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Review Paper

Comparison of Complete and Incomplete data Using Fuzzy Estimation Approach

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ABSTRACT: *An approach is proposed to construct fuzzy Estimation of Parameters and confidence intervals in Triangular and Trapezoidal shaped fuzzy numbers. In this paper, a real-time data is also considered to illustrate the use of the fuzzy approach to estimate the fuzzy estimator and the fuzzy confidence interval. In Survival data, identified the importance of the fuzzy approach using Buckley's methods and the fuzzy confidence interval approach.*

KEYWORDS: *Fuzzy sets, Point Estimation, Fuzzy confidence intervals, Survival analysis.*

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I. INTRODUCTION

In recent decades, statistical methods in fuzzy environments have been extensively explored, both theoretically and practically. In many real-world situations, data cannot be precisely measured or collected, making classical statistical methods inadequate. To address this, fuzzy set theory-first introduced by Zadeh in 1965-has emerged as a powerful toll for modelling imprecise or vague data.

One of the fundamental concepts in statistics is the confidence interval, which plays a key role in statistical inference. In fuzzy environments, the concept extends naturally to fuzzy confidence intervals, which account for uncertainty in a more flexible and realistic way. Some authors have investigated the issue through certain approaches. [12] and [13] introduced two classes of fuzzy confidence intervals for the fuzzy process capability indices based on ranking functions. [14] Presented an approach to confidence interval for fuzzy models by combining a fuzzy identification methodology with some methods from applied statistics. The main idea is to determine the confidence interval defined by the lower and upper fuzzy bounds which construct the band that contains all the output measurements. Using extension principle, [15] investigated the problem of confidence interval based on fuzzy data. Besides the above studies, [2] developed a method to estimate an unknown parameter in a statistical model. He used a set of confidence intervals producing a triangular fuzzy number for the estimation of the interested parameter. [5] proposes applying fuzzy set theory to statistical confidence intervals for unknown fuzzy parameters by formulating and solving optimization problems involving fuzzy random variables. In the presence of censored observations, ascertaining the median survival time is always obscure; hence, fuzzy methodology has been adopted ([7]). [1] builds fuzzy estimators like mean, varience, proportions.

The prime familiar nonparametric procedures such as the Kaplan-Meier [8] estimator for calculating the Survival probabilities, [9] proposed a fuzzy product limit estimator that could be utilized even with extensive censoring. They demonstrated that it is more trustworthy than the classical estimator. [6] proposed an approach for a Fuzzy Kaplan-Meier estimator based on fuzzified survival times and compared it to the traditional method. [10] applied Weibull smoothening and fuzzy linear regression approach to estimate survival probabilities that varies over an interval, compared to the Kaplan-Meier's constant probabilities of survival in the intervals.

Usually, for estimating a parameter for the distribution, statistic is used as a point estimator and with the help of this statistic, confidence interval for the parameter is obtained using statistical approach. This way of estimating parameters and its confidence intervals are based on probabilistic approach.

Instead of getting single measure and confidence interval to make an inference about the unknown parameter, the fuzzy approach is developed to obtain fuzzy estimator and and fuzzy confidence interval based on possibilistic approach from a set of confidence intervals. Fuzzy confidence interval benefits from both probabilistic and possibilistic sources of information.

In this paper, some real time data were considered to explain the application of the fuzzy approach for estimating fuzzy estimator and fuzzy confidence interval. Survival data also considered and identified the importance of fuzzy approach using Buckely's Methods and Fuzzy Confidence Interval approach.

This paper is organized as follows. Section 2 is devoted to the theoretical approach to point estimation. In Section 3, explores the fundamental principles and techniques involved in fuzzy estimation, focusing on both theoretical foundations and practical methods. Section 4 delves into survival analysis, exploring Kaplan-Meier (KM) methods and interval estimation techniques to assess time-to-event data and uncertainty in statistical inference. Section 4 examines the implementation of real-time data techniques for both complete and survival data, highlighting practical approaches and challenges in real-world applications. Summarizes the findings of the study; suggestions and future work are given in the final section.

II. THEORETICAL APPROACH TO POINT ESTIMATION

2.1 Point Estimation

In statistics, point estimation and interval estimation are two ways to estimate a population parameter using sample data. Statistic is used to estimate an unknown parameter ϴ of the distribution, it is called an estimator. A particular value of the estimator is called the estimate of ϴ. A single value estimate of a population parameter, such as a sample mean. A point estimate is a "best guess" or "best estimate" of the unknown population parameter. Sample Mean is the best estimate for Population Mean. A point estimate provides the single value as an estimate.

2.2 Interval Estimation and Confidence Interval

2.2.1 Interval estimation

A range of values where the population parameter is expected to lie, taking into account margin of error. The most common type of interval estimate is a confidence interval.

2.2.2 Confidence level

The confidence level of an interval is denoted by $1-\alpha$, where α is a value between 0 and 1. For example, if you want to be 95% confident that the parameter is inside the interval, alpha is 5%.

Let us take, the value of level of significance α (5% or 1%) and determine two constants say, c_1 and c_2 such that:

$p(c_1 < \theta < c_2 \mid t) = 1-\alpha$, where t is the estimate

The Quantities c_1 and c_2 are Known as Confidence limits and the interval $[c_1, c_2]$ within which the unknown value of the Population parameter is expected to lie, is called the confidence interval. $1-\alpha$ is called the confidence coefficient.

III. FOUNDATIONS AND METHODS OF FUZZY ESTIMATION

In 1965, Zadeh[16] created fuzzy sets to manage data and information with non-statistical uncertainty. It was created with the goal of representing uncertainty and ambiguity mathematically and providing defined methods for dealing with the imprecision that is inherent in many issues. Fuzzy systems have two primary properties that improve their effectiveness in particular applications.

- Fuzzy systems are appropriate for uncertain or approximate reasoning, particularly for systems with difficult to derive mathematical models.
- Fuzzy logic enables decision making with estimated values when information is inadequate or uncertain.

3.1 Fuzzy Number

Let X be a Universal set, then a fuzzy subset $\overline{A}$ of n is defined by its membership function, written $\overline{A}(X)$, which produces values in [0,1] for all x in X. So, $\overline{A}(X)$ is a function mapping X into [0,1].

If $\overline{A}(X_0) = 1$, then we say X_0 belongs to $\overline{A}$, if $\overline{A}(X_1) = 0$ we say X_1 does not belong to $\overline{A}$, and if $\overline{A}(X_2) = 0.6$ we say the membership value of X_2 in $\overline{A}$ is 0.6. When $\overline{A}(X)$ is always equal to one or zero we obtain a crisp (non-fuzzy) subset of X.

3.2 α – Cut

In fuzzy set theory, an alpha cut is a method used to transform a fuzzy set into a crisp set with well-defined boundaries. If $\overline{A}$ is a fuzzy subset of some set X, then an α -cut of $\overline{A}$, written $\overline{A}[\alpha]$ is defined as $[\alpha] = \{x \in X \mid \overline{A}(x) \geq \alpha\}$ for $0 \leq \alpha \leq 1$.

3.3 Triangular Shaped Fuzzy

Let R be the set of real numbers. A triangular shaped fuzzy number $\overline{N} \in F(R)$ is a fuzzy subset of R satisfying

(i) $\overline{N}\ (X)=1$ For exactly one x∈ R;

(ii) For α ∈ (0, 1], the α-cut of $\overline{N}$ is a bounded and closed interval denoted by $\overline{N}(\alpha)\ = [n_1(\alpha),\ n_2(\alpha)]$ Where $n_1(.)$ and $n_2(.)$ are the increasing and decreasing continuous functions, respectively.

3.4 Trapezoidal Shaped Fuzzy Number

A Trapezoidal shaped fuzzy number $\overline{M} \in F(\mathbb{R})$ is a fuzzy subset of $\mathbb{R}$ satisfying

(i) $\overline{M}_{(X)} = 1$, for any $x \in [x_1, x_2]$ such that $x_1, x_2 \in \mathbb{R}$ and $x_1 \leq x_2$;

(ii) For $\alpha \in (0,1]$,

The α -cut of $\overline{M}$ is a bounded and closed interval denoted by $\overline{M}(\alpha) = [m_1(\alpha), m_2(\alpha)]$ where $m_1(.)$ and $m_2(.)$ are the increasing and decreasing continuous functions, respectively. Triangular shaped fuzzy numbers is a special case of the trapezoidal shaped fuzzy number.

3.5 Fuzzy Point Estimation – Buckley's Method

Let $X_1, X_2, \ldots, X_n$ random sample with observed values $x_1, x_2, \ldots, x_n$ from a distribution with probability density/mass function. $f(x;\theta)$ Where θ is a single unknown crisp parameter of interest. Construct $100(1-\alpha)\%$ confidence intervals for the crisp parameter θ, $0 \leq \alpha \leq 1$. Such confidence interval denoted by $[\theta_1(\alpha), \theta_2(\alpha)]$.

$[\theta_1(\alpha), \theta_2(\alpha)]$ is a point estimation. Statistical confidence intervals of θ are employed to construct the fuzzy confidence interval. When α increases, the length of $[\theta_1(\alpha), \theta_2(\alpha)]$ decreases, and vice versa. Place the confidence intervals one on top of the other form $\alpha = 0$ to $\alpha = 1$ to construct a triangular shaped fuzzy number $\overline{\theta}$.

Alpha cuts of $\overline{\theta}$ as follows

$\overline{\theta}[0] = \theta$

$\overline{\theta}[\alpha] = [\theta_1(\alpha), \theta_2(\alpha)]$, for $0 < \alpha < 1$

$\overline{\theta}[1] = \theta_1(1) = \theta_2(1)$

Estimate a crisp parameter by constructing a fuzzy confidence interval.

3.6 Fuzzy Confidence Interval Approach (Parchami, 2022[11])

Let $X_1, X_2, \ldots, X_n$ random sample with observed values $x_1, x_2, \ldots, x_n$ from a distribution with probability density/mass function. $f(x;\theta)$, Assume that is an unknown crisp parameter and a fuzzy confidence interval is constructed for θ. A $100(1-\gamma)\%$ fuzzy confidence interval for θ, denoted by $c_\theta^{-100(1-\gamma)\%}$, (as a fuzzy subset of θ) is constructed by the following intervals as its α-cuts:

$C_\theta^{-(100(1-\gamma)\%}[0] = \theta$,

$c_\theta^{-100(1-\gamma)\%}[\alpha] = [\theta_1(\alpha\gamma), \theta_2(\alpha\gamma)]$,for $0 < \alpha < 1$

$c_\theta^{-100(1-\gamma)\%}[1] = [\theta_1(\gamma), \theta_2(\gamma)]$,

Where $[\theta_1(\alpha), \theta_2(\alpha)]$ is the usual $100(1-\alpha)\%$ confidence interval for θ. $c_\theta^{-100(1-\gamma)\%}[1]$ is the crisp $100(1-\gamma)\%$ confidence interval for θ.

Note:

[1]. α is used for α-cut of the fuzzy confidence interval (which is a crisp subset of θ for each $\alpha \in [0,1]$, while the notation γ is used for the confidence level of the fuzzy confidence interval.

[2]. Buckley's point estimate is a particular case of the proposed fuzzy interval estimate when $\gamma = 1$.

[3]. Interpretation of a fuzzy set is based on the possibility. Moreover, the occurrence possibility of an event is defined as the supremum of the possibility of single elements of the set.

$poss(A) = \sup\left\{poss\left(\{x\}\right); x \in A\right\}$.

IV. SURVIVAL ANALYSIS

Survival Analysis is the study about time-to-event data. This study is different from other studies because of Censoring. Survival analysis techniques are mainly concerned with predicting the probability of response, probability of survival, mean life time, and comparing survival functions.

The survival function is conventionally denoted by S(t), which is defined as:

$S(t) = P(T > t)$ For all $t \geq 0$

Where t is some specified time, T is a random variable denotes the time that the event occurs. This function gives the probability that the unit will survive time t or we can say that the event will occur after time t. For the survival function, it is usually assumed that $S(0) = 1$, and $lim_t\left(n \to \infty\right)[S(t) = 0]$.

4.1 Kaplan-Meier Survival Estimate

The Kaplan-Meier (KM) method is a non-parametric method used to estimate the survival probability from observed survival times ([8]).

The Kaplan-Meier estimator is given by:

$$\hat{S}(t) = \prod_{t_i \leq t}\left(1 - \frac{d_i}{n_i}\right)$$

Where d_i s the number of events at time point i, n_i is the number at time point *i* and t_i is the time point with an event or censoring.

4.2 Greenwood's Formula ([4])

Standard error of the Kaplan-Meier estimate of the survival function, defined to be the square root of the estimated variance of the estimate.

$$se\{\hat{S}(t)\} = \hat{S}(t)\left\{\sum_{j=1}^{k}\frac{d_j}{n_j(n_j - d_j)}\right\}^{\frac{1}{2}} \quad \text{for} \quad t_k \le t \le t_{(k+1)}$$

A $100(1-\alpha)\%$ confidence interval for S(t), for a given value of t, is the interval from

$$\hat{S}(t) - z_{\frac{\alpha}{2}}\, se\left\{\hat{S}(t)\right\} \text{ to } \hat{S}(t) + z_{\frac{\alpha}{2}} se\{\hat{S}(t)\}$$

When the estimated survival function is close to zero or unity, symmetric intervals are inappropriate, since they can lead to confidence limits for the survivor function that lie outside the interval (0,1). Replace any limit that is greater than unity by 1.0, and any limit that is less than zero by 0.0.

4.3 Log-Log Transformation

Log-log transformations and confidence intervals are often used in statistical analysis. Log-log transformations are commonly used in survival analysis to estimate survival probabilities and construct confidence intervals. To avoid the survival function lies outside (0,1) log-log transformation is used

$$se\left\{\log\{-\log \hat{S}(t)\}\right\} = \frac{1}{\{\log \hat{S}(t)\}^2}\left\{\sum_{j=1}^{k}\frac{d_j}{n_j(n_j - d_j)}\right\}^{\frac{1}{2}}$$

$100(1-\alpha)\%$ limits of the form $\hat{S}(t)^{\exp\left\{\pm z_{\frac{\alpha}{2}} se\left\{\log\{-\log \hat{S}(t)\}\right\}\right\}}$

Where is the upper point of the standard normal distribution.

4.4 Estimating Median of Survival Times

Estimated median survival time $\bar{t}(50)$, is defined to be the smallest observed survival time for which the value of the estimated survivor function is less than 0.5.

$\bar{t}(50) = \min\left\{t_i \mid \hat{s}(t_i) < 0.5\right\}$, where t_i is the observed survival time for the i^{th} individual, $i = 1, 2, ..., n$.

4.5 Estimating Percentiles of Survival Times

A similar procedure to that described for median can be used to estimate other percentiles of the distribution of survival times. The p^{th} percentile of the distribution of survival times is defined to be the value $t(p)$ which is such that $F\left\{t(p)\right\} = p/100$.

In terms of survival function, t(p) is such that $S\left\{t(p)\right\} = 1-(p/100)$, so that

$$S\left\{t(10)\right\} = 0.9, S\left\{t(90)\right\} = 0.1$$

Using estimated survival function, the estimated p^{th} percentile is the smallest observed survival time, $\bar{t}(p)$, for which $\hat{s}(\bar{t}(p)) < 1 - \left(\frac{p}{100}\right)$.

4.6 Confidence Interval for the Median and Percentiles

The standard error of $\bar{t}(p)$, the estimated p^{th} percentile, is therefore given by

$$se\{\bar{t}(p)\} = \frac{1}{\bar{f}\{\bar{t}(p)\}} se\left[\hat{S}(\bar{t}(p))\right]$$

The standard error of $\hat{S}(\bar{t}(p))$ is found using Greenwood's formula.

Probability density function at $\bar{t}(p)$ is obtained by

$$\bar{f}\ \{\bar{t}(p)\} = \frac{\hat{S}(\hat{u}(p)) - \hat{S}(\hat{l}(p))}{\hat{l}(p) - \hat{u}(p)},$$

Where $\hat{u}(p) = max\left\{t_{(j)} \mid \hat{S}\left(t_{(j)}\right) \geq 1 - \frac{p}{100} + \epsilon\right\}$ and

$\hat{l}(p) = min\left\{t_{(j)} \mid \hat{\boldsymbol{S}}\left(t_{(j)}\right) \leq 1 - \frac{p}{100} - \epsilon\right\}$, for $j = 1, 2, \ldots, r$ and small values of ϵ .

The $100(1-\alpha)\%$ confidence interval for $t(p)$ has limits of

$\hat{t}(p) \pm Z_{\frac{\alpha}{2}} se\{\bar{t}(p)\}$ Where $Z_{\frac{\alpha}{2}}$ is the upper $\frac{\alpha}{2}$ point of the standard normal distribution.

V. DATA IMPLEMENTATION

In this section, two data sets (complete and survival) are provided to elaborate on the methods given in above sections. All calculations done using the software R (4.5.0).

Consider $X \sim N(\mu, \sigma^2 = 100)$ and mean of the 64 random samples to be 28.6. Then a(1-β)100% confidence interval for μ is

$\left[\theta_1(\beta), \theta_2(\beta)\right] = \left[28.6 - Z_{\frac{\beta}{2}}\left(\frac{10}{\sqrt{n}}\right), 28.6 + Z_{\frac{\beta}{2}}\left(\frac{10}{\sqrt{n}}\right)\right]$ When $Z_{\frac{\beta}{2}} = 1.96$ at 5% level of significance, then the confidence limits are 26.15 and 31.05.

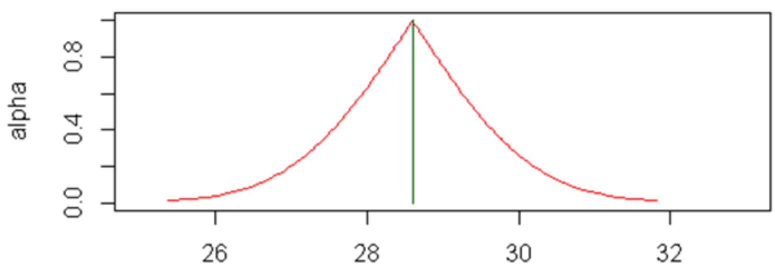


Figure 1. Fuzzy Estimator of "$(\bar{\mu})$" for known variance, $0.01 \leq \beta \leq 1$.

Figure 1 shows the membership function of $\overline{\mu}$ in complete data based on Buckley's method. To complete the picture, we draw short vertical line segments, from the horizontal axis up to the graph, at the end points of the base of the fuzzy number $\overline{\mu}$. The base $\overline{\mu}[0]$ is a 99% confidence interval for μ.

When $Z_{\frac{\beta}{2}} = 1.96$ at 5% level of significance, then the confidence limits are 26.15 and 31.05

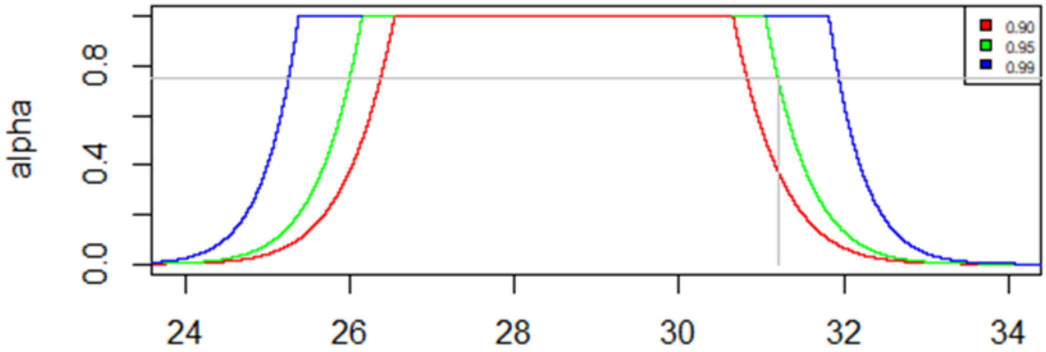


Figure 2. Fuzzy confidence interval in place of "($\overline{\mu}$)" for known variance, $0.01 \le \beta \le 1$.

Figure 2 indicates the nested fuzzy confidence intervals based on the Fuzzy Confidence Interval approach ([12]) in the parameter space. Constructed fuzzy intervals have probabilistic-possibilistic interpretations. From the figure 2, red, green and blue line indicates the level of confidence interval at 0.90, 0.95 and 0.99 respectively (The same applies to the following Figures 4, 6, 8, 10). For instance, at confidence level 0.95, it is completely possible that the unknown parameter population mean between 26.15 and 31.05. With possibility 0.75, population mean value is more than 31.2.

Fuzzy Estimator of "μ" for unknown variance, $0.01 \le \beta \le 1$

- $n = 25, \overline{x} = 28.6, s^2 = 3.42$
- Then a $(1-\beta)100\%$ confidence interval for μ is
- $\left[\theta_1(\beta), \theta_2(\beta)\right] = \left[28.6 - t_{\frac{\beta}{2}, n-1}\left(\frac{s}{\sqrt{n}}\right), 28.6 + t_{\frac{\beta}{2}, n-1}\left(\frac{s}{\sqrt{n}}\right)\right]$
- When $t_{\frac{0.05}{2}, 24} = 2.06$ at 5% level of significance, then the 95% confidence limits are 27.83 and 29.36

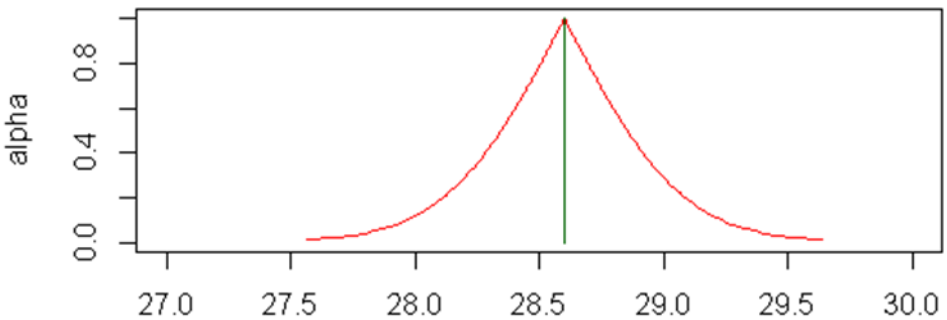


Figure. 3. Fuzzy Estimator of "$\overline{\mu}$ " for unknown variance, $0.01 \le \beta \le 1$

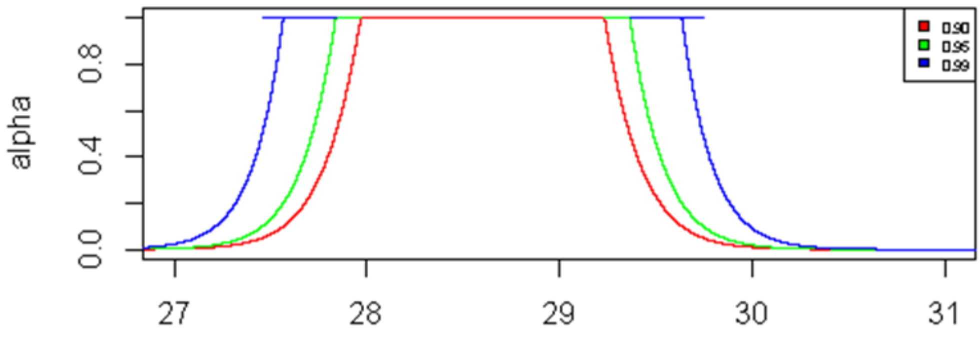


Figure 4. Fuzzy confidence interval for "$\bar{\mu}$" for unknown variance, $0.01 \leq \beta \leq 1$

Figure 3 & 4 shows another example applied to the above methods. Here the authors used small sample test namely t- test. At 95% confidence limits are 27.83 and 29.36 which is confound the above figure.

Survival Data

The following Table 1 refer to the number of weeks from the commencement of use of a particular type of intrauterine device (IUD). Data are given for 18 women, all of whom were aged between 18 and 35 years and who had experienced two previous pregnancies. Discontinuation times that are censored are labelled with an asterisk. This data is taken from the text book ([4]) Modelling Survival Data in Medical. In this 9 women's are attain event remaining are censored.

Table 1.Time in weeks to discontinuation of the use of an IUD

10	13*	18*	19	23*	30	36	38*	54*
56*	59	75	93	97	104*	107	107*	107*

Table 2 shows the probability of survival over time, along with the number of events, censored cases, and individuals at risk at each time point. The standard error and the lower and upper bounds of the 95% confidence interval are also provided. It is observed that the 10th week probability of survival in this study is 94% with standard error 0.054, associated lower and upper bounds of the 95% confidence interval are 0.839 and 1.000 respectively. The median survival time for this study is 93 weeks.

Table 2. Survival Probabilities and its confidence limits Using Greendwood's formula

Time	Number at risk	Number of event	Survival	Standard Error	CI	CI
10	18	1	0.944	0.054	0.839	1
19	15	1	0.881	0.079	0.727	1
30	13	1	0.814	0.098	0.622	1
36	12	1	0.746	0.111	0.529	0.963
59	8	1	0.653	0.13	0.397	0.908
75	7	1	0.559	0.141	0.283	0.836
93	6	1	0.466	0.145	0.182	0.751
97	5	1	0.373	0.143	0.093	0.653
107	3	1	0.249	0.139	0	0.522

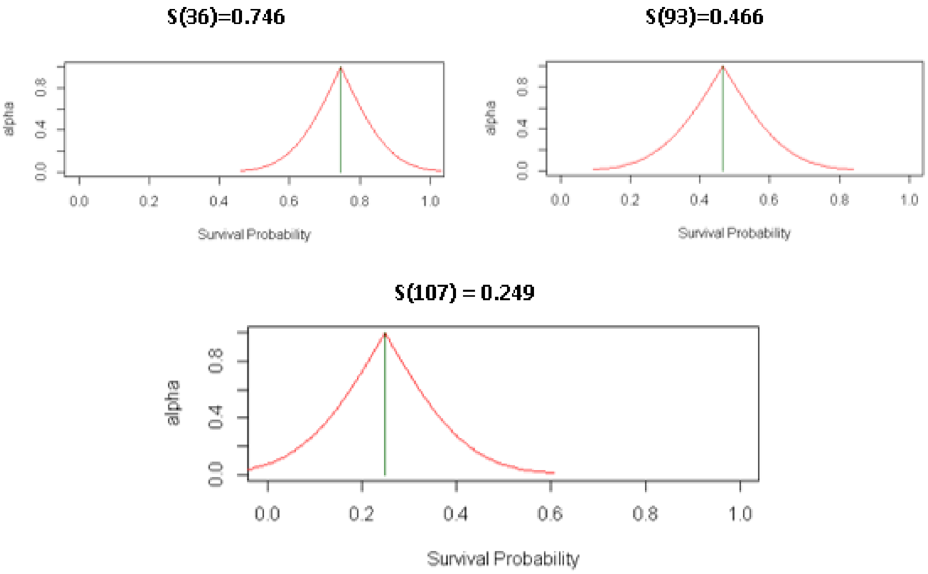


Figure 5: Fuzzy Estimate for different survival function estimates

Figure. 5 shows that the membership function for different survival function estimates based on Buckley's method. At 93 weeks, by stacking the alpha-cuts from $\alpha = 0.1$ to $\alpha = 0.8$, a triangular-shaped fuzzy point estimate for the parameter can be generated, with its membership function shown in Figure 5. Similarly, the estimates at 36 and 107 weeks are also represented in Figure. 5.

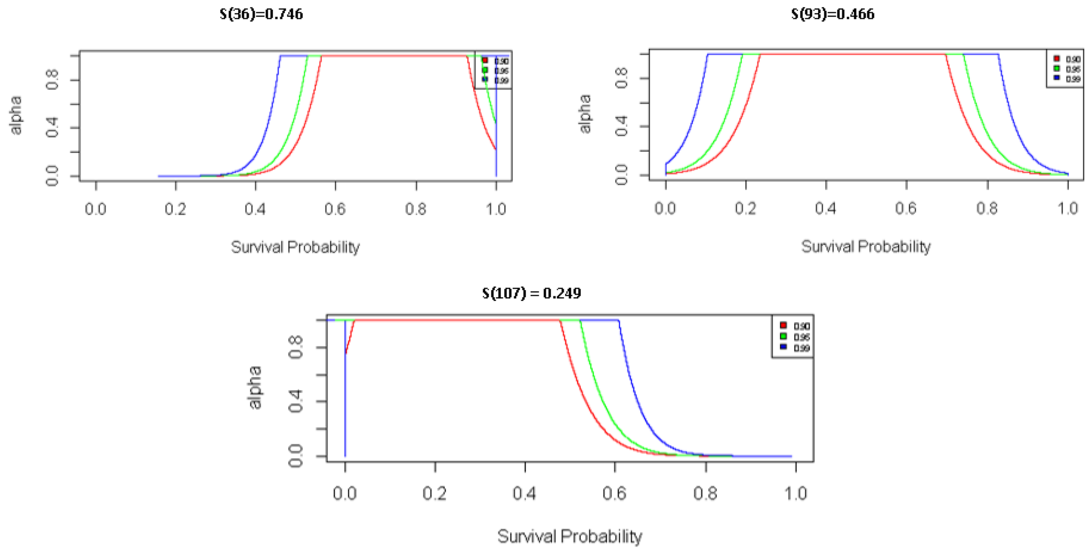


Figure 6: Fuzzy Confidence interval

Three membership functions of fuzzy confidence intervals 0.90(red line), 0.95(Green line), and 0.99(blue line).this is applied for KM method with confidence interval using Greenwood’s formula are in Figure 6. To make a comparison between different fuzzy confidence intervals and studying the effect of value provide fuzzy confidence intervals for 10%, 5%, and1%. The results are three fuzzy confidence intervals and related membership functions are shown in Figure6.

Table 3. Survival Probabilities and its confidence limits Using $\log(-\log S)$ formula

Time	Number at risk	Number of event	Number of censor	Probability	Standard Error	CI	CI
0	18	1	0	0.944	0.057	1	0.844
13	17	0	1	0.944	0.057	1	0.844
18	16	0	1	0.944	0.057	1	0.844
19	15	1	0	0.881	0.09	1	0.739
23	14	0	1	0.881	0.09	1	0.739
30	13	1	0	0.814	0.12	1	0.643
36	12	1	0	0.746	0.148	0.998	0.558
38	11	0	1	0.746	0.148	0.998	0.558
54	10	0	1	0.746	0.148	0.998	0.558
56	9	0	1	0.746	0.148	0.998	0.558
59	8	1	0	0.653	0.2	0.965	0.441
75	7	1	0	0.559	0.252	0.917	0.341
93	6	1	0	0.466	0.311	0.858	0.253
97	5	1	0	0.373	0.383	0.791	0.176
104	4	0	1	0.373	0.383	0.791	0.176
107	3	1	2	0.249	0.56	0.745	0.083

Table 3 represents the survival probability, with the number of events, censored cases, and individuals at risk at each time point. The standard error and the lower and upper bounds of the 95% confidence interval are also provided when using in $\log(-\log S)$ method. It is observed that the 75th week probability of survival in this study is 56%. At the same time, number at risk are 7 and the event is one. The standard error 0.252, associated lower and upper bounds of the 95% confidence interval are 0.917 and 0.341 respectively. The median survival time for this study have 93 weeks.

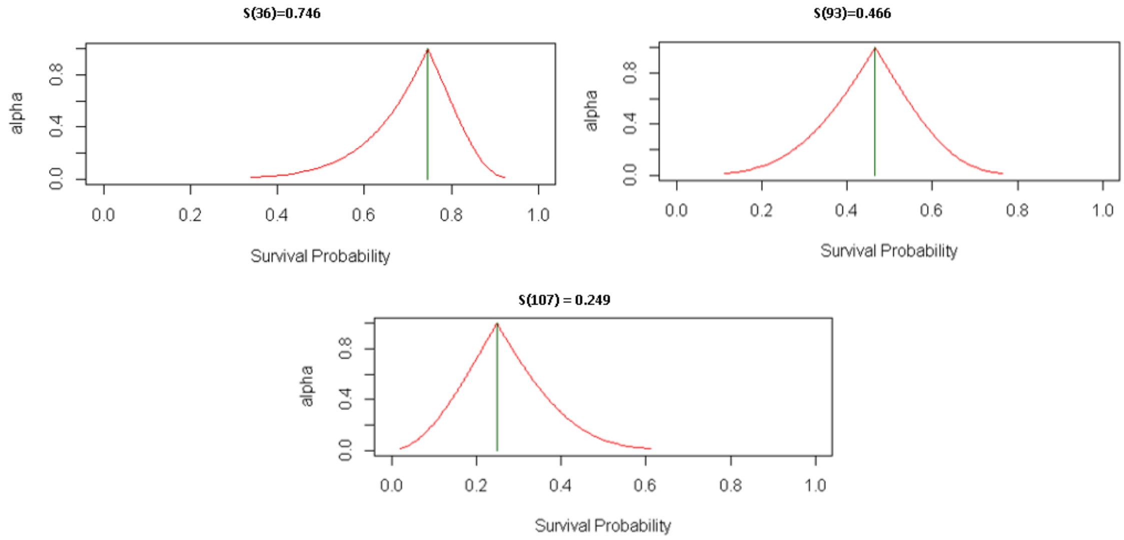


Figure 7. Fuzzy survival estimate Using log(-log S) method

Figure 7 shows that The membership function for different survival function estimates Using $\log(-\log S)$ method based on Fuzzy Point Estimation. At the time 107 weeks by placing the alpha-cuts of one on top of the other from $\alpha = 0$ to $\alpha = 0.6$, it is possible to produce a triangular shaped fuzzy point estimate for parameter, where its membership function is depicted in Figure1.similarly at time point 36 and 93 weeks are also given in the figure 7.

Three membership functions of fuzzy confidence intervals 90% (red line), 95% (Green line), and 99% (blue line).this is applied for km method with confidence interval using Log-Log transformation are presented in Figure 8. To compare different fuzzy confidence intervals and examine the impact of various significance levels, fuzzy confidence intervals are calculated for significance levels of 10%, 5%, and 1%. The resulting fuzzy confidence intervals and their corresponding membership functions are presented in the figure8.

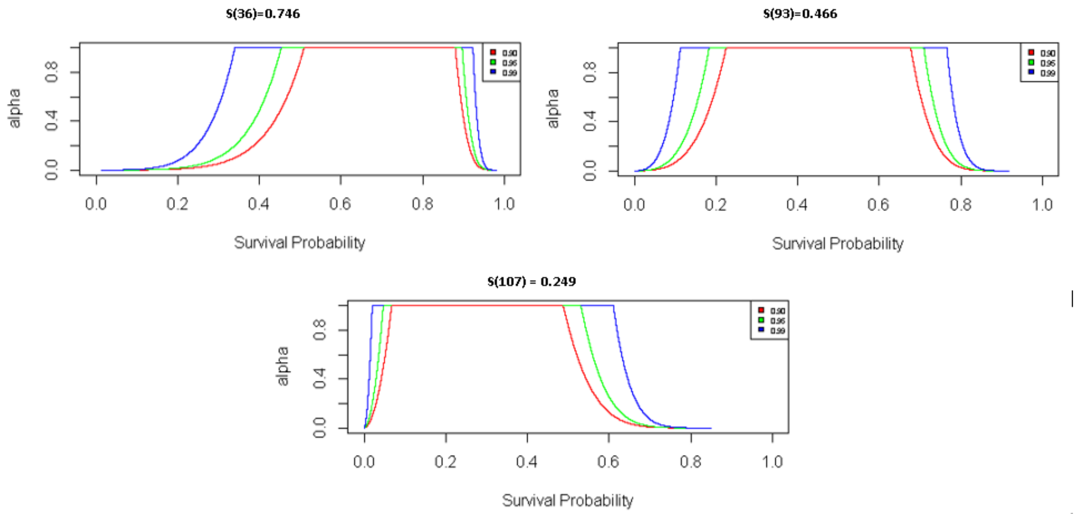
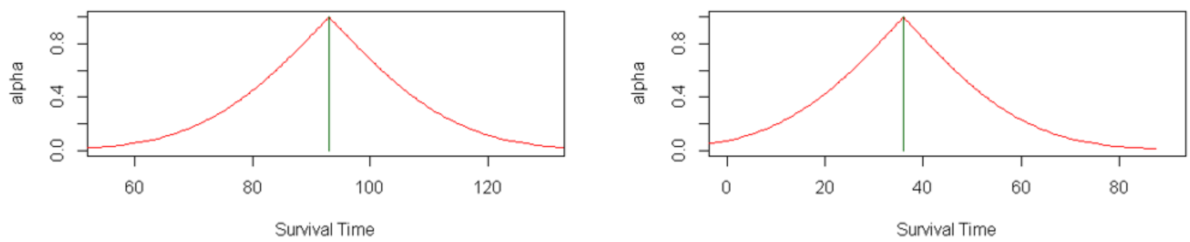


Figure. 8. Fuzzy Confidence interval

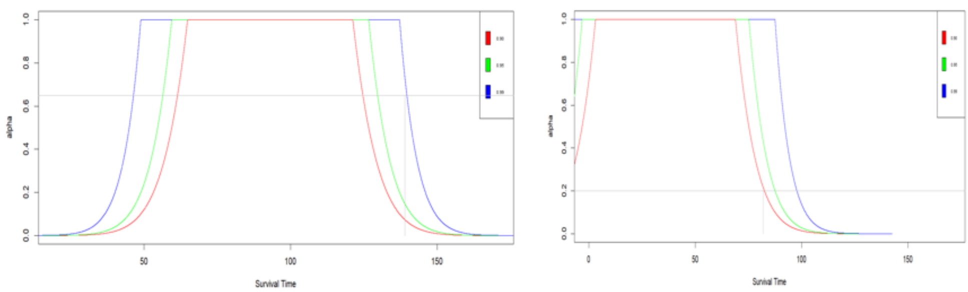


(a) (b)

Figure 9. Fuzzy Estimate of (a) Median Survival Time (93 Weeks) and (b) 25th Percentile Survival Time (36 Weeks)

Figure 9 shows that the membership function for different survival function estimates based on Buckley's method. This fuzzy estimate of Median Survival Time at 93 Weeks is shown in left side of figure and 25th Percentile Survival Time at 36 Weeks is shown in right side of the figure.

From figure 10(a) At Confidence level 0.99, with possibility 0.65, the median survival time is more than 139 Weeks. At confidence level 0.90: With Possibility 0.20, the 25th Survival Time is more than 82 Weeks are shown in figure 10(b).



(a) (b)

Figure 10. Fuzzy Confidence interval for Median Survival Time (a) and 25th Survival Time (b)

VI. SUMMARY

The Buckley fuzzy estimation approach is applied to both complete and incomplete data. A method for constructing fuzzy confidence intervals for unknown crisp statistical parameters is also employed, specifically for the survival function and survival time. This fuzzy confidence interval approach enables a comparative analysis of Greenwood's formula and the $\log(-\log S)$ transformation method. Notably, there are no inherent limitations in constructing fuzzy confidence intervals for any unknown parameter. The interpretation of these intervals incorporates both probabilistic and possibilistic perspectives. Compared to Buckley's fuzzy point estimation method, the fuzzy confidence interval approach offers a more general and flexible framework. Its broad applicability is demonstrated through analyses based on real-time data across various domains.

VII. FUTURE WORK

This paper proposes a fuzzy approach for parameter estimation and the construction of confidence intervals using Triangular and Trapezoidal fuzzy numbers. The methodology is illustrated with two real-time datasets and applied within the framework of survival analysis using Buckley's method and the fuzzy confidence interval approach. Furthermore, the study extends the fuzzy estimation technique to other statistical models, including regression and Bayesian models, in a fuzzy environment. Potential applications are also explored in diverse domains such as medical survival data, engineering reliability analysis, and financial risk modelling.

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