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Research Paper

Magneto-Diffusive Convection in Dufour-Soret Induced Couple Stress Nanofluid Layer

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Abstract
Rayleigh Benard triply diffusive magneto convection in a couple stressed nanofluid layerincorporated with Dufour -Soret effects has been studied in this paper. The effects ofSoretparameter and magnetic field on the stability of the system have been investigated.A comparison has been done with the results obtained for a nanofluid without couple stress factor.

2010 Mathematics Subject Classification: *76E06, 76E25, 76R50, 80A19, 80A21.*

Keywords - *Nanofluid, Magnetic field, Couple Stress, Double diffusion, Dufour-Soret parameter.*

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I. INTRODUCTION

With the growing importance of non-Newtonian fluid in modern technology and industries, Stokes[6] gave the theory of couple-stress fluid. Because of use of couple-stress fluid in study of the mechanisms of lubrication of synovial joints, it has become the object of scientific research. Walicki and Walicka [7] modelled synovial fluid as a couple-stress fluid in human joints.

For the detection and cure of certain diseases the use of magnetic field is being made with help of magnetic field instruments. Magneto-hydrodynamics has several scientific and practical applications in astrophysics, geophysics, space sciences etc. Thermosolutal convective problem for acouple-stress fluid through a porous medium under influence of vertical magnetic field and vertical rotation has been studied by Kumar [2] and was found that magnetic field and couple-stress have both stabilizing and destabilizing effects on stability of system.

In fluid flow problems, the phenomenon of generation of the concentration flux by temperature gradient is termed as Soret effectand the energy flux caused by a composition gradient is called the Dufour or diffusion thermo effect. Study on the Soret induced convective instability of a regular Newtonian fluid saturated in a porous medium has been done by many researchers. Wang and Tan [8] analysed the convective instability in Benard cells in a non-Newtonian fluid incorporating Soret factor. The impact of Soret parameter induced by the temperature gradient studied by Singh et al. [5],Bahlowl et al. [1] and Mansour et al. [3] also have much research work on Soret effect in different forms of stability.

Recently triple diffusive convection with Soret-Dufour effects in a Maxwell nanofluid saturated in Darcy porous medium was studied by Singh et al. [5] demonstrating the effects of different parameters on heat transfer.Triple diffusive magneto convection in nanofluid layers incorporating Dufour-Soret factor was studied by the author [4] and the effects of all parameters on stability of system were investigated analytically as well as graphically.

The above literature survey shows that no investigation has been doneto study the effect of magnetic field on triple diffusive convection in a couple stressed nanofluid layer with Dufour-Soret factor. Keeping in mind the importance of couple stress fluids, the present study attempts to studythe effect of vertical magnetic field on Dufour-Soret induced triple diffusive convection in a couple stressed nanofluid layer. To find the behaviour of different parameters in presence of couple stress, a comparison has been made with nanofluid without couple stress factor.

II. MATHEMATICAL FORMULATIONS

A couple stressednanofluid layer is confined between two infinite horizontal surfaces separated by a distance a, with z-axis vertically upward. Lower surface is maintained at higher temperature T_l^* and upper surface is maintained at temperature T_r^*. A uniform vertical magnetic field $M^* = (0, 0, M_0^*)$ is applied (See Fig. 1)

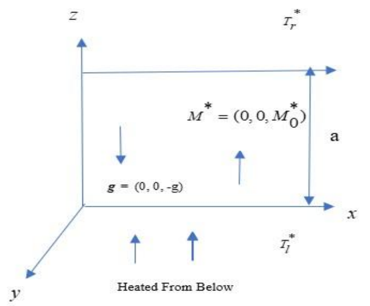

Fig. 1: Physical configuration of the problem

The governing equations for conservation of mass, momentum, energy and concentration of salt and nanoparticles in non-dimensional form on defining $\left(X,Y,Z\right)=\dfrac{(x^*,y^*,z^*)}{a}$, $t=\dfrac{t^*\alpha_m}{\sigma a^2}$, $\left(u_{1d},u_{2d},u_{3d}\right)=\dfrac{\left(u_{1d}^*,u_{2d}^*,u_{3d}^*\right)a}{\alpha_m}$, $p=\dfrac{p^*K}{\mu\alpha_m}$, $\psi=\dfrac{\psi^*-\psi_0^*}{\psi_0^*}$, $\left(M_X,M_Y,M_Z\right)=\dfrac{(M_X^*,M_Y^*,M_Z^*)}{M_0^*}$, $S=\dfrac{S^*-S_r^*}{S_l^*-S_r^*}$, $T=\dfrac{T^*-T_r^*}{T_l^*-T_r^*}$, $\lambda=\dfrac{\lambda^*\alpha_m}{a^2}$ and $S=\dfrac{S^*-S_r^*}{S_l^*-S_r^*}$ are as follows:

$$\nabla.\boldsymbol{q}=0\ , \qquad (1)$$

$$(\boldsymbol{q}-\square\nabla^2\boldsymbol{q})=\left(1+\frac{\lambda}{\sigma}\frac{\partial}{\partial t}\right)\left[\left(-\nabla p-R_m\hat{\boldsymbol{e}}_z-R_n\psi\hat{\boldsymbol{e}}_z+R_aT\hat{\boldsymbol{e}}_z+\frac{R_s}{Ln}\boldsymbol{S}\hat{\boldsymbol{e}}_z\right)+\frac{P_1}{P_{1M}}QD_a\left(\nabla\times\boldsymbol{M}\right)\times\boldsymbol{M}\right], \qquad (2)$$

$$\frac{\partial T}{\partial t}+\left(\boldsymbol{q}.\nabla\right)T=\nabla^2T+\frac{N_b}{Le}\nabla\psi.\nabla T+\frac{N_aN_b}{Le}\nabla T.\nabla T+N_{tc}\nabla^2S\ , (3)$$

$$\frac{1}{\sigma}\frac{\partial S}{\partial t}+\frac{1}{\in}\boldsymbol{q}.\nabla S=\frac{1}{Ln}\nabla^2S+N_{ct}\nabla^2T\ , \qquad (4)$$

$$\frac{1}{\sigma}\frac{\partial\psi}{\partial t}+\frac{1}{\in}\left(\boldsymbol{q}.\nabla\right)\psi=\frac{1}{Le}\nabla^2\psi+\frac{N_a}{Le}\nabla^2T\ , \qquad (5)$$

$$\frac{1}{\sigma}\frac{\partial\boldsymbol{M}}{\partial t}+\frac{1}{\in}\left(\boldsymbol{q}.\nabla\right)\boldsymbol{M}=\frac{1}{\in}(\boldsymbol{M}.\nabla)\boldsymbol{q}+\frac{P_1}{P_{1m}}\nabla^2\boldsymbol{M}\ , \qquad (6)$$

Here $R_a(=\frac{\rho g\beta Ka\left(T_l^*-T_r^*\right)}{\mu\alpha_m})$, $R_n(=\frac{\left(\rho_P-\rho\right)\psi_0^* gKa}{\mu\alpha_m})$, $R_m(=\frac{\rho_P\psi_0^*+\rho(1-\psi_0^*)gKa}{\mu\alpha_m})$, $R_s(=\frac{\rho\beta_c gaK(S_l^*-S_r^*)}{\mu S_d})$ are thermal, concentration, basic density and solutal Rayleigh Darcy number respectively, $P_1(=\frac{\mu}{\rho\alpha_m})$ and $P_{1m}(=\frac{\mu}{\rho\eta})$ are Prandtl numbers, $Q(=\frac{\mu_e M_0^{*2}a^2}{4\pi\mu\eta})$ is Magnetic Chandrasekhar number, $D_a(=\frac{K}{a^2})$ is Darcy number, $\square(=\frac{\mu_{cs}}{\mu l^2})$ is couple-Stress parameter, $N_{ct}(=\frac{S_{ct}(T_l^*-T_r^*)}{\alpha_m(S_l^*-S_r^*)})$ is Soret parameter, $N_{tc}(=\frac{S_{tc}(S_l^*-S_r^*)}{\alpha_m(T_l^*-T_r^*)})$ is Dufour parameter, $N_a(=\frac{B_t(T_l^*-T_r^*)}{B_d T_r^*\psi_0^*})$ and $N_b(=\frac{\left(\rho c\right)_P\psi_0^*\in}{\left(\rho c\right)_F})$ are modified diffusivity ratio and modified particle density increment respectively, $Le(=\frac{\alpha_m}{B_d})$ and $Ln=\frac{\alpha_m}{S_d}$ are Lewis numbers for nanofluid and salt respectively.The boundary conditionsare given as

$\boldsymbol{q}=0$, T=1, S =1, $\frac{\partial\psi}{\partial Z}+N_a\frac{\partial T}{\partial Z}=0$ at Z = 0 (7)

$\boldsymbol{q}=0$, T=0, $S=0$, $\frac{\partial\psi}{\partial Z}+N_a\frac{\partial T}{\partial Z}=0$ at Z = 1 (8)

On the basic state, we superimpose perturbations in the form

Let $\boldsymbol{q}=\boldsymbol{q'}$, $p=p_{bs}+p'$, $T=T_{bs}+T'$, $S=S_{bs}+S'$, $\psi=\psi_{bs}+\psi'$, and $\boldsymbol{M}=\hat{\boldsymbol{e}}_Z+\boldsymbol{M'}$,

where the primes denote infinitesimal small quantities. Ignoring the products of primed quantities and their derivatives, following linearised form of equations is obtained:

$$\left(\frac{1}{\sigma}\frac{\partial}{\partial t}-\frac{P_1}{P_{1M}}\nabla^2\right)\left[(\nabla^2-\square\nabla^4)u'_{3d}-\left(1+\frac{\lambda}{\sigma}\frac{\partial}{\partial t}\right)\left(R_a\nabla_H^2T'-R_n\nabla_H^2\psi'+\frac{R_s}{Ln}\nabla_H^2S'\right)\right]$$
$$=(1+\frac{\lambda}{\sigma}\frac{\partial}{\partial t})Q\frac{P_1}{P_{1M}}\frac{D_a}{\in}\nabla^2\frac{\partial^2u'_{3d}}{\partial Z^2} \quad (9)$$

$$\frac{\partial T'}{\partial t}-u'_3=\nabla^2T'-\frac{N_aN_b}{Le}\frac{\partial T'}{\partial Z}-\frac{N_b}{Le}\frac{\partial\phi'}{\partial Z}+N_{tc}\nabla^2S', \quad (10)$$

$$\frac{1}{\sigma}\frac{\partial S'}{\partial t}-\frac{u'_3}{\in}=\frac{1}{Ln}\nabla^2S'+N_{ct}\nabla^2T' , \quad (11)$$

$$\frac{1}{\sigma}\frac{\partial\psi'}{\partial t}+\frac{1}{\in}(\boldsymbol{q'}.\nabla)\psi'+\frac{1}{\in}N_au'_3=\frac{1}{Le}\nabla^2\psi'+\frac{N_a}{Le}\nabla^2T' , \quad (12)$$

$u'_3=0, T'=0, S'=0, \frac{\partial\psi'}{\partial Z}+N_a\frac{\partial T'}{\partial Z}=0$ at Z = 0 and Z = 1 (13)

III. LINEAR STUDY

Following the linear stability theory by Chandrasekhar [5], the perturbations are taken of the form

$$\left(\psi',T',u'_3,S'\right)=\left[\Phi(Z),\Theta(Z),\Omega(Z),\Psi(Z)\right]e^{st+iLX+iMY}, \quad (14)$$

where L and M are dimensionless wave numbers in X and Y directions respectively.On substituting the above values and employing Galerkin method to solve equations (9)-(12) together with the boundary condition (13) and taking first estimation as N=1, we have $\Omega=A_1\sin\pi Z$, $\Theta=B_1\sin\pi Z$, $\Phi=-N_aC_1\sin\pi Z$,

$\Psi = D_1 \sin \pi Z$ .Taking the determinant of above matrix equation as zero, the following Rayleigh number is obtained

$$R_a = \frac{\sigma}{\alpha^2}\left[\frac{\begin{array}{l} R_s \alpha^2 (\lambda s + \sigma)(\sigma A \chi^2 + s)(\sigma \chi^2 + sLe)(\chi^2 + s)\{\chi^2 (\in N_{ct} - 1) - s\} \\ -R_n N_a \alpha^2 (\lambda s + \sigma)(A \chi^2 \sigma + s)[(\chi^2 \sigma + sLn)\{\chi^2 (\in + Le) + sLe\} - \chi^4 N_{tc} Ln\sigma (1 + LeN_{ct})] \\ + \in (\chi^2 \sigma + sLn)(\chi^2 \sigma + sLe)(\chi^2 + s)\{A\sigma \chi^4 + B\pi^2 \chi^2 (\sigma + \lambda s) + s\chi^2\}\{(\chi^2 \sigma + sLn)(\chi^2 + s) - \chi^4 N_{tc} N_{ct} Ln\sigma \end{array}}{(\sigma \chi^2 \in + sLn \in - \chi^2 N_{tc} Ln\sigma)(\sigma \chi^2 + sLe)(\lambda s + \sigma)(A \chi^2 \sigma + s)}\right],$$

(15)

where, $A = \frac{P_1}{P_{1m}}, B = Q\frac{P_1}{P_{1m}}\frac{D_a}{\in}$, $\chi^2 = \pi^2 + \alpha^2$.

Taking s=0 in equation (15),

$$R_a^{st} = \frac{\chi^2 (\in \chi^2 + QD_a \pi^2 + □ \in \chi^4)(1 - LnN_{ct} N_{tc}) - R_n N_a \alpha^2 \{\in + Le - LnN_{tc}(1 + LeN_{ct})\} - R_s \alpha^2 (1 - \in N_{ct})}{\alpha^2 (\in - LnN_{tc})} \quad (16)$$

The above relation expresses the stationary Rayleigh number as a function of the parameters □ , Ln , Le , Q , N_a , R_n , $\in$, D_a R_s , N_{ct} , N_{tc} and dimensionless wave number α .

To obtain critical Rayleigh number putting $\frac{dR_a^{st}}{d\alpha} = 0$, critical wave number is given by equation

$$2□ (\alpha^2)^3 + (3\pi^2 □ + 1)(\alpha^2)^2 - (\pi^4 + □ \pi^6 + \frac{QD_a}{\in}\pi^4) = 0 \quad (17)$$

which shows that critical wave number depends on Couple Stress parameter, Darcy number, Porosity and Magnetic field.

IV.RESULTSAND DISCUSSION

The effect of magnetic field and Soret factor on the stability of the systemfor stationary convection has been studied.It has been found that the magnetic field has stabilizing nature whereas the destabilizing behaviour of Soret parameter in absence of magnetic field [17] does not persist here in presence of magnetic field and converts to the stabilizing agent.

In Fig.2 and Fig.3, a comparison between the two Rayleigh numbers for a nanofluid with Dufour-Soret effect and a couple stress nanofluid with Dufour-Soret effect for stationary convection has been made for fixed values of the concerned parameters. It is found that for stationary convection, for $\in = 0.5$, $Le = 100$, $N_a = 10$, $Ln = 50$, $R_s = 5$ $N_{ct} = 1.1$, $N_{tc} = 0.1$, $D_a = 1.2$. It has been observed that

R_a^{st} (Couple Stress nanofluid with Dufour-Soreteffect) > R_a^{st} (Nanofluid with Dufour-Soret induced convection)

It confirms the early convection in a nanofluid in comparison to convection sets in a nanofluid with couple stress

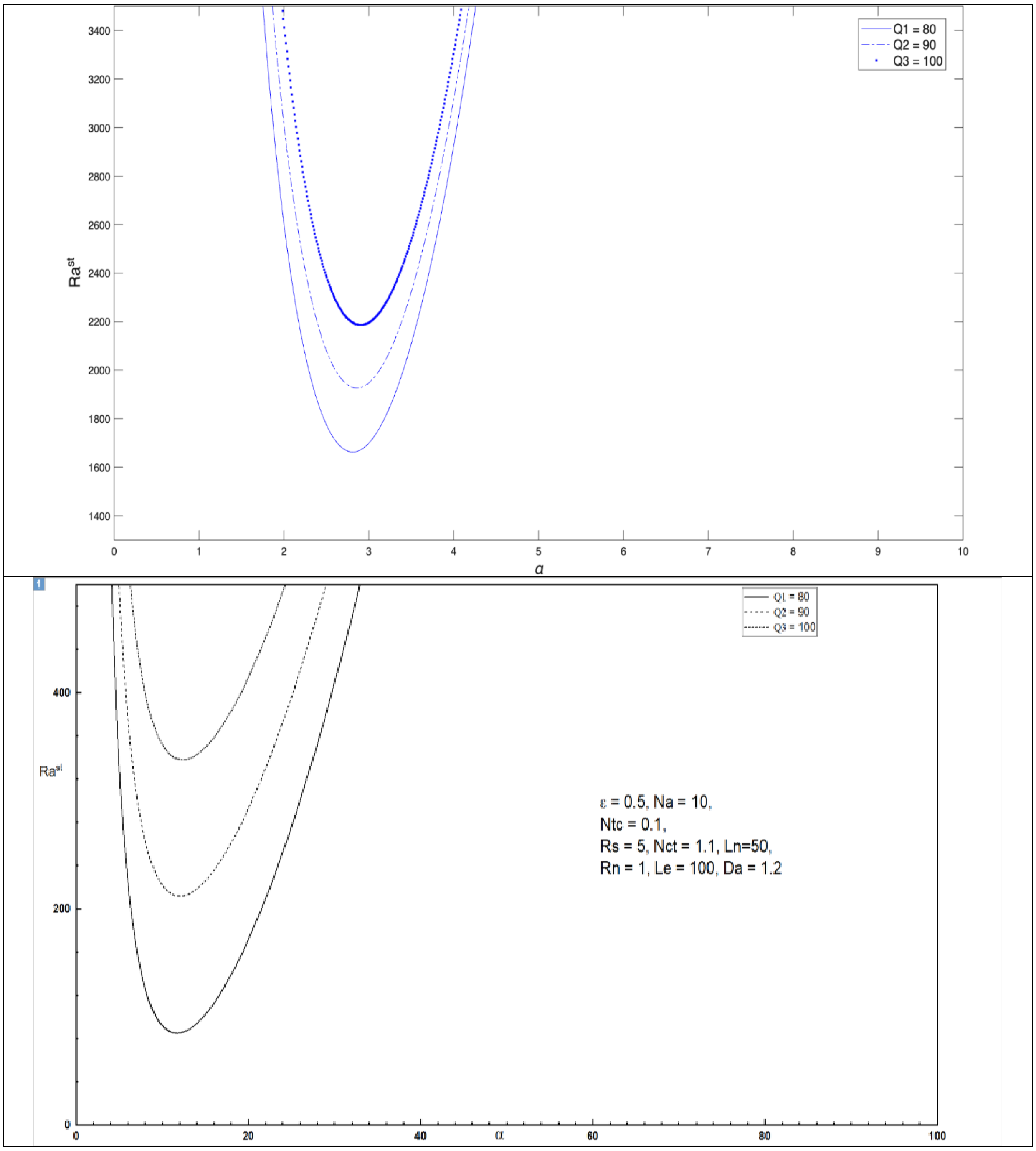

Fig. 2: Comparison of Linear stationary convection for a couple-stress nanofluid and ananofluid for different values of Q

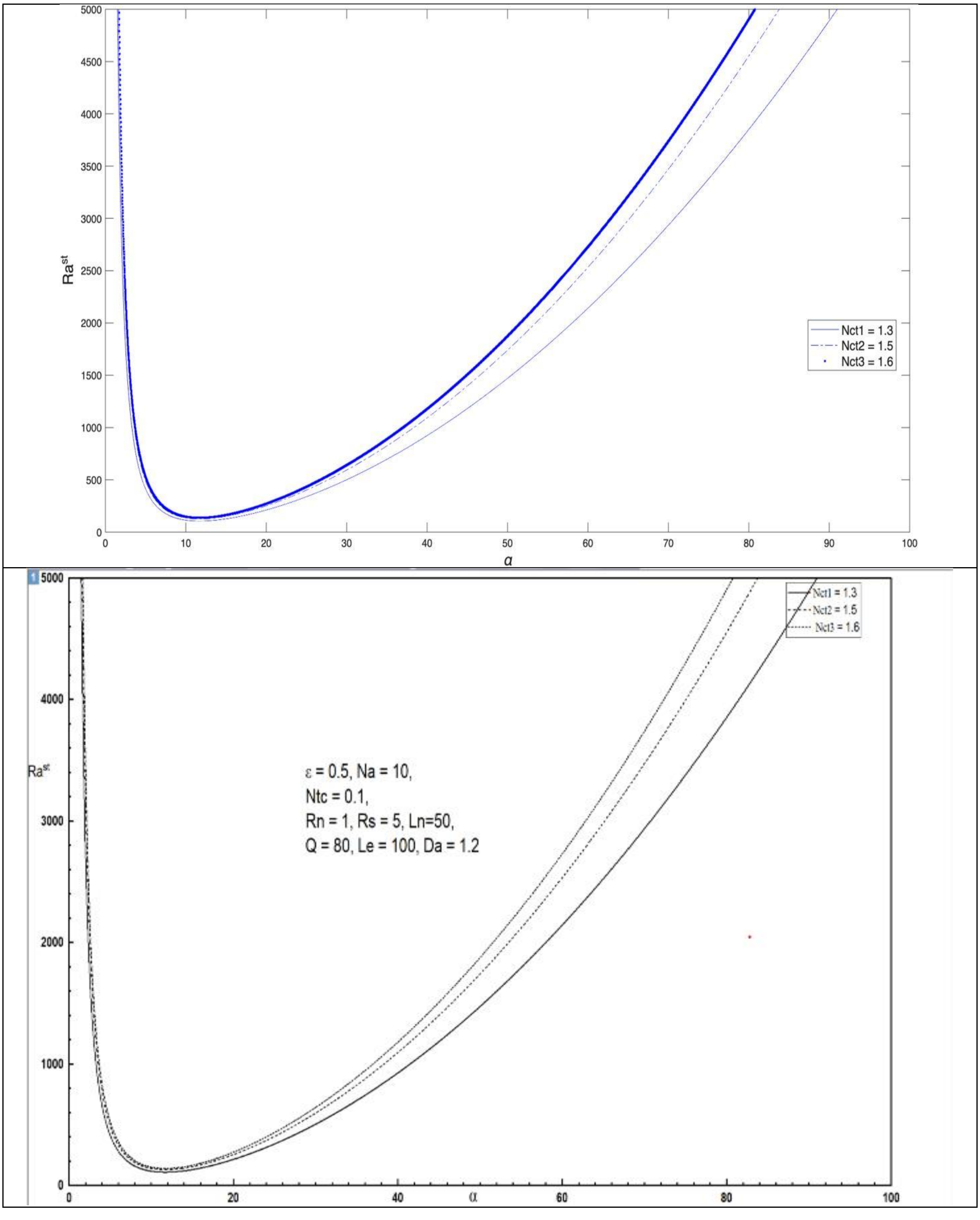

Fig. 3: Comparison of Linear stationary convection for a couple-stress nanofluid and a nanofluid for different values of N_{ct}

V. CONCLUSION

This paper presents an analytical study of linear Rayleigh Benard triply diffusive magnetoconvection in a Maxwell nanofluid layer with Soret-Dufour effects. We reach at the following conclusions:

- For the stationary mode the Soret parameter, Dufour parameter, couple stress and magnetic field have a stabilizing effect.

- A comparison between the two Rayleigh numbers for a nanofluid with Dufour-Soret effect and a couple stress nanofluid with Dufour-Soret effect for stationary convectionconfirms the early convection in a nanofluid in comparison to convection sets in a nanofluid with couple stress.

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