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Research Article

An Efficient Algorithm to Find Minimum Extended Dominating Set in Cactus Graphs: Application In Wireless Sensor Networks

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ABSTRACT: *In a graph G and a positive integer k, a vertex set S, a subset of V is referred as a k-extended dominating set of G if for each vertex u of V, either the distance between u and at least one member of S is maximum one or there exist minimum k distinct vertices $v_1, v_2, ...v_k \in S$ such that u is situated exactly two distances from each vertex v_i, $i = 1, 2, ...k$. The minimum number of elements among all minimum k-extended dominating sets of a graph G is termed as the k-extended domination number, denoted by $\gamma^k(G)$. When $k = 2$, the set is known as an extended dominating set, and the corresponding minimum size is the extended domination number. A subset S of $V(G)$ is called an extended connected dominating set if it is an extended dominating set and the subgraph induced by S is connected. In this article, we first study the concept of extended domination number and extended dominating sets specifically for cycle graphs. Using these results, we design an efficient algorithm that finds a minimum extended dominating set for cactus graphs in $O(n)$ time. We also verify that our algorithm gives the correct output and analyze its time complexity. Finally, we show how our results can be applied to solve a real-world problem in a wireless sensor network using the idea of extended dominating sets.*

KEYWORDS: *Domination; Extended domination; Extended connected domination; cactus graph*

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I. INTRODUCTION

Graph theory provides a foundational framework for representing and examining complex systems found in numerous fields, including computer science, biology, and social network analysis. A key concept within this area is the dominating set, which is instrumental in exploring the structure and dynamics of graphs. Dominating sets are useful in real life for improving net- work design, picking the best spots for setting up services or buildings, and making computer programs work more efficiently.

In the graph structure $G = (V, E)$, the node point set/vertex set is V and the link set/edge set is E. We also assume that G is connected and has neither self-loop nor parallel edges. Also, the notation $N_G(w)$ we use to indicate the open neighborhood of the node $w \in V(G)$, that is, $N_G(w) = \{u : (u, w) \in E(G)\}$. In addition, the notation $N_G[w]$ we use to refer to the closed neighborhood of the node point w, where $N_G[w] = N_G(w) \cup \{w\}$. Domination always attracts research persons. In, G, if each node point in $V - D$ is linked to at least one node point of D in V by an edge, then the subset D is known as the dominating set (DS) of G. A vertex $v \in D$ dominates all members of $N_G[v]$. The goal of solving the domination problem is to compute DS D with minimum node points. Claude

Berge [1] first explored the preliminary idea of the $\gamma(G)$. The terms D-number & DS were first mentioned in Ore's research paper [2]. In the year 1998, Haynes et al. [3] published a fundamental book on domination covering around 12000 research papers on domination. When a DS is connected, it is called a connected dominating set (CDS); that is, any two nodes in the D-Set can be connected directly or through intermediate nodes in the DS. The use of CDS as a connected virtual backbone has been widely used for the broadcast process [4], for the search in a reduced space, and for the coverage of points in sensor networks [5]. Due to the promiscuous receiving mode of wireless sensors, when each node in a CDS forwards the packet once, all nodes in the network will receive the packet. Sampathkumar et al. [6] first initiated the term "connected domination number". Lots of domination's variations such as secured domination, perfect domination, connected domination [7], edge domination, extended domination, *k*-hop domination [8, 9, 10], k-tuple domination, weighted domination, paired-domination, independent-domination, roman domination [11], inverse roman domination [12], total domination [13], isolate domination [14], *k*-efficient domination [11], restrained domination [15], independent line-set domination [16] have been briefly discussed in the literature [17, 3, 18, 19]. Slater referred a *k*-HDS as a *k*-basis [10]. Including these, many researchers proposed many characteristics of graphs concerning domination in ([20, 21, 3, 22, 23, 24, 25, 26]).

For a graph G and a positive integer k, a vertex subset S of V is called a k-extended dominating set of G if every vertex u of V satisfies one of the following conditions: the distance between u and at least one member of S is at most one or there are at least k different vertices $v_1, v_2, ...v_k \in S$ such that the distance between u and v_i, $i = 1, 2, ...k$, is two. The k-extended domination number $\gamma^k(G)$ of G is the minimum size over all k-extended dominating sets in G. When $k = 2$, they are called the extended dominating set and the extended domination number of G, respectively. In other words, a DS is called an extended dominating set (EDS) if for every node in the network, it is in the set, it has a neighbor in the set, or it has two 2-hop neighbors in the set. γ^2 is called the minimum extended domination number of graphs. A extended dominating set with minimum cardinality is called a minimum EDS (MEDS, is short), we denote it by the symbol D_{min}.

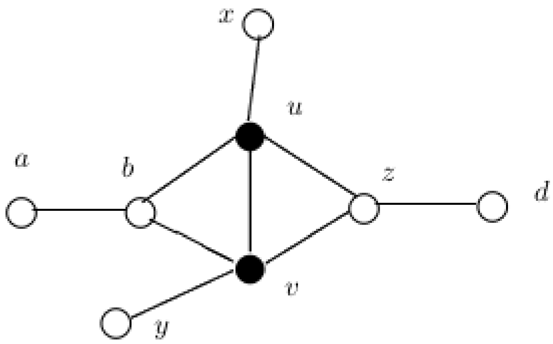

Figure 1: A connected graph.

In Figure 1, $\{u, v, b, z\}$ forms a CDS as well as dominating set, where as $\{u, v\}$ forms an extended dominating set as the nodes a, d are situated in 2 distances from both u and v.

In this article, we focus on the extended dominating sets in graphs. We also focus on designing algorithms for finding a minimum extended dominating set of cycle and cactus graphs.

1.1 Cactus Graphs

Let G=(V,E) be a finite, simple, connected and undirected graph. A vertex u is called a cut-vertex if the removal of u and all edges incident to u disconnects the graph. A connected graph without a cut-vertex is called a non-separable graph. A block of a graph is a maximal non- separable sub graph. In graph theory, a cactus graph is a connected graph in which every block is either an edge or a cycle, in other words, every edge in cactus graph belongs to at most one simple cycle. A block which is an edge, we refer it as an edge-block. Cactus graph were first studied under the name of the Husimi trees, bestowed on them by Frank Harary and George Eugene Unlenbeck in honour of previous work of these graphs by Kodi Husimi. Cactus graphs have been extensively studied and used as models for many real-world problems. This graph is one of the most useful discrete mathematical structures for modelling problems arising in the real-world. An early application of cactus graphs was associated with the representation of op-amps [40, 41, 42]. Also, the applications of cactus graphs can be found in [44, 45]. Recently, cactus graphs have been used in comparative genomics as a way of representing the relationship between different genomes or parts of genomes [43]. This graph is a subclass of planner graph and superclass of tree.

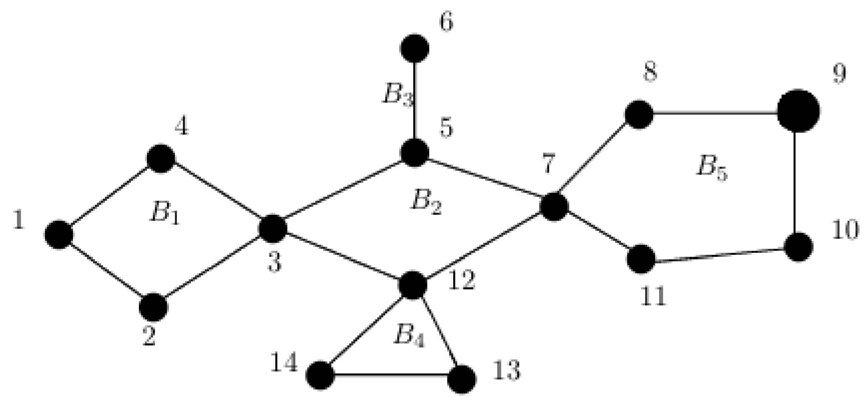

Figure 2: A cactus graph.

Now recall an important properties of cactus graphs given below.

Lemma 1: [39] *For a cactus graph G,* $|E| \le n + c - 1$, *where c is the number of cycles in G.*

Lemma 2: [46] *The time complexity to find the blocks of a connected cactus graph is* $O(n)$ *time.*

Lemma 3: [46] *The time complexity to find the cut vertices of a connected cactus graph is* $O(n)$ *time.*

1.2 Notations:

Here, we present some notations that are used throughout the paper.

Abbreviation/symbol	Meaning
DS	Dominating set
EDS	Extended DS
CDS	Connected DS
D_{min}	Minimum EDS
$N_G(v)$	Open neighbourhood set of v
$N_G[v]$	Closed neighbourhood set of v
d(x, y)	Distance between two vertices x and y

II. Literature review

The idea of Extended Connected Dominating Sets (ECDS) was introduced by Guha et al. [34] in 1998 while studying approximation algorithms for connected dominating sets in wireless networks. Earlier, in 1985, Fink et al. [35] in 1985. introduced the concept of k-extended dominating sets in graphs. Later, in 2002 [36], Wu proposed a routing method based on extended dominating sets for ad hoc wireless networks with unidirectional links. In 2006 [37], Wu and colleagues further explored the use of extended dominating sets in ad hoc networks, focusing on cooperative communication. More recently, in 2023 [38], Gao et al. calculated the exact extended domination numbers for paths and cycles, and also found some bounds for these numbers in planar graphs with small diameters.

III. Applications

Extended dominating sets are instrumental in optimizing complex systems across diverse fields. In communication networks, they enable the efficient placement of critical nodes to enhance message delivery and maintain robustness against node failures. For facility location problems, such as determining optimal sites for hospitals or service centers, extended dominating sets provide strategies that ensure broad and equitable coverage. Within social networks, they assist in pinpointing influential users, supporting applications in targeted outreach and the analysis of information diffusion. Overall, leveraging extended dominating sets contributes to more effective, resilient, and well-structured designs in both technological and social systems.

Main outcome

In this article, we study some new properties on extended dominating set of cactus graphs. We also find the extended domination number of cycles. Then, we present an algorithm for finding the minimum extended dominating set of cactus graphs. We also verify the accuracy of the output of our proposed algorithm. In addition to these, we analyze the compilation time of our designed algorithm. Finally, we present a real application of our studied results.

Structure of this paper

The next section explores the extended domination number of cycles. Section 3 presents an observation on cactus graph and an efficient algorithm for finding a minimum extended connected dominating set in cactus graphs. In this section, we verify the correctness of the proposed algorithm. We also show that the proposed algorithm requires $O(n)$ time. In Section 4, we also present a real-world application of the results discussed in this article. This problem connected to wireless sensor networks. We solve this problem using cactus graph model based on EDS. Finally, we conclude in Section 6.

IV. MINIMUM EXTENDED DOMINATING SET OF CYCLES

Here, we present a minimum extended dominating set of cycles.

Lemma 4: The extended domination number of the cycle C_n is $\lceil n/4 \rceil$, for all n in $\in$ N-{4}.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of a cycle C_n. If $n = 3$, then any vertex can extended dominates other vertices. So, $\gamma^2{}_e = 1 = \lceil n/4 \rceil$ and we can take $D_{min} = \{v_1\}$.

Again if $n = 5, 6, 7, 8$, then, in each case, the vertices $v_1, v_2, \ldots, v_5, v_6$ are extended dominated by v_1 and v_5; the vertices v_7, v_8 are extended dominated by v_5 and v_1. Therefore, $\gamma^2{}_e = 2 = \lceil n/4 \rceil$ and we can take $D_{min} = \{v_1, v_5\}$.

Again if $n = 9, 10, 11, 12$, then, in each case, the vertices $v_1, v_2, \ldots, v_5, v_6$ are extended dominated by v_1 and v_5; the vertices v_7, $v_8 v_9$, v_{10} are extended dominated by v_5 and v_9; the vertices v_{11}, v_{12} are extended dominated by v_9 and v_1. So, $\gamma^2{}_e = 3 = \lceil n/4 \rceil$ and we can take $D_{min} = \{v_1, v_5, v_9\}$. Proceeding in this way, we can show that for cycle C_n, $\gamma^2{}_e = \lceil n/4 \rceil$for all $n \in N\text{-}\{4\}$.

From the above lemma, we can conclude the following results.

Corollary 1: *For any cycle* C_n, $n(\geq 5) \in N$, D_{min} *is*

$\{v_1, v_5, \ldots, v_n\}$, *if* $n = 4m + 1, m \in N$;

$\{v_1, v_5, \ldots, v_{n-1}\}$, if $n = 4m + 2, m \in N$

$\{v_1, v_5, \ldots, v_{n-2}\}$, if $n = 4m + 3, m \in N$

$\{v_1, v_5, ..., v_{n-3}\}$, if $n = 4m + 4, m \in N$.

Note 1: For, C_3, $\gamma^2_e = 1$ and any vertex makes a D_{min}.

Note 2: For, C4, $\gamma^2_e = 2$ and any two vertices make a D_{min}.

V. EXTENDED DOMINATING SET OF CACTUS GRAPHS

First we present an observations on the induced subgraph of cactus graphs.
Let G be a connected cactus graph. Now, if we build up the induced subgraph H of G with its cut vertices, then the following result is true.

Lemma 5: The highest degree of vertices in H is two.

Proof. Let H be the induced subgraph of a connected cactus graph G. So, H is connected. Also, any two blocks always shares only single cut vertex. So, the degree of vertices of H are either 1 or 2. Hence, the result is proved.

To find a minimum extended dominating set of cactus graphs, we apply our studied results for cycles. To accomplish this, we design an algorithm as described below.

Algorithm MIN-EDS-CG

Input: Cactus graph G.

Output: A minimum extended connected dominating set of cactus graph of G.

Step 1: Initially set $D_{min} = \emptyset$.

Step 2: Find all the cut vertices of the cactus graph G and include them in D_{min}. (By Lemma 3)

Step 3: Build the induced subgraph H of G with D_{min}.

Step 4: Find the degree of vertices of H.

Step 5: Find all the cycles of G. (By Lemma 2)

Step 6: Select each cut vertex that lies on a cycle (of length greater than or equal to 4) whose edges are not in H, then find MEDS of that cycle, include the member of MEDS as member of D_{min}, and mark all vertices which are extended dominated by the members of MEDS. (Lemma 4)

Step 7: Select the cut vertices (not in D_{min} and whose degrees are 1 in H) as members of D_{min}, mark all unmarked vertices of G that are extended dominated by the new members of D_{min}.

Step 8: Select each block, one by one, with 3 or less unmarked vertices and whose edges are not in H.
If its cut vertex's (that is in H also) one neighbor in H is marked, then select its other unmarked neighbor as member of D_{min} and marked all unmarked vertices of G that are extended dominated by new selected member of D_{min}.
Else if its both neighbors in H are unmarked, then select these two neighbor as member of D_{min} and marked all unmarked vertices of G that are extended dominated by new selected members of D_{min}.
Else End.

Step 9: If all unmarked vertices of H are adjacent to a marked cut vertex or are situated at two distances from two marked cut vertices, then Stop.
Else Select an unmarked cut vertex (of H which is situated at four distances from any marked cut

vertices) as a member of D_{min} and marked all unmarked vertices that are extended dominated by the new member of D_{min} and go to Step 7.
End

End MIN-EDS-CG.
If we apply the above Algorithm **MIN-EDS-CG** for the cactus graph G of Figure 2, then minimum extended dominating set D_{min} = {3, 4, 7, 11} of G.

5.1 Correctness, time complexity

Lemma 5: *The Algorithm* **MIN-EDS-CG** *gives a minimum extended dominating set of cactus graph G.*

Proof. In Step 6 of the MIN-EDS-CG algorithm, we find MEDS of each cycle (block) and these cycles are distinct (may share only one cut vertex). So, selected members of D_{min} must be needed. Again, in Step 7, we select those those cut vertices as members of D_{min} which are essential. Therefore, updated D_{min} is minimum. In Step 8, we select the members of D_{min} in such a way that they must be needed and they extended dominate maximum number of vertices of G. Thus, the final D_{min} is minimum.

Theory 1: The Algorithm **MIN-EDS-CG** runs in $O(n)$ time to find a minimum extended dominating set of cactus graphs.
Proof. Step 1 requires unit time. In Step 2, all the cut vertices of the cactus graph can be computed in $O(n)$ time, and the number of these vertices is less than n. So, Step 2 requires $O(n)$ time. Again, we know that the number of edges of a cactus graph is $n - 1 + c$, where c is the number of cycles, so Step 3 can build up H in $O(n)$ time. Since the degree of each vertex of H is either 1 or 2, so, Step 4 needs $O(n)$ time. In a cactus graph, all blocks (cycles or edges) can be counted in $O(n)$ time. So, Step 5 requires $O(n)$ time. Since, the cycles of G are distinct (can share only one cut vertex), so, MEDS of each cycle of length (≥ 4) can be computed in $O(n)$ time. So, Step 6 finishes in $O(n)$ time. Again, number of blocks with three or fewer vertices is less than n, so, the updation of D_{min} in Step 8 can be done in $O(n)$ time. Since, the number of vertices of the induced graph H is less than n, Step 9 need $O(n)$ time. Hence, the total time complexity of the algorithm MIN-EDS-CG is $O(n)$ time.

VI. REAL APPLICATION

Here, we present a real-world application of cactus graphs based on the Extended Dominating Set (EDS) arises in the domain of wireless sensor networks (WSNs) and ad hoc communication systems, where network efficiency, fault tolerance, and energy savings are critical.
Wireless sensor networks often form sparse and tree-like or cycle-restricted topologies to sim- plify routing and reduce interference. Again, a cactus graph is an ideal abstraction for such networks where cycles exist but are limited in structure (each edge belongs to at most one cy- cle). To ensure reliable message transmission and minimal energy consumption, a small number of leader nodes or cluster heads must be chosen to relay messages across the network.
Stepwise representation of applying an Extended Dominating Set (EDS) approach to a cactus graph, especially useful for a wireless sensor network modeled as a cactus-shaped topology
Step 1: Problem set up Consider a wireless sensor networks with cactus-shaped topology, where every sensor node is either: cluster head (in the set), directly connected to a cluster head (1-hop), or indirectly supported by two nodes that can reach a cluster head within 2-hops. This type of network is ideal for real-world scenarios like environmental monitoring or infrastructure surveillance.

Step 2: Model the network as a cactus graph

Represent the sensor or communication network as an undirected cactus graph $G(V, E)$. This graph has a special property: every block is either a simple cycle or a bridge (single edge).

Our aim is to construct an Extended Dominating Set (EDS) such that every node in the cactus graph is either: in the set, or has a 1-hop neighbor in the set, or has two 2-hop neighbors in the set.

Step 3: Decompose the Graph into Blocks

We use Tarjan's algorithm [46] to find blocks (cycles or edges) in O(n + m) time. Store each block as either: a cycle (for local redundancy), or a bridge (edge that connects parts of the network).

Step 4: Initialize the Extended Dominating Set $D_{min} = \phi$.

Step 5: Algorithm selection to find MEDS of cactus graph

We select the MIN-EDS-CG algorithm to MEDS of the cactus graph as it runs in $O(n)$ time. **Step 6: Analysis** Extended dominating set (EDS) improves energy efficiency and robustness in the network discussed in below.

1) Energy Efficiency:

Fewer Active Nodes: EDS selects only a small subset of nodes (dominators) to handle commu- nication tasks like broadcasting or routing.

Reduces Redundant Transmission: Since non-dominator nodes rely on nearby dominators (1-hop or 2-hop), they avoid sending unnecessary messages, saving battery.

Localized Communication: Dominators relay data locally, minimizing long-distance transmis- sions, which are more energy-consuming.

2) Robustness of the network:

2-Hop Redundancy: The EDS ensures that even if a node is not directly connected to a dom- inator, it has two 2-hop neighbors in the set. This provides backup paths if one dominator fails.

Fault Tolerance: If one dominator or its link fails, other nearby dominators maintain network coverage, preventing disconnection.

VII. CONCLUSION

In graph theory, domination concepts are key tools for analyzing the vulnerability and resilience of communication networks, which are commonly represented as graphs. There are several variations of domination, each characterized by the structure and rules of its dominating set. One such variant, the Extended Dominating Set (EDS), stands out for its practical importance. In this article, we explore extended domination number and extended dominating set of cycles.

We also design an $O(n)$ time algorithm to find a minimum extended dominating set of cactus graphs based on our studied results. We also check the correctness of the output of our proposed algorithm. In addition, we analyze the time complexity of our algorithm. Finally, we apply our studied results to solve a wireless sensor network based on EDS.

In future, we have a plan to develop an optimal algorithm for finding a minimum ECDS in more complex network models, such as communication networks, social networks.

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