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Research Paper

Applying Derivatives to The Design of Economic Modelling Problems for Teaching Mathematics for Economics at Thai Nguyen University of Agriculture and Forestry

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Abstract
Derivatives are widely used to examine marginal quantities and solve optimization problems in economics. Yet calculating a derivative is only one part of an economic modelling task. Students must also identify relevant variables, formulate assumptions, interpret the mathematical result and decide whether it makes sense in the original setting. This paper develops a set of derivative-based problems for teaching Mathematics for Economics at Thai Nguyen University of Agriculture and Forestry. The study draws on research concerning students' understanding of derivatives, mathematical modelling competence and undergraduate mathematics teaching. Three illustrative problems are developed: marginal cost analysis, profit maximization and the choice of fertilizer input in a simplified agricultural production model. Each problem includes a mathematical formulation, a verified solution, an economic interpretation and suggested learning activities. A five-stage teaching sequence and an assessment rubric are also proposed. The contribution lies in the organization of familiar mathematical ideas into a coherent set of teaching tasks, with particular attention to economic interpretation and agricultural applications. No classroom intervention or empirical evaluation was conducted; the proposed materials therefore require further review and testing before claims about their educational effectiveness can be made.

Keywords: *derivatives; mathematics for economics; mathematical modelling; profit optimization; agricultural economics education.*

I. INTRODUCTION

1.1. Derivatives in mathematics for economics

Calculus provides economists with a language for describing how one quantity changes in relation to another. Derivatives are used in marginal analysis, the study of cost and revenue functions, and the optimization of economic decisions. Their usefulness, however, depends on more than computational accuracy. An economically meaningful solution requires a suitable model, assumptions that can be defended, and an interpretation consistent with the quantities being studied.

Voßkamp, Feudel and Monaghan [1] examine the particular ways in which calculus is used in economics and the difficulties these uses may present for students. Their discussion draws attention to an important feature of mathematics for economics: mathematical concepts are often introduced alongside disciplinary interpretations that are not immediately apparent from the formal notation. A derivative may be taught as the slope of a tangent or an instantaneous rate of change, but it later appears in economics as marginal cost, marginal revenue or marginal product.

The transition from school mathematics to university courses adds another difficulty. Landgärds-Tarvoll [2], in a review of research on mathematics for economics, discusses challenges associated with this transition. Students entering university may have encountered differentiation as a collection of rules without having developed a sufficiently connected understanding of rates of change, functions and applications. Such differences in mathematical preparation matter when a course expects students to move quickly from symbolic manipulation to economic modelling.

For students studying agriculture-related disciplines, the use of derivatives can be illustrated through questions about production costs, sales revenue and the allocation of inputs. These contexts are suitable for the present study because they allow mathematical results to be interpreted as economic decisions. Their inclusion

does not imply that the examples represent observed production conditions or have already been adopted in the university's curriculum.

1.2. From differentiation to economic interpretation

A persistent issue in calculus education is the distinction between carrying out a mathematical procedure and understanding the concept on which it rests. Tall and Vinner [3] distinguish between a formal concept definition and the broader collection of mental associations that students develop around a mathematical idea. Their work concerns limits and continuity, but the distinction offers a useful perspective when considering why a student may differentiate a function correctly yet struggle to explain what the result means.

Orton [4] examined students' understanding of differentiation and rates of change, identifying difficulties that cannot be reduced to errors in applying differentiation rules. More directly related to economics education, Feudel and Biehler [5] investigated students' understanding of the derivative in the context of mathematics for economics. Their subsequent study [6] addressed the economic interpretation of the derivative in marginal cost problems.

The distinction between marginal cost and the cost of producing one additional unit is particularly important. If total cost is nonlinear, the derivative at a given output level and the difference between total costs at two consecutive output levels need not be identical. Treating them as interchangeable can conceal the approximation involved in marginal analysis.

Similar concerns arise when students interpret other economic measures. Mkhatshwa [7], studying price elasticity of demand in business calculus, examined students' covariational reasoning together with procedural and conceptual knowledge. Although price elasticity is not one of the problems developed in this paper, the study reinforces the need to distinguish calculation from interpretation in economic contexts.

These studies suggest that a derivative-based teaching problem should ask students to do more than obtain a numerical answer. It should require them to explain the quantity represented by the derivative, examine the assumptions under which the calculation is meaningful, and relate the result to the economic question.

1.3. Previous teaching approaches and the purpose of this study

Connecting derivatives with economics is not a new teaching idea. In Vietnam, Lê Thái Bảo Thiên Trung and Phạm Hoài Trung [8] discussed different meanings of the derivative and designed teaching activities linking those meanings with economic knowledge. Their work provides an important local precedent and makes it necessary to define the contribution of the present paper more narrowly.

Recent work has also addressed the organization of mathematics courses for economics students. Barile [9] discusses the use of prior-knowledge checks, structured instructional support and practical applications to help students connect mathematics with economics. Feudel and Skill [10], through a textbook analysis, examine how differentials are understood and presented in economics. Together, these studies indicate that both the selection of examples and the language used to explain them deserve attention.

The present study does not propose a new theory of differentiation or claim to introduce economic applications into mathematics teaching for the first time. Its purpose is to develop a small, connected set of teaching problems that makes the modelling process explicit. Three questions guide the design: How can the mathematical meaning of a derivative be distinguished from its economic interpretation? How can optimization problems be structured so that students must check the conditions of their solutions? And how can an agricultural production example be used without presenting hypothetical calculations as evidence about actual farming practices?

The resulting materials are intended as a teaching proposal for Mathematics for Economics at Thai Nguyen University of Agriculture and Forestry. Their practical suitability remains to be established through curriculum review and classroom evaluation.

II. THEORETICAL BASIS AND RESEARCH METHODS

2.1. Mathematical meaning of the derivative

For a real-valued function (f), the derivative at an interior point (x_0), when it exists, is defined as

$$f'^{(x_0)} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

The expression is the limit of an average rate of change. Geometrically, it gives the slope of the tangent to the graph at the point under consideration. The distinction between the limiting rate and a change over a finite interval is fundamental to the economic applications discussed below. The mathematical definition is standard calculus;

references [1], [5] and [6] are used here to support its interpretation and teaching in economics, rather than to attribute the definition to those authors.

Let (C(Q)) be a differentiable total cost function, where (Q) denotes output. Marginal cost is defined by: $MC(Q) = C'(Q) = \frac{dC(Q)}{dQ}$

If output changes by a finite amount ΔQ, the corresponding change in total cost is

$$D\Delta C = C(Q + \Delta Q) - C(Q)$$

For a differentiable cost function and a sufficiently small change in output, the first-order approximation is $\Delta C \approx C'(Q)\Delta Q$

Equation (4) is an approximation, not a general identity. Its accuracy depends on the size of the change and the behaviour of the cost function over the interval. This distinction is directly relevant to the teaching issues examined by Feudel and Biehler [6] and to the discussion of differentials in economics by Feudel and Skill [10].

Derivatives are also used to locate candidates for optimal economic decisions. Let revenue and cost be represented by (R(Q)) and (C(Q)), respectively. Profit is defined as their difference. For an interior optimum of a differentiable profit function, the first-order necessary condition is

$$\pi'(Q^*) = R'(Q^*) - C'(Q^*) = 0$$

This condition is commonly expressed as equality between marginal revenue and marginal cost. It is not sufficient on its own to establish a global maximum. The shape of the profit function, the feasible domain and the boundary values must also be considered.

These mathematical requirements inform the design of the three problems presented in Section 3. Each problem asks students to calculate a derivative, but the solution is considered incomplete until the result has been checked and interpreted.

2.2. Mathematical modelling as a teaching framework

Mathematical modelling involves moving between a situation outside mathematics and a mathematical representation of that situation. The process requires decisions about what to include, what to simplify and how to judge whether the resulting model is adequate for its purpose.

Blomhøj and Jensen [11] describe modelling competence in terms of the ability to carry out a modelling process and discuss the component abilities involved. Maaß [12] also examines modelling competencies, including activities associated with constructing, interpreting and validating mathematical models. Blum and Leiß [13] investigate how students and teachers deal with modelling problems, drawing attention to the work required to move through different stages of a modelling task.

These contributions support the view that solving an equation is only one part of modelling. Students must first decide what the equation should represent. They must then return to the original situation to interpret the answer and consider whether the assumptions remain reasonable.

For the present study, this perspective is adapted into five stages:

1. Identify the economic question and the quantities involved.
2. Specify variables, assumptions, functions and constraints.
3. Use differentiation to analyse the model and verify the solution.
4. Interpret the result and examine the model's limitations.
5. Summarize the method and consider a related situation.

The five stages are an instructional arrangement developed for this paper. They should not be treated as a reproduction of a single framework from [11]–[13] or as an independently validated measure of modelling competence. The sequence also allows movement back to earlier stages. For example, if an optimum lies outside the feasible domain, the student must reconsider the mathematical formulation or the constraints rather than report the stationary point as the final decision.

2.3. Principles for problem design

Four principles were used to construct the teaching problems.

First, the mathematics must be correct. Each problem requires a clearly defined function, a stated domain and an appropriate method for checking its solution. An interior stationary point cannot be presented as an optimum without further justification.

Second, the economic interpretation must be consistent with the model. Variables and parameters need clear meanings and units. In particular, a derivative measured in currency per unit of output must not be described as a total monetary amount.

Third, the problems should require progressively more decisions from students. The first task provides the cost function and asks for two related calculations. The second requires the construction of a profit function from price and cost relationships. The third asks students to compare two different objectives within the same agricultural production model.

Fourth, the teaching tasks should leave room for explanation and criticism. Students should be asked whether a result is feasible, which assumptions support it, and what additional information would be needed before using the model for a real decision.

These principles are consistent with the concerns about mathematical interpretation discussed in [5]–[10] and with the modelling perspective in [11]–[13]. Their particular combination is a design decision of the present study.

2.4. Research approach and development procedure

The study uses a conceptual and design-oriented approach. It draws on published work concerning differentiation, mathematics for economics, modelling competence and university teaching. The literature is discussed thematically; no systematic review protocol, exhaustive database search or quantitative synthesis is claimed.

The development process began by identifying three economic questions that can be represented by differentiable functions. Mathematical expressions and parameter values were then selected to make the relevant distinctions visible. Each solution was checked algebraically, including the feasible domain and the conditions for a maximum where appropriate. The completed problems were subsequently translated into proposed teaching activities and assessment criteria.

The numerical values were chosen for illustration. They were not obtained from student assessments, business accounts, field experiments or agricultural production records. The study therefore reports the design and mathematical verification of teaching materials, not the results of a classroom intervention.

III. RESULTS AND DISCUSSION

3.1. Development and organization of the derivative-based problem set

The principal outcome of this study is a set of three teaching problems designed to connect differentiation with economic modelling. The problems were selected to represent distinct uses of the derivative: measuring a marginal quantity, identifying an optimal level of output, and choosing an economically preferable level of agricultural input. Although all three problems rely on elementary differentiation, they place different demands on students' interpretation of mathematical results.

The sequence begins with a total cost function that is already given. Students are not required to construct the model; instead, they must examine what its derivative represents and distinguish an instantaneous rate of change from a finite increase in cost. The second problem requires students to combine a price function and a cost function to construct a profit function. The mathematical task is therefore accompanied by a modelling decision: selecting the appropriate objective. The third problem introduces two possible objectives within the same agricultural setting. Students must distinguish maximum physical production from maximum economic return and consider why the mathematically optimal decision depends on the objective being pursued.

This progression was informed by research on students' understanding of differentiation in economic contexts. Feudel and Biehler [5] examine the relationship between students' conceptions of the derivative and its economic applications, while their study of marginal cost [6] highlights the importance of distinguishing mathematical definitions from the ways marginal quantities are interpreted in economics. The problem set also draws on mathematical modelling research, particularly the view that solving an equation is only one stage of a broader process involving assumptions, mathematical representation, interpretation and validation [11]–[13].

The three problems were deliberately constructed using relatively simple polynomial functions. This choice reduces the computational demands of differentiation and allows attention to be directed towards the interpretation of results. The simplicity of the functions should not, however, be mistaken for a claim that the models adequately describe real production processes. The coefficients are hypothetical, and the models are intended to make particular mathematical and economic relationships visible in a teaching setting.

Table 1. Structure and intended learning demands of the proposed problems

Problem	Mathematical content	Economic reasoning	Expected learning outcome
1. Marginal cost	Derivative, finite difference, first-order approximation	Distinguishing marginal cost from the cost increase over a finite interval	Explain what the derivative measures and when it approximates an observed change
2. Profit maximization	Construction of an objective function, first- and second-order derivatives	Distinguishing maximum revenue from maximum profit	Formulate, solve and verify a constrained optimization problem
3. Fertilizer input	Marginal product, concavity, comparison of two objective functions	Distinguishing maximum physical output from maximum economic return	Compare alternative objectives and evaluate the limits of a simplified production model

The problems share a common structure: an economic situation is described, the mathematical model is specified or constructed, the relevant derivatives are calculated, the solution is checked, and the result is interpreted in the original context. This structure is intended to make the reasoning process visible rather than leaving students with a collection of unrelated computational exercises.

3.2. Mathematical analysis and instructional design of the proposed problems

3.2.1. Problem 1: Marginal cost and the interpretation of a derivative

Economic situation and mathematical formulation

Consider a hypothetical producer whose total cost depends on the quantity of output according to $C(Q) = 0.02Q^2 + 10Q + 500, \quad 0 \leq Q \leq 200$Here, (Q) represents the quantity produced, measured in product units, while (C(Q)) denotes total production cost in notional currency units. The constant term represents a fixed component of cost, the linear term contributes a constant amount to marginal cost, and the quadratic term causes marginal cost to rise with output.

The quadratic specification is useful for teaching because it allows the derivative and a finite cost difference to be compared without introducing complicated algebra. It also avoids a special case in which the two quantities would be identical. The function is an author-designed example, not an estimated cost relationship.

The problem asks students to determine marginal cost at an output of 100 units, calculate the exact additional cost of increasing output from 100 to 101 units, and explain why the two results are not necessarily equal.

Solution and mathematical verification

Differentiating the total cost function yields: $MC(Q) = C'(Q) = 0.04Q + 10$

At (Q=100), $MC(100) = 0.04(100) + 10 = 14$

The result is measured in currency units per unit of output. It represents the instantaneous rate at which total cost changes with output at the specified production level.

The exact increase in total cost from 100 to 101 units is calculated directly:

$$\Delta C = C(101) - C(100) = 14.02$$

The derivative gives 14, whereas the finite increase is 14.02 currency units. The discrepancy can be explained by examining the cost function over a general change in output.

For a change of (h) units from the initial output of 100, direct substitution and simplification give

$$\Delta C = C(101) - C(100) = 14.02$$

The first-order approximation based on the derivative at (Q=100) is (14h). The term (0.02h^2) is omitted in that approximation. For (h=1), this term equals 0.02, which accounts exactly for the difference between the marginal-cost estimate and the finite cost increase.

This additional calculation helps students see why the derivative is an approximation to a finite change rather than an alternative formula that should always produce the same number. As (h) approaches zero, the omitted term becomes negligible relative to the first-order term. The difference between the two calculations is therefore a consequence of the mathematical structure of the function, not a contradiction.

A second way to interpret the result is to observe that marginal cost increases with output. At the beginning of the interval, (MC(100)=14), while at its end, (MC(101)=14.04). The average marginal cost across the interval is 14.02, equal to the exact increase in total cost for a one-unit change. This equality follows from the linear marginal cost function in the present quadratic example and should not be generalized to arbitrary nonlinear marginal cost functions.

Economic interpretation

The distinction between marginal cost and the cost of producing one additional unit is not merely terminological. In economic analysis, marginal cost is defined through differentiation, whereas the cost associated with a discrete increase in output is obtained by subtracting two total cost values. The two concepts are closely related, but the relationship depends on the scale of the change and the behaviour of the cost function. Feudel and Biehler [6] investigated students' understanding of the economic interpretation of the derivative in marginal cost problems. Their work provides a direct reason for treating this distinction as a learning objective rather than assuming that students will infer it from differentiation rules alone. The units also require attention. A derivative of total cost with respect to output is expressed in currency units per unit of output, whereas a finite cost difference is expressed in currency units. When the output change is exactly one unit, the numerical values can be compared directly, but the underlying quantities retain different meanings.

Proposed classroom activities

The instructor can begin by presenting the total cost function and asking students to predict whether the derivative at (Q=100) will equal the additional cost of producing the 101st unit. Students should be asked to justify their predictions before calculating. The first calculation is performed using differentiation. The second uses direct

evaluation of the cost function. After comparing their answers, students work in pairs to explain the discrepancy. The instructor then introduces a smaller output change, such as (h=0.1), and asks students to examine whether the approximation becomes more accurate. A final discussion can focus on the meaning of the word "marginal" and why an instantaneous rate of change is useful in economic analysis. This discussion is intended to move students beyond the mechanical application of differentiation rules. The expected learning product is a short mathematical explanation containing the derivative, the finite difference, the correct units and a reasoned account of the approximation. Assessment should focus not only on numerical accuracy but also on whether the student distinguishes the two economic quantities.

3.2.2. Problem 2: Profit maximization under a price–quantity relationship

Economic situation and model construction

The second problem considers a producer whose selling price decreases as the quantity sold increases. The assumed price function is $P(Q) = 100 - Q, \quad 0 \le Q \le 80$
and the total cost function is $C(Q) = Q^2 + 20Q + 200$
Output is measured in product units, price in currency units per product unit, and total cost in currency units. The feasible interval is part of the model and must be considered when identifying the optimal output. Unlike the first problem, the mathematical objective is not given directly. Students must recognize that profit is the difference between revenue and cost. They must therefore construct the revenue function before forming the profit function. This distinction matters in teaching. A student may be able to differentiate a quadratic function correctly while still selecting the wrong economic objective. The task is designed to reveal whether students understand the difference between maximizing sales revenue and maximizing profit.

Solution and verification of the optimum

Revenue is the product of selling price and quantity: $R(Q) = P(Q)Q = 100Q - Q^2$
Profit is obtained by subtracting total cost from revenue:

$$\pi(Q) = R(Q) - C(Q) = 80Q - 2Q^2 - 200$$

Differentiating the profit function gives $\pi'(Q) = 80 - 4Q$
The first-order condition requires $\pi'(Q)$, giving: $Q^* = 20$
The second derivative is: $\pi''(Q) = -4 < 0$
The profit function is strictly concave over the feasible interval. Consequently, the stationary point at (Q=20) is the unique global maximum. The result is not established solely by solving the first-order equation; it also depends on the shape of the function and the feasibility of the solution.
At the optimal output,

$$P(20) = 80, \quad R(20) = 1600, \quad C(20) = 1000, \quad \pi(20) = 600$$

The marginal revenue and marginal cost functions are

$$MR(Q) = 100 - 2Q, \qquad MC(Q) = 2Q + 20$$

At (Q=20), both marginal quantities equal 60 currency units per product unit. The equality between marginal revenue and marginal cost is therefore satisfied at the optimum.
To check the result independently, the profit at the boundaries can be evaluated. At (Q=0), the producer incurs a loss of 200 currency units because the fixed cost remains. At (Q=80), the profit is negative 6,600 currency units. Both values are below the profit of 600 currency units at (Q=20). This confirms the global comparison on the stated interval, although strict concavity already establishes uniqueness.

Comparison between maximum revenue and maximum profit

The revenue function has derivative (R'(Q)=100-2Q). Its stationary point occurs at (Q=50), where revenue reaches 2,500 currency units. However, total cost at this output is 3,700 currency units, giving a loss of 1,200 currency units. Thus, the output that maximizes revenue is not the output that maximizes profit.
This comparison is economically important because the two objectives answer different questions. Revenue maximization concerns total sales receipts. Profit maximization concerns the amount remaining after production costs are deducted. A larger output may increase revenue while also increasing cost by an even greater amount.
The example also shows why the condition (MR=MC) should be interpreted within a complete optimization problem. It is a necessary condition for an interior optimum under differentiability, but students must still check the second-order condition or another argument establishing optimality, together with the feasible domain.

Instructional implications

The instructor can present the price and cost functions without initially writing the profit function. Students first identify the objective and explain why revenue alone is insufficient for choosing the economically preferable output.

After constructing the model, students solve the optimization problem and verify the result. They then calculate the revenue-maximizing output and compare it with the profit-maximizing output.
The comparison provides a natural opportunity for discussion. Students may be asked why a producer would reject the output level that generates the highest sales revenue. They can also examine what happens when the fixed cost changes. In the present model, a change in fixed cost alters the level of profit but does not alter the profit-maximizing output, provided that the feasible domain and the variable components of revenue and cost remain unchanged.
This extension is useful because it encourages students to distinguish changes that affect the value of the objective function from changes that affect the location of its optimum.
Assessment should examine whether students construct the correct objective, apply differentiation accurately, justify the maximum and interpret the result economically. The final answer should include an explanation, not only the value (Q=20).

3.2.3. Problem 3: Fertilizer input and the distinction between production and economic objectives
Economic situation and modelling assumptions
The third problem considers a simplified agricultural production process. Output depends on the amount of fertilizer used, according to the hypothetical production function

$$Y(x) = 10 + 4x - 0.2x^2, \quad 0 \le x \le 10$$

Here, (x) denotes fertilizer input in notional input units, while (Y(x)) denotes agricultural output in notional product units. The selling price of output is assumed to be constant at 5 currency units per product unit, and fertilizer costs 4 currency units per input unit.
The model is deliberately simple. It contains one variable input and assumes that all other production conditions remain unchanged. The quadratic form allows the marginal product of fertilizer to decline as input increases. The numerical coefficients were selected for illustration; they were not estimated from agricultural observations.
The central question is whether the fertilizer level that produces the greatest physical output also generates the greatest economic return. This question is suitable for derivative-based teaching because it requires students to recognize that the optimal solution depends on the objective function.

Maximum physical output
Differentiating the production function gives $Y'(x) = 4 - 0.4x$
The derivative is positive for (0\leq x<10) and equals zero at (x=10). The second derivative is (Y"(x)=-0.4), indicating strict concavity. Within the specified feasible interval, physical output reaches its maximum at the upper boundary: Y(10)=30
The location of this maximum deserves attention. Although the derivative is zero at (x=10), the point is a boundary of the feasible interval rather than an interior point. The conclusion that it maximizes output follows from the behaviour of the function over the entire interval.

Maximum economic return
To consider the economic decision, the model must include both the value of agricultural output and the cost of fertilizer. With a fixed output price of 5 and fertilizer cost of 4 per input unit, the objective function is $\pi(x) = 5Y(x) - 4x = 50 + 16x - x^2$
This expression measures revenue after fertilizer expenditure but before any other production costs. If other costs are constant with respect to fertilizer input, subtracting them would change the numerical value of profit without changing the optimal input level.
Differentiating gives $\pi'(x) = 16 - 2x$
The first-order condition ($\pi'(x)$=0) yields (x=8). Because the second derivative is $\pi'(x)$=-2<0), the objective function is strictly concave. The solution (x=8) is therefore the unique global maximizer within the feasible interval.
At this input level,$Y(8) = 29.2, \quad \pi(8) = 114$
At the fertilizer level that maximizes physical output, $\pi(10) = 110$
The comparison shows that increasing fertilizer from 8 to 10 input units raises physical output by 0.8 product units but reduces the economic objective by 4 currency units.
The first-order condition can also be written as $5Y'(x) = 4$
The left-hand side is the value of the marginal product of fertilizer, while the right-hand side is the marginal cost of fertilizer input. At the interior optimum, the additional value generated by a small increase in fertilizer equals its additional cost.

Why the two optima differ

The difference between (x=10) and (x=8) is not caused by conflicting mathematical methods. The two calculations optimize different functions.

Maximum physical output depends only on the production relationship (Y(x)). Maximum economic return depends on both the production relationship and the relative prices of output and fertilizer.

Between (x=8) and (x=10), additional fertilizer still increases physical output, but the value of the additional production is lower than the cost of the additional input. As a result, using more fertilizer is technically productive but economically disadvantageous under the assumptions of the model.

This distinction can be made clearer by examining the marginal product at the two input levels. At (x=8), marginal product equals 0.8 product units per input unit, and its value at the assumed output price is 4 currency units. At (x=10), marginal product is zero, while the marginal fertilizer cost remains 4 currency units. Therefore, continuing to increase fertilizer towards the output-maximizing boundary reduces economic return. The problem offers a useful example of why an optimization model must begin with a clearly stated objective. A mathematically correct answer to the wrong objective does not resolve the original economic question.

Sensitivity to price assumptions

The assumed selling price and fertilizer cost determine the economic optimum. If the output price decreases while fertilizer cost remains unchanged, the value of additional production falls, making a lower fertilizer input more attractive. Conversely, an increase in the output price can support a higher input level, provided that the resulting solution remains within the feasible interval.

This relationship can be examined without introducing empirical data. Students can replace the fixed prices with symbolic parameters and reason about the effect of changing them. The purpose is not to estimate an actual fertilizer recommendation, but to make the dependence of the optimal decision on economic conditions explicit.

Limitations of the agricultural model

The production function does not describe a particular crop, location or farming system. It omits interactions among nutrients, differences in soil fertility, weather variability, irrigation, crop management and environmental effects. It also assumes a fixed selling price and a constant unit cost of fertilizer.

These simplifications are acceptable for a teaching illustration if they are clearly identified. They would not be sufficient for recommending fertilizer application rates to farmers. A practical decision would require a production function supported by relevant field data, a fuller cost specification and consideration of agronomic and environmental constraints.

Proposed classroom activities

The instructor can begin with the question: Does applying enough fertilizer to achieve maximum yield necessarily produce the highest economic return?

Students first identify the production function and determine the input that maximizes physical output. They then construct the economic objective and solve the second optimization problem. The results are compared in a table showing fertilizer input, physical output and economic return.

A further discussion asks students to explain why the two optima differ and to identify the assumptions that would need to be tested before the model could inform a real agricultural decision.

This final stage is important because it separates the ability to solve a mathematical model from the ability to judge whether the model is suitable for practical use.

Assessment should examine the correctness of both solutions, the distinction between the objectives, the economic interpretation of marginal product and the student's ability to identify model limitations.

3.3. Proposed teaching process and assessment framework

The three problems are intended to be used as a connected sequence rather than isolated exercises. The first concentrates on conceptual interpretation, the second on constructing and solving an optimization model, and the third on comparing economic objectives and evaluating assumptions.

The proposed teaching process consists of five stages. It draws on mathematical modelling research by Blomhøj and Jensen [11], Maaß [12], and Blum and Leiß [13], but the particular organization presented here is the authors' instructional proposal.

Stage 1: Understanding the economic situation

Students identify the economic question, relevant quantities and decision variable. The instructor should ask what the problem is trying to determine before any differentiation begins.

In the marginal cost task, the question concerns a rate of change. In the profit problem, the question concerns an optimal output level. In the fertilizer problem, the question changes depending on whether the objective is physical output or economic return.

Recognizing these distinctions is necessary for selecting the appropriate mathematical approach.

Stage 2: Constructing the mathematical model

Students define variables, functions, parameters and constraints. When the model is already provided, they should explain what its terms represent. When only component relationships are given, they must construct the objective function.

The instructor should pay particular attention to units and feasible domains. A solution that lies outside the stated domain cannot be accepted simply because it satisfies a derivative equation.

Stage 3: Solving and verifying

Students differentiate the relevant functions, identify stationary points and determine whether those points satisfy the conditions of the problem. The instructor should require verification rather than accepting a first-order calculation as a complete solution.

Verification may involve a second derivative, the monotonicity of the function, comparison with boundary values or a direct argument based on the shape of the objective function.

Stage 4: Interpreting and evaluating

Students return to the economic setting and explain what the mathematical result means. They should identify whether the result represents a rate, a quantity, a monetary amount or an input decision.

They should also consider whether the assumptions are reasonable for the intended use. The fertilizer example provides a particularly clear opportunity to distinguish a classroom model from a model that could support an agricultural recommendation.

Stage 5: Consolidating and extending

The instructor asks students to summarize the reasoning used and examine a modified situation. Changes in cost coefficients, output prices or input prices can be used to explore how the solution depends on model parameters.

This stage is intended to encourage students to recognize relationships among problems rather than memorize separate solution procedures.

Table 2. Proposed teaching activities and expected learning products

Stage	Instructor activity	Student activity	Expected product
Understand	Present the economic situation and clarify the question	Identify variables and objectives	Statement of the problem
Model	Question assumptions and constraints	Construct or interpret functions	Mathematical model
Solve	Guide verification of the solution	Differentiate and check candidate solutions	Verified mathematical result
Interpret	Ask for economic justification	Explain the meaning of the result	Economic interpretation
Extend	Modify an assumption or parameter	Reconsider the model and its outcome	Revised reasoning

The process is consistent with the general emphasis on modelling activities in [11]–[13]. It also allows structured support of the kind discussed by Barile [9]. Such support may be reduced as students become more familiar with the tasks, although the appropriate amount of support would need to be established in actual teaching. The inclusion of discussion and collaborative reasoning is also compatible with broader evidence on active learning in undergraduate science, technology, engineering and mathematics education [14]. However, that evidence should not be treated as direct proof that this particular teaching sequence is effective.

Assessment framework

Assessment should reflect the full modelling process rather than focus exclusively on the final numerical answer. Five criteria are proposed: interpretation of the problem, construction of the model, mathematical solution, economic explanation and critical evaluation.

Table 3. Proposed assessment rubric

Criterion	Main evidence	Maximum score
Problem interpretation	Correct identification of quantities, variables and objectives	4
Model construction	Appropriate functions, assumptions and constraints	4
Mathematical solution	Correct differentiation and verification	4
Economic explanation	Interpretation consistent with the model and units	4
Critical evaluation	Recognition of assumptions and limitations	4
Total		**20**

The rubric is intended to support instructional planning. It has not been tested for reliability, validity or consistency between assessors. Before formal use, the criteria would need more detailed performance descriptors and review by instructors familiar with the course.

3.4. Discussion of the proposed design in relation to previous research

The proposed problem set addresses several issues identified in earlier research on calculus and mathematics for economics. The marginal cost task responds directly to the distinction between the derivative and a finite cost increase examined by Feudel and Biehler [6]. The profit problem emphasizes the construction of an

economic objective and the verification of a maximum, while the agricultural problem requires students to compare objectives and consider the limitations of a model.

The sequence is also consistent with the broader modelling perspective developed by Blomhøj and Jensen [11], Maaß [12], and Blum and Leiß [13]. These studies provide a rationale for treating model construction, mathematical processing and interpretation as connected activities. The present study applies that perspective to a limited set of derivative-based economic problems.

The approach should nevertheless be distinguished from earlier work. Lê and Phạm [8] had already designed activities linking the derivative with economics in a Vietnamese teaching context. Barile [9] discusses practical applications and structured instructional support for mathematics in economics. The present contribution is therefore not the use of economic examples itself. It is the construction of a connected problem set in which each example has an explicit mathematical solution, economic interpretation, instructional purpose and assessment focus.

The agricultural problem adds a disciplinary context that is relevant to an agriculture-oriented university. Its value as a teaching example lies in the contrast between physical production and economic return. The example makes it possible to show that an optimum is defined relative to a chosen objective and that the mathematical result must be interpreted within the assumptions of the model.

At the same time, the simplicity of the models limits what can be concluded from them. Polynomial functions with selected coefficients are useful for illustrating mathematical principles, but they do not capture the complexity of real production decisions. The agricultural example, in particular, should not be interpreted as an agronomic model validated for fertilizer management.

There are also educational limitations. The study has not examined how students respond to the problems, which stages they find difficult or whether the proposed sequence improves their ability to use derivatives. The assessment rubric has not been tested, and the teaching activities have not been reviewed against the actual course timetable or learning outcomes.

These limitations define the next stage of work. The problem set should first be reviewed by instructors in mathematics and economics to assess mathematical accuracy, economic clarity and curriculum fit. A subsequent classroom study could examine students' written solutions and explanations, with particular attention to the distinction between procedural calculation and economic interpretation.

The present findings should therefore be understood as the development and mathematical verification of a teaching design. They provide a structured set of materials for further review and investigation, not evidence of educational effectiveness.

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