primary school using RhodeScript Theory tools such as problem-position construction, realistic mathematics, representations, ICT use and interdisciplinarity. More specifically, this paper aims to present the application of the above mathematical tools and their use in teaching practice, specifically in the assimilation of the above concepts, in order to improve students' performance, increase their participation, their interest and enhance collaboration among them. Moreover, the objectives that are attempted to be fulfilled through this project are to make the learners and teachers realize that learning the steps of solving exercises and problems related to the concepts of digit position value and scale can be achieved through experiential and interdisciplinary approaches, which will make the students solve them in a more constructive and profitable way.*

Keywords: *RhodeScript theory, digit position value, scale, 4th and 6th grade grades*

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I.INTRODUCTION

International assessment programs show poor student performance in mathematics in many countries around the world with different educational systems, including Greece. Therefore, many researchers worldwide have studied students' difficulties in mathematics and have proposed methods to reduce them. In particular, the findings of the researches show the critical aspects of how mathematics is taught (Chen & Li, 2009; Rønning, 2013? Howe et al., 2015; Vlachou & Avgerinos, 2018; Psaras et al., 2020; Psaras, 2020), the use of visual representations when teaching mathematical concepts (Shahbari & Peled, 2015; Dreher & Kuntze, 2015; Vlachou & Avgerinos, 2016, 2019), and the attitudes and beliefs of teachers and future teachers towards mathematics (Avgerinos & Vlachou, 2013; Şahin, Gökkurt & Soylu, 2016; Thanheiser et al., 2016; Whitacre & Nickerson, 2016).

Considering the above, this paper presents the didactic approach to two mathematical concepts, digit place value and scale, for grades 4 and 6 of primary school using mathematical tools of RhodeScript Theory, such as problem place construction, realistic mathematics, visual representations, ICT and interdisciplinarity. The aim of the research is to present the application of the above mathematical tools and their use in teaching practice, specifically in the assimilation of the above concepts, in order to improve students' performance, increase their participation, interest and enhance collaboration among them.

II.THEORETICAL FRAMEWORK

The difficulties of pupils

The subjects that were studied are Geometry in grade 6 and Numbers in grade 4, specifically in the chapters "59. Extrapolate-Micronize shapes" and "36. Know numbers up to 100,000" respectively. Based on the literature review conducted and especially the teachers' guide, the following difficulties faced by students in the chapters studied emerged. In particular, difficulties were recorded in relation to the following:

Geometric Transformations: Spatial and geometric concepts, although considered quite simple due to their supervisory approach and everyday experience, are accompanied by misconceptions that do not allow for the meaningful and in-depth understanding that is necessary both for the development of more formal geometric concepts and for dealing with problems of everyday life. Research shows that although pupils do not have difficulty in (holistic) recognition of characteristic shapes such as squares, triangles, etc., they remain, without an appropriate teaching approach, at this level regardless of their age. They perceive shapes holistically and fail to describe their elements and properties or approach them in a general way (Clements & Sarama, 2020). It is also known that, initial ease of recognition decreases when shapes are not familiar or do not have a specific shape, orientation or size, and more generally when they do not correspond to stereotypical shapes, such as the shapes below (Jones & Tzekaki, 2016).

Natural numbers: Despite the fact that natural numbers occupy a large part of the compulsory education curriculum, relevant research continues to document students' difficulties in developing their sense of natural number. One of the most prominent, and relevant to the present research, is the difficulty in understanding the value of digits (Mcguire & Kinzie, 2013; Chan, Au and Tang, 2013; Moeller, 2011). Several students are unable to clearly manage the institutional value of digits, even at the beginning of their high school years, which may make it difficult for them to perform operations with natural numbers. The relevant literature suggests the use of activities that highlight structural and semantic elements of the decimal system of writing and reading numbers from the very first grades of primary school. Difficulties have also been identified with regard to the structural properties of natural numbers. In order to be able to appreciate the structural properties of numbers, the pupil must be able to perceive numbers as self-existent entities, independent of the context in which they appear. Research shows that the first steps in this direction can be identified from the age of 6 years. However, generalization of these properties is not expected before the age of 9 years. By the end of their time in primary school, pupils are expected to be able to at least recognize and accept the above characteristics (Potari, 2016).

The RhodeScript Theory

The instructional approach for interpreting and utilizing the scale in grade 6 and identifying digit position in six-digit numbers as well as linking verbal and symbolic number writing in grade 4 presented in this paper is based on the use of mathematical tools of RhodeScript Theory, as researched and applied by Avgerinos et al. (2018). RhodeScript Theory is a theory structured on eleven basic mathematical practices, that is, on 10+1 tools. The teaching framework was named "RhodeScript", a word derived from the initial letters of the names of the mathematical tools in English:

1. Representations
2. History of mathematics
3. Open problems
4. Breach of Didactical contract
5. Estimation and mental computation
6. Spatial ability and geometric transformations
7. Counterexamples
8. Realistic Mathematics Education
9. Interdisciplinarity
10. Problem posing

+1. Technology

The RhodeScript Theory aims to enhance mathematical literacy by incorporating a variety of practices, methods, and tools that encourage students to understand mathematical concepts in different ways within contexts that are meaningful to them. This enables students to engage in knowledge discovery processes by externalizing and exchanging multiple problem-solving strategies.

Specifically, in the teaching interventions of this article, Representations were utilized as a tool. This tool serves as a means of visualizing and modeling mathematical concepts to promote conceptual understanding and facilitate the construction of mathematical knowledge. Representations make mathematical ideas more concrete and accessible for reflection, helping students to articulate their thoughts and understand complex concepts (Coulombe & Berenson, 2001). There are two types of representations: external and internal (Lesh, Post & Behr, 1987). In this paper, we will focus only on external representations (symbols, shapes, diagrams, texts), which involve the live representation and visualization of mathematical concepts in real-time, making them more understandable and accessible to students. Another tool used is Problem Construction, which involves creating new problems and questions to explore a given situation by the students themselves. This approach is linked to creativity and has a positive impact on students' ability to solve word problems, aiming for a multifaceted and deeper analysis and understanding of the cognitive aspects involved (Silver, 1994).

Furthermore, Interdisciplinarity is an important tool that connects content from different disciplines—in this case, mathematics and geography. Combining one or more fields is particularly beneficial in the educational process as it breaks down the perception of school knowledge as distinct subjects, instead treating it as an integrated whole (Matsagouras, 2002). Realistic Mathematics is another mathematical tool used in this study. This approach, conceptualized by Freudenthal (1973, 1983), views mathematics as a human activity that should be connected to reality, influenced by and influencing society, and accessible to students to the greatest extent possible. Finally, the software Wordwall was utilized, providing the ability to create interactive activities and thereby integrating Information and Communication Technologies (ICT) into the teaching process.

III.THE RESEARCH

Purpose of the Research

This study, in light of the difficulties faced by students with geometric transformations—particularly the scaling of shapes and understanding place value in six-digit numbers—represents an attempt to approach these concepts through the use of the mathematical tools of the RhodeScript Theory. The aim is to reduce students' difficulties in these areas. Specifically, the objectives of the study are to investigate:

- To what extent the application of the RhodeScript Theory in teaching shape transformations can reduce the difficulty for sixth-grade students in understanding the concept of scale.
- To what extent the application of the RhodeScript Theory in teaching whole numbers can reduce difficulties for fourth-grade students in understanding place value in six-digit numbers.

Research Methodology

Sample: The research population consisted of 35 fourth- and sixth-grade students from the 12th Elementary School of Rhodes in Greece.

Research Tools: To achieve the research objectives, teaching interventions totaling four instructional hours were conducted in April 2023. The Lesson Study method (Lewis, C. C., Perry, R. R., & Hurd, J., 2009) was used for observing the teaching interventions, and evaluation sheets were prepared by the researchers and distributed to students after the interventions.

IV.THE TEACHING INTERVENTION

In this section, four of the activities implemented in two teaching interventions—each lasting two hours and conducted with sixth and fourth-grade students—are presented. These interventions aimed to improve students' performance in problem-solving using the mathematical tools of the RhodeScript Theory, as researched and applied by Avgerinos et al. (2018). The teaching proposal is structured into the following eight phases: pre-assessment, motivation, discovery, reinforcement, extension, evaluation, organization of activities, and student activities.

Activities by Grade Level

Sixth Grade

Activity 1 (Extension Phase): Mapping

- Mathematical Tools of RhodeScript Theory: problem construction, realistic mathematics.
- Activity Details: Duration of 10 minutes, individual work.
- Student Activity Description – Teaching Management Suggestion: Students are provided with a worksheet featuring the following map and a table with necessary elements. They are then asked to create as many problems as possible based on the provided information.

Map Scale	1:40
Red Line	15 cm
Actual Distance from School to Beach	650 m

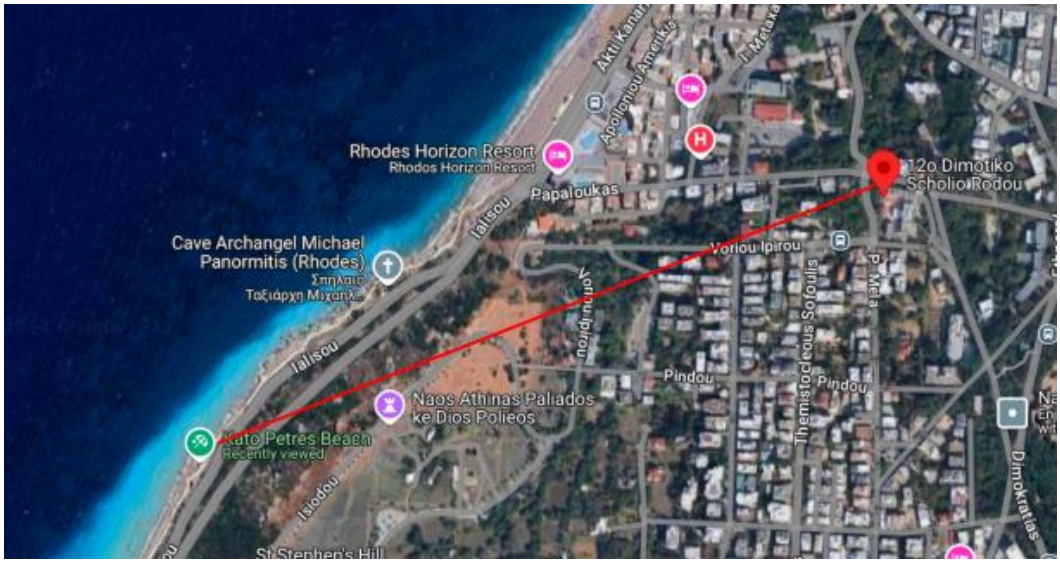


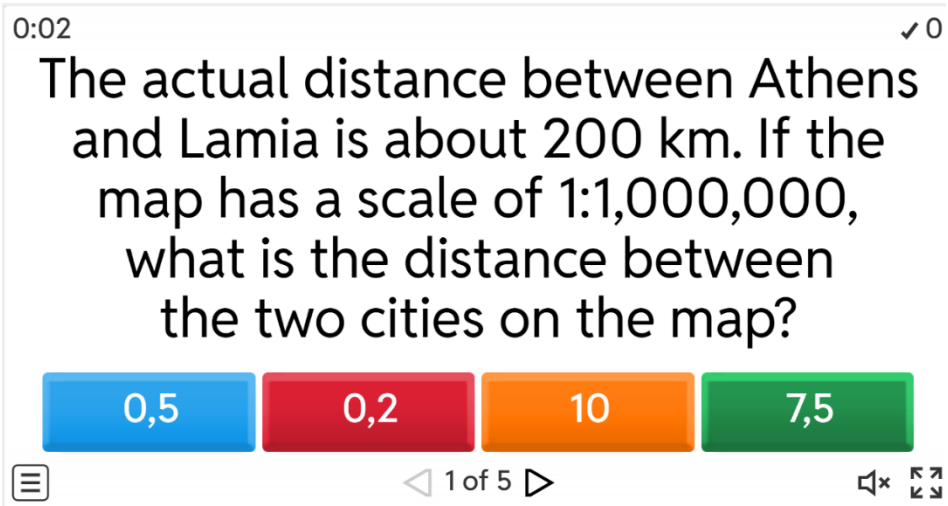
Image 1. Map of Rhodes

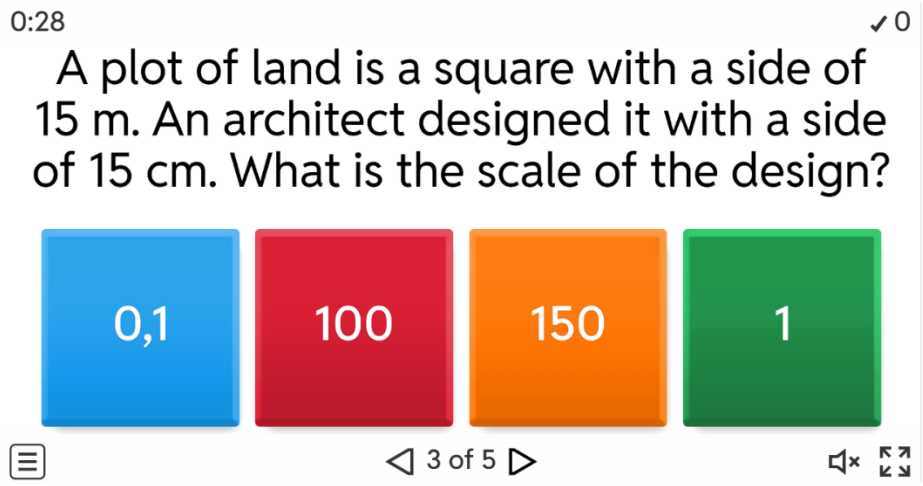
Activity 2 (Evaluation): Let's Play!

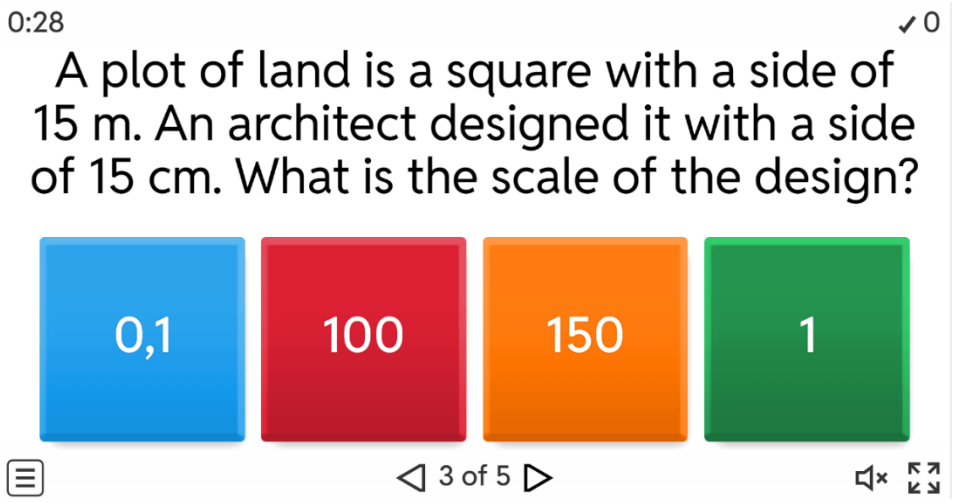
- Mathematical Tools of the RhodeScript Theory: Representations, ICT
- Activity Details: Duration 20 minutes, Group Work Mode
- Description of Student Activity – Teaching Management Proposal: The teacher displays the quiz on the board. The students answer each question individually within a set time, and after discussing it in the plenary session, the correct answer is chosen.

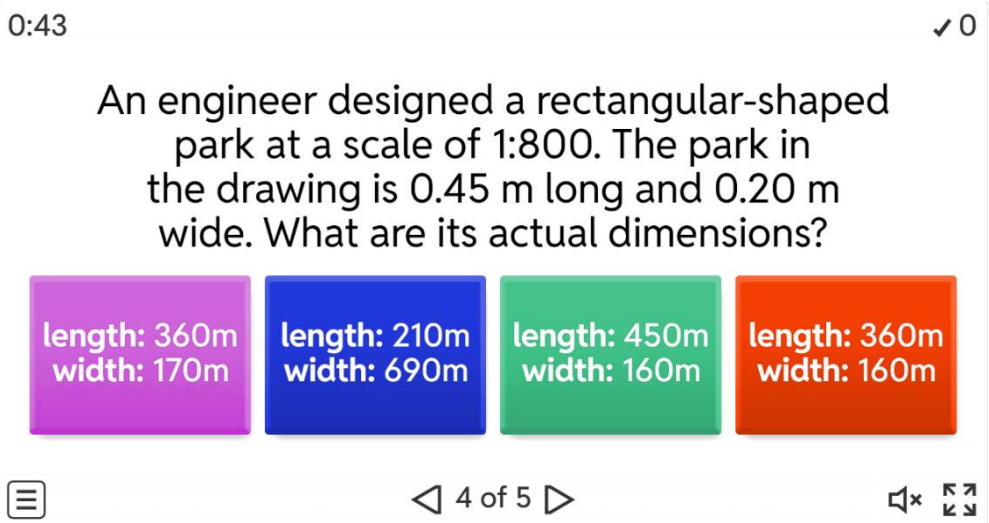
Wordwall link: https://wordwall.net/resource/83776117

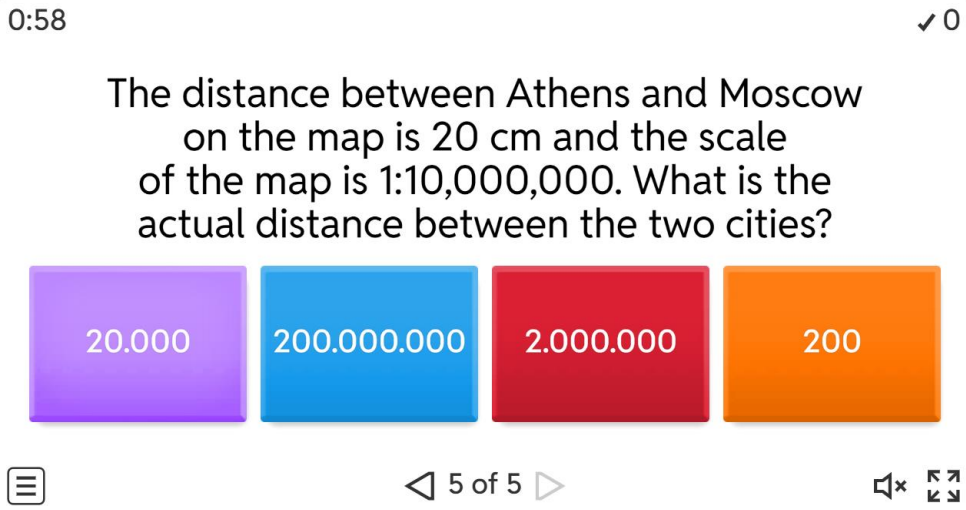
Screenshots of all the questions of the Wordwall quiz "The scale of a map":




0:28
✓0
A plot of land is a square with a side of 15 m. An architect designed it with a side of 15 cm. What is the scale of the design?
0,1
100
150
1
3 of 5


0:28
✓0
A plot of land is a square with a side of 15 m. An architect designed it with a side of 15 cm. What is the scale of the design?
0,1
100
150
1
3 of 5


0:43
✓0
An engineer designed a rectangular-shaped park at a scale of 1:800. The park in the drawing is 0.45 m long and 0.20 m wide. What are its actual dimensions?
length: 360m width: 170m
length: 210m width: 690m
length: 450m width: 160m
length: 360m width: 160m
4 of 5



Fourth Grade

Activity 3 (Discovery): We know Greece

- Mathematical tools of RhodeScript Theory: Realistic Mathematics, Interdisciplinarity, Representations
- Activity Elements: Duration 20 minutes, Mode of Work: individual

- Student Activity Description - Instructional Management Suggestion: give students the following map of Greece (on which the populations of specific cities have been marked based on data from the 2021 population census) and after a quick look, ask what the orange numbers under each city name might be. Through the discussion, the term "population census" and its meaning is mentioned. Then the students individually solve the first subquestion, in which a connection to geography is made because we ask about county capitals. After discussing the answers, students individually solve the second subquestion.

1) With the help of the map above, fill in the table:

County	Capital	Population in numbers and letters	
Chania		**111.375**	
	Alexandroupoli		
Ioannina			
	Volos		**One hundred thirty-nine thousand six hundred seventy**
Thessaloniki		**319.045**	

2) Answer the following questions based on the map above:

i. According to the following table, is Volos or Chania more populated?

……..ii. Which cities have approximately the same population?

……..iii. Write the populations of the cities in descending order:
……..
iv. Which city has a population larger than Volos and smaller than Athens?
……..
v. Write one city that has a population greater than 100,000 and one with a smaller population:
……..

Activity 4 (Engagement): Solve the riddle

- RhodeScript Theory mathematical tools: representations.
- Activity Elements: Duration 8 minutes, Working method: individual
- Description of student activity - Suggested teaching management:
- We give students the following problem where they have to solve the riddle while practicing their knowledge of digit position value.

As Helen was walking through the forest, she noticed something shining. She approached closer and saw a chest filled with gold coins, but it was locked. To find the six-digit code that unlocks the chest, you need to solve the following riddle. Can you help her?

The riddle:

(a) In the position of the **hundreds of thousands**, the number is 3.

(b) The digit of the **tens of thousands** is three times greater than the digit of the **hundreds of thousands.**

(c) The digit of the **thousands** is the difference between the digit of the **tens of thousands** and the digit of the **hundreds of thousands**.

(d) The digit of the **tens of thousands** is the smallest odd number.

(e) The digit of the **hundreds** is double the digit of the **tens of thousands.**

(f) The digit of the **ones** is the sum of the digits of the hundreds and the hundreds of thousands.

The code:

_ _ _ _ _ _

Image 4. The riddle

V.RESULTS

During the course of the teaching intervention in grade 6 and specifically in the extension phase, with the implementation of task 1, which concerned the concept of scale using problem construction and realistic

mathematics, it was observed that the majority of students successfully used all the data on the board and with the help of the map constructed problems with different questions. In the evaluation phase with the application of project 2, a digital quiz was used that addressed the same concept using ICT and representations. It was observed that students actively participated and showed interest due to the fact that the learning was in the form of a game. The activity developed fair play and a sense of cooperation among the students. First, groups of four were formed and after solving each problem individually, they came to the correct result as a group. Then, the representative of each group announced the solution to the rest of the class, and in case of a mistake, the students of the group that found the correct answer solved the exercise in detail on the blackboard.

During the course of the instructional intervention in grade 4, and specifically in the discovery phase with the application of Task 3, the concept of digit place value in six-digit numbers was analyzed using realistic mathematics, interdisciplinarity, and representations. More specifically, it was observed that the majority of students did not have difficulty in grasping the concept, but those who did have difficulty were able to master the knowledge through the exercises and clarification of their questions. Furthermore, students who were lagging behind in Mathematics were given the opportunity to showcase their knowledge in Geography, boosting their confidence and increasing their participation. In the last phase, that of consolidation, with the application of task 4, which concerned the same concept, using representations, a particular interest and participation was observed by all the children in solving the puzzle, without encountering any difficulty.

VI.DISCUSSION-CONFERENCES

In this paper, a teaching proposal for teaching two concepts to 4th and 6th grade primary school students using RhodeScript Theory mathematical tools such as problem-posing construction, realistic mathematics, representations, ICT use and interdisciplinarity was presented in order to increase their participation, interest and enhance collaboration among them.

The results recorded after observing the implementation of projects 1 and 2 in grade 6 are encouraging in terms of the use of problem construction, realistic mathematics, representations and New Technologies, such as digital quizzes, for understanding scale in mathematics, as a large proportion of students successfully completed the activities in a context of collaboration, fair competition and play. Along the same lines were the results of the implementation of projects 3 and 4 where through the use of realistic mathematics, interdisciplinarity and representations the concept of digit place value in six-digit numbers was taught. Most of the 4th grade students did not show any particular difficulty in consolidating the mathematical concept and all of them participated in solving the tasks, showing interest and active participation when it came to experiential activities and group work.

It is important to note that, the way of teaching a mathematical concept, although very important, is not the only factor responsible for students' difficulties in the forms of activities mentioned, as textbooks and teachers' knowledge involve a significant reason for students' difficulties in mathematics. The harmonious blending of these three elements of books, teaching, and teacher should therefore be the pillar that will support the effort to reduce students' difficulties in understanding the value of digit place value in six-digit numbers and the concept of scale.

Considering the above, we conclude that the utilization of RhodeScript theory in the teaching of mathematics in elementary school classrooms can not only be applied in a variety of ways and to various concepts of the subject matter, but also has encouraging results in terms of easier understanding by students.

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