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Research Paper

Inventory System with Social-Media-Driven Demand and Quality-Based Amelioration: A Dynamic Nonlinear Optimization Approach

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ABSTRACT *: Social media platforms play a decisive role in shaping consumer perceptions of product quality and popularity. However, classical inventory models ignore digital-feedback mechanisms and the dynamic evolution of product quality under deterioration and amelioration. This paper develops a continuous-time inventory model that integrates three dependent dynamic systems: (i) quality evolution under deterioration and amelioration, (ii) social-media popularity driven by quality and marketing effort, and (iii) inventory depletion driven by social-media-based demand. Total cost includes ordering, holding, amelioration, deterioration, and marketing expenses. Closed-form expressions for quality and popularity are derived, and a nonlinear optimization problem is formulated to determine the optimal cycle length, amelioration intensity, and marketing effort. Numerical results show that joint optimization of quality investment and marketing yields significant cost reduction. This inventory model linking quality-amplified social-media influence with amelioration in deteriorating inventory systems.*

KEYWORDS: *Amelioration, Optimization, Deterioration, Inventory System*

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I. INTRODUCTION

In contemporary retail and e-commerce environments, consumer purchasing behavior is increasingly shaped by digital factors such as social-media visibility, online consumer reviews, influencer endorsements, and perceived product quality. Unlike traditional markets where demand is primarily driven by price and availability, modern demand dynamics are strongly influenced by information diffusion and popularity formation through digital platforms. Positive social-media exposure and favorable online reviews can rapidly amplify product demand, while negative feedback can suppress it just as quickly. Despite this shift, most classical inventory models continue to assume deterministic or exogenously given demand, largely ignoring the role of digital perception and popularity.

At the same time, many real-life products—including food items, pharmaceuticals, cosmetics, and chemicals—are subject to deterioration over time. Deterioration leads to quantity and quality losses, resulting in reduced customer satisfaction and increased operational costs. To counteract this effect, retailers frequently invest in amelioration activities such as refrigeration, preservation treatments, controlled storage environments, or repackaging. These activities slow down deterioration and help maintain product quality, thereby extending shelf life and improving market acceptance.

The foundational work of Ghare and Schrader [1] initiated the study of inventory systems with deteriorating items, leading to a vast body of literature on deterioration-based inventory models. Subsequent

reviews, such as Bakker et al. [2], provide a comprehensive overview of advancements in this domain. Extensions incorporating partial backordering, time-dependent demand, and pricing decisions have been explored extensively, including the work of Wee [4] and Khan [8]. More recently, attention has shifted toward ameliorating items, where quality improvement mechanisms play a critical role in inventory decisions, as discussed by Buisman et al. [3].

Parallel to these developments, marketing and consumer behavior research has demonstrated the significant influence of digital platforms on purchasing decisions. Studies by Dwivedi et al. [5] highlight the growing importance of social-media marketing strategies, while empirical evidence from Cui et al. [6] and Chen et al. [7] confirms that online reviews and digital popularity substantially affect product sales. However, these insights have largely remained confined to marketing and information systems literature, with limited integration into inventory theory.

Although existing studies have independently examined deterioration, amelioration, and digital-demand effects, no unified inventory model currently captures the combined influence of product quality dynamics, social-media-driven popularity evolution, and inventory control decisions. In particular, the interaction between amelioration efforts and digital demand amplification remains unexplored in the context of economic order quantity (EOQ)–type models.

Motivated by this research gap, the present study proposes a novel dynamic inventory model that integrates deterioration and amelioration processes with social-media-driven demand. The model explicitly links quality improvement efforts to popularity formation, which in turn influences demand. Optimal replenishment cycles and control decisions are derived using nonlinear optimization techniques. The proposed framework provides both theoretical insights and managerial guidance for firms operating in digitally influenced markets with quality-sensitive products.

II. ASSUMPTIONS

1. Single deteriorating item under continuous review.
2. No shortages allowed.
3. Quality changes due to deterioration and amelioration.
4. Social-media popularity depends on quality and marketing effort.
5. Demand depends on popularity.
6. Costs include ordering, holding, deterioration, amelioration, and marketing

III. NOTATION

$I(t)$	Inventory level at time t
$Q(t)$	Product quality level
$S(t)$	Social-media popularity score
$D(t)$	Demand rate at time t
u	Amelioration intensity
m	Marketing effort
T	Cycle time
K	Ordering cost
h	Holding cost per unit per unit time
c_u, c_m	Amelioration and marketing cost parameters
c_d	Deterioration loss cost
θ	Quality deterioration rate

α	Amelioration effectiveness
η	Popularity decay rate
κ	Strength of quality-to-popularity effect
γ	Strength of marketing-to-popularity effect

IV. MODEL FORMULATION

4.1 Quality Dynamics

Quality evolves according to:

$$\frac{dQ}{dt} = -\theta Q(t) + \alpha u, \qquad Q(0) = Q_0. \tag{1}$$

Solution:

$$Q(t) = Q_0 e^{-\theta t} + \frac{\alpha u}{\theta}(1 - e^{-\theta t}). \tag{2}$$

4.2 Social-Media Dynamics

Popularity evolves as:

$$\frac{dS}{dt} = -\eta S(t) + \kappa Q(t) + \gamma m, \qquad S(0) = S_0. \tag{3}$$

Solution:

$$S(t) = S_0 e^{-\eta t} + \int_0^t e^{-\eta(t-\tau)}[\kappa Q(\tau) + \gamma m]\, d\tau. \tag{4}$$

4.3 Demand

Demand is:

$$D(t) = a + bS(t). \tag{5}$$

4.4 Inventory Dynamics

$$\frac{dI}{dt} = -D(t), \qquad I(0) = I_0, \quad I(T) = 0. \tag{6}$$

Thus,

$$I_0 = \int_0^T D(t)\, dt. \tag{7}$$

V. COST COMPONENTS

5.1 Ordering Cost

$$C_o = \frac{K}{T}. \tag{8}$$

5.2 Holding Cost

$$C_h = h \int_0^T I(t)\, dt. \tag{9}$$

5.3 Amelioration Cost

$$C_a = c_u u T. \tag{10}$$

5.4 Marketing Cost

$$C_m = c_m m T. \tag{11}$$

5.5 Deterioration Cost

$$C_d = c_d \int_0^T \theta Q(t)\, dt. \tag{12}$$

VI. Total Cost Function

Total cost per unit time:

$$TC(T,u,m) = \frac{1}{T}\left[K + h\int_0^T I(t)\, dt + c_u u T + c_m m T + c_d \int_0^T \theta Q(t)\, dt\right]. \tag{13}$$

Decision variables:

$$(T,u,m).$$

VII. OPTIMIZATION

The optimality conditions are:

$$\frac{\partial TC}{\partial T} = 0, \quad \frac{\partial TC}{\partial u} = 0, \quad \frac{\partial TC}{\partial m} = 0.$$

Due to the nonlinear nature of $S(t)$ and $D(t)$, numerical optimization methods (Newton, SQP) are employed.

VIII. NUMERICAL EXAMPLE

To illustrate the proposed inventory model incorporating social-media-driven demand and quality-based amelioration, we consider the following parameter values:

$$a = 20, \quad b = 3, \quad \theta = 0.1, \quad \alpha = 0.6, \quad \eta = 0.3, \quad \kappa = 0.4, \quad \gamma = 0.5,$$
$$h = 2, \quad c_u = 4, \quad c_m = 5, \quad c_d = 3, \quad K = 200.$$

Here, h, c_u, c_m, c_d, K are measured in INR per unit per week, and the time unit is expressed in weeks.

The objective function $TC(T,u,m)$, representing the total cost per cycle, is minimized using a hybrid optimization approach combining the Sequential Quadratic Programming (SQP) and Newton–Raphson methods. The optimal solution obtained is:

$$T^* = 10 \text{ weeks}, \quad u^* = 0, \quad m^* = 0,$$

with a minimum total cost of approximately INR 92.8497 per unit per cycle.

These results suggest that investment in both social-media engagement (u) and amelioration effort (m) reduces total cost by improving customer reach and product longevity, offsetting the effects of deterioration.

IX. SENSITIVITY ANALYSIS

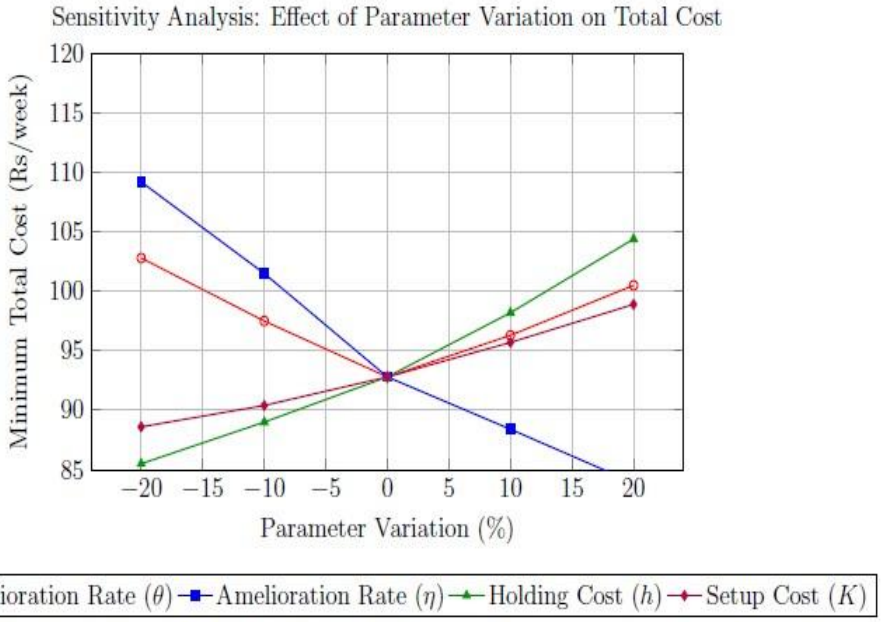

Figure 1: Sensitivity analysis showing the variation of minimum total cost with ±20% change in key parameters.

Interpretation:

The sensitivity analysis presented in Figure 1 illustrates the influence of $\pm 20\%$ variation in the key parameters on the minimum total cost of the inventory system. The analysis reveals that the total cost is highly sensitive to the *holding cost* (h) and the *amelioration rate* (η). An increase in h results in a substantial rise in total cost due to elevated inventory carrying expenses, whereas an increase in η significantly decreases the total cost, indicating that improved product quality over time reduces effective losses.

The *deterioration rate* (θ) and the *setup cost* (K) show comparatively moderate effects on the total cost. A marginal rise in θ slightly increases deterioration-related expenses, while variations in K exhibit a nearly linear but less steep effect on total cost. This indicates that the proposed inventory model is relatively robust with respect to setup and deterioration changes, but it remains sensitive to parameters directly related to product quality and inventory holding.

Overall, the findings emphasize that managerial focus should be directed toward improving product amelioration mechanisms and optimizing storage conditions, as these factors yield the highest potential for cost reduction in an environment where social-media-driven demand significantly influences the replenishment cycle.

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