Quest Journals
Journal of Research in Applied Mathematics
Volume 12 ~ Issue 2 (February 2026) pp: 54-65
ISSN (Online): 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

Stability Analysis on Double Diffusive Convection for a Newtonian Fluid with Thermal Diffusion and Internal Heat Generation/Absorption

E. Amos, K. T. Davies, I. Davies, L. Ebiwareme
Department of Mathematics, Rivers State University, Nkpolu- Oroworukwo, Nigeria

ABSTRACT

The investigation, Stability Analysis on Double Diffusive Convection with Thermal Diffusion and Internal Heat Generation and Absorption was carried out for Newtonian fluid systems. The Boussinesq approximation was used for the density variation with temperature and concentration. Also, the Rosseland approximation was adopted for the radiative flux. Governing equations that incorporate the coupling effect of thermal diffusion and internal heat generation and absorption is developed. The governing equations are non- dimensionalised, and the emerging nonlinear partial differential equations governing the flow are solved using the regular perturbation, normal mode representation and linear stability analysis processes. In order to compare our mathematical solutions to findings from existing literature, profiles were developed using Mathematica software to test for the sensitivity of the pertinent parameters on the onset of instability of the system. The results showed that for a Newtonian fluid, the increase in Lewis number, Le, internal heat generation, Q, and magnetic field, M, parameters has a delaying effect on the onset of instability, with higher values of pertinent parameters resulting in a greater stabilization of the system, while an increase in the Soret, porosity and radiation parameters increases the onset of instability, that is, higher Soret, porosity and radiation parameters destabilizes the system. Our results are in agreement with existing literatures. In conclusion, we were able to investigate the stability behaviour of Double Diffusive Convection for Newtonian fluid systems experiencing the coupling effects of thermal diffusion and internal heat generation and absorption using linear stability analysis.

Keywords: *Stability Analysis, Double Diffusive Convection, Thermal Diffusion, Porous Medium, Internal Heat Generation and Absorption, Newtonian Fluid.*

*Received 03 Feb., 2026; **R**evised 10 Feb., 2026; **A**ccepted 12 Feb., 2026*

I. INTRODUCTION

Double diffusive convection is a fluid dynamics phenomenon that describes a form of convection caused by two different density gradients which have different rates of diffusion as a result of density variations within them due to the composition of the fluid or by thermal expansion, that is difference in temperature. When a fluid is heated from below thermal expansion takes place, molecules from below which becomes less dense as a result of the heating moves upward away from the heat source while molecules which are above (cold) replaces them downwards because they denser. This process is a repeated cycle called heat transfer through convection [1].

Double diffusive convection is relevant in nature and industrial applications. Such complex flow structures are often found in a wide variety of fields namely: geosciences, oceanography, astrophysics and so on [2]. Thermal diffusion plays an important role in the study of compositional variation in hydrocarbon reservoirs, hydrodynamic instability of mixtures and mineral migrations and mass transport in turning matters [3]. Internal heat generation and absorption as pertains to moving fluids is key in several physical problems dealing with chemical reactions and dissociating fluids. The analysis of the convection problem is also considered to be of great importance in structures due to transportation of fluid molecules from one point of a media to another caused by heating, hence notable researches have been carried out in this area also.

[4] carried out an investigation for a fully developed unsteady magnetohydrodynamic free convection flow of a viscous incompressible and electrically conducting Newtonian fluid through porous medium bounded by an infinite vertical porous plate, in a rotating system in the presence of heat source and thermal diffusion. It was observed that the velocity boundary layer condenses with the increasing value of Schmidt number, flow reversal is prevented for low speed of rotation, high value of chemical reacting species and high value of magnetic parameter.

[5], considered similarities solutions of natural convection with internal heat generation and came to the conclusion that the presence of internal energy generation leads to increased flow and in some cases, temperature that exceed the wall temperature, especially for fluids with Pr < 1.0.

[6], studied double diffusive natural convection heating from below the wall in a closed cavity was studied with direct numerical simulation method. The flow characteristics are presented as isotherms, iso concentrations, and streamlines. The results show that Ra_T has signification strong effects on average Nusselt number, with a more considerable Ra_T, the value of Nusselt number higher. These were also found to increase with increasing buoyancy ratio for aiding flow and decrease as buoyancy ratio decreases for opposing flows.

[7], considered the effects of concentration based internal heat and vertical magnetic field on the onset of double diffusive convection in a horizontal porous layer using normal mode analysis. They concluded that the concentration based internal heat, γ, hastens the onset of instability while the magnetic field, H_a, and solutal Rayleigh number, R_s delays the onset of instability in the system for stationary and oscillatory convections. They also presented the influence of Lewis number, L_e and porosity, ε. [8] inquired into the effects of soret and magnetic field on thermosolutal convection in a porous medium with concentration based internal source. They employed linear stability analysis to determine the onset of instability. It was established that the stability of the system occurred for values of internal heat, $\gamma_c < 0.7$, while instability sets in for $\gamma_c \geq 0.7$ for all values of Hartmann number, H_a, and Soret, S_r.

[9], examined the internal heat source and reaction effects on the onset of thermosolutal convection in a local thermal non-equilibrium porous medium, where the temperature of the fluid and the solid skeleton may differ. Their results detect that utilizing the internal heat source, reaction, and non-equilibrium have pronounced effects in determining the convection stability and instability thresholds.

[10], suggested that possible heat generation effects may alter temperature distribution and therefore, particle deposition rate. The effect of internal generation and absorption is applicable in reactor safety analysis, metal waste, spent nuclear fuel, fire and combustion studies, strength of radioactive materials and so on.

[11], investigated double-diffusive convection with chemical reaction in a Darcy–Brinkman porous layer, heated and salted below. The effect of the thermal contribution of the chemical reaction was found to be more effective on the convective instability in comparison with the solutal contribution of the chemical reaction. The reaction parameters k_1 k_2, enhance the onset of convection in the stationary mode, however, in oscillatory mode, the onset of convection is seen to be delayed.

The impact of viscous dissipation and radiation on unsteady magnetohydrodynamic free convection stream past an endless vertically warmed plate in some optically thin surroundings with time dependent suction was studied by [12]. They sort for the impact of key parameters on the temperature and velocity profiles. Their results show that increase in Grashof number and Eckert number results in an increase in the velocity profile while increase in magnetic field, radiation and Darcy parameters leads to decrease in velocity. Also, increasing the Eckert number gives rise to an increment in the temperature while increase in radiation and magnetic parameters results in a decrease in temperature during the cooling of the plate.

[13], numerically studied micropolar fluid behavior on unsteady two dimensional oscillatory electrically conducting viscous, incompressible Boussinesq fluid flow along an unbounded vertical plate with periodical temperature variation about an average non-zero constant value with item. This study is useful in the production of electro-conductive polymer.

In a research carried out by [14], on unsteady MHD free convection warmth and mass exchange flow of Newtonian flow past boundless vertical plate with homogenous chemical reaction and heat absorption, they non-dimensionalized the leading equations and solved them using multiple perturbation method which were subject to some boundary conditions. This result showed that an increase in the radiation absorption parameter led to an increase in velocity, temperature and skin friction.

[15] numerically studied the problem of unsteady free convection with heat and mass transfer from an isothermal vertical plate in a porous media. With the aid of boundary layer, Boussinesq approximation and Darcy-

Brinkman Forcheimer model, the power law fluid was modelled. This research revealed that the friction factor, heat and mass transfer tend to a steady state value as the time tends towards infinity.

The literatures reviewed, shows that double diffusive convection in porous media is a multifaceted phenomenon influenced by coupled physical processes such as temperature gradients, solutal gradients and radiative heat transfer. While foundational theories by [16], [17] and [18] remain indispensable, contemporary advancements now allow richer modeling of real systems involving thermal radiation and internal energy sources.

3.1 Mathematical Formulation:

Consider an infinite horizontal porous layer of thickness H filled with binary fluid and confined between two rigid plates, see Figure 3.1. The horizontal plates are located at $Z' = 0$ and $Z' = H$ and heated from below, therefore different temperatures T_1 and T_2 and solutal mass concentration C_1 and C_2 respectively exist at the bottom and top of the rigid plates such that $\Delta T = T_1 - T_2\ (> 0)$ and $\Delta C = C_1 - C_2\ (> 0)$.

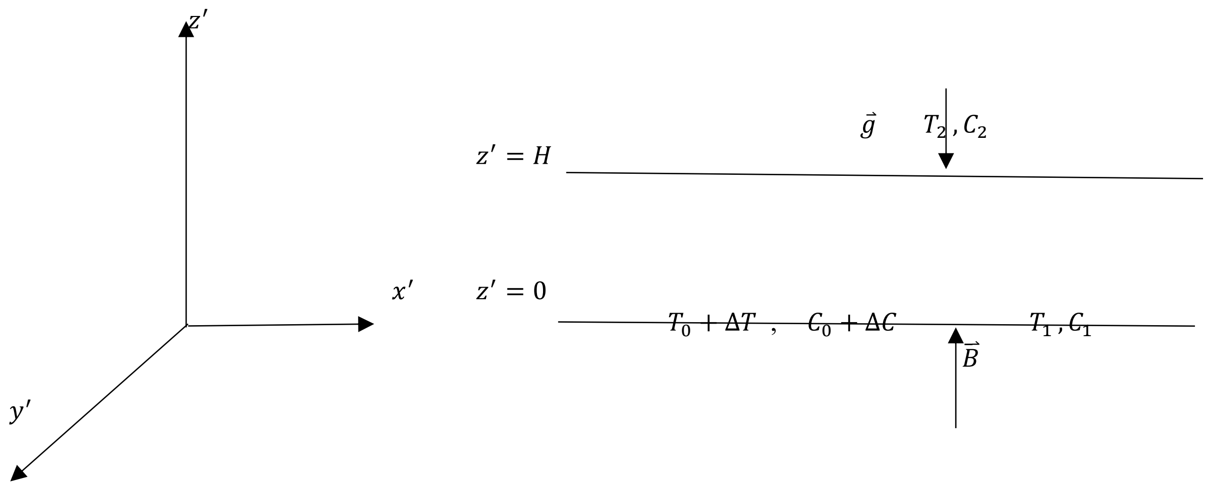

Fig 1: The Physical Configuration.

We apply a magnetic field of strength $\vec{B}$ perpendicular to the plates. The saturating fluid is assumed incompressible, Newtonian and electrically conducting. The Boussinesq approximation is a key simplification for problems in which fluid varies in temperature (or composition), driving a flow of fluid and heat/mass transfer [19]. The onset of thermal and solutal convection is under the Boussinesq approximation, consequently we adopt the state equation [20],

$$\rho = \rho_0[1 - \beta_T(T' - T_0) + \beta_c(C' - C_0)] \tag{1}$$

We take into consideration the absorbing and emitting characteristics of the fluid radiating in a non-scattering medium. Under the assumptions and considerations above, the governing equations are presented as:

$$\vec{\nabla}.\vec{V}' = 0 \tag{2}$$

$$\rho_0\left(\frac{\partial \vec{V}'}{\partial t'} + \vec{V}'.\vec{\nabla}'\vec{V}'\right) = -\nabla' P' + \mu \nabla'^2.\vec{V}' - \mu\frac{\vec{V}'}{k} + \vec{F}_{em} + \rho g \tag{3}$$

$$\rho_0 C_P\left(\frac{\partial T'}{\partial t'} + \vec{V}'.\nabla' T'\right) = k\nabla'^2 T' - \nabla'.q'_r + Q_0(T' - T_0) \tag{4}$$

$$k\left(\frac{\partial C'}{\partial t'} + \vec{V}'.\nabla' C'\right) = \nu\left(D\nabla'^2 C' + D^* C_0 \nabla'^2 T'\right) \tag{5}$$

where $\vec{V}'$, T_0 and C_0 are velocity, reference temperature and concentration respectively.

The boundary conditions are:

$$\vec{V}' = 0, \quad T' = T_0 + T_1 - T_2, \quad C' = C_0 + C_1 - C_2 \quad \text{at} \quad Z' = 0 \tag{6a}$$

$$\vec{V}' = 0, \quad T' = T_0, \quad C' = C_0, \quad \text{at} \quad Z' = H \tag{6b}$$

According to the Boussinesq approximation, density variations only pertains to the buoyancy term in the body force term, ρg, consequently Equation (3) becomes:

$$\rho_0\left(\frac{\partial \vec{V}'}{\partial t'} + \vec{V}'.\vec{\nabla}'\vec{V}'\right) = -\nabla'P' + \mu\nabla'^2.\vec{V}' - \mu\frac{\vec{V}'}{k} - g\beta_T\rho_0(T'-T_0)\hat{e}_z + g\beta_c\rho_0(C'-C_0)\hat{e}_z + \vec{F}_{em} \quad (7)$$

By the absorbing and emitting characteristics of the fluid, we use the Roseland approximation [21] for the radiative flux, hence in equation (4), we have:

$$q'_r = -\frac{4\sigma*\nabla'T'^4}{3\delta} \quad (8)$$

We then assume that the difference in temperature within the fluid and the porous medium is sufficiently small. By this assumption, T'^4 can be expressed as a linear function of the temperature, T', and expanded about the free stream temperature, T_0, using Taylor series and neglecting higher order terms to obtain [20]:

$$T'^4 \approx 4{T_0}^3T' - 3{T_0}^4$$

We then have

$$q'_r = -\frac{4\sigma *}{3\delta}\nabla'(4{T_0}^3T' - 3{T_0}^4)$$

$$q'_r = -\frac{16\sigma*}{3\delta}{T_0}^3\nabla'T' \quad (9)$$

Next, we consider the electromagnetic force, $F_{em} = J_e \times \vec{B}$ in equation (7). The electromagnetic force is defined by:

$$J_e = \sigma(-\nabla'Q_e + \vec{V}' \times \vec{B}) \quad (10)$$

With electrically insulated boundaries, the electric potential Q_e is constant [22]. Thus, the induced magnetic field is negligible and equation (10) reduces to

$$\vec{J} = \sigma(\vec{V}' \times \vec{B}) \quad (11)$$

and consequently, F_{em} in equation (7) becomes

$$\sigma(\vec{V}' \times \vec{B}) \times \vec{B} = -\sigma B^2\vec{V}' \quad (12)$$

Substituting Equations (9) and (12) into Equations (7) and (4) we obtain the governing equations as:

$$\rho_0\left(\frac{\partial \vec{V}'}{\partial t'} + \vec{V}'.\vec{\nabla}'\vec{V}'\right) = \nabla'P' + \mu\nabla'^2.\vec{V}' - \vec{g}\beta_T\rho_0(T'-T_0)\hat{e}_z + \vec{g}\beta_c\rho_0(C'-C_0)\hat{e}_z - \mu\frac{\vec{V}'}{k} - \sigma B^2\vec{V}' \quad (13)$$

$$\rho_0C_P\left(\frac{\partial T'}{\partial t'} + \vec{V}'.\nabla'T'\right) = k\nabla'^2T' + \frac{16\sigma*}{3\delta}{T_0}^3\nabla'^2T' + Q_0(T'-T_0) \quad (14)$$

$$k\left(\frac{\partial C'}{\partial t'} + \vec{V}'.\nabla'C'\right) = \nu\left(D\nabla'^2C' + D^*C_0\,\nabla'^2T'\right) \quad (15)$$

subject to the boundary condition in equation (6a) and (6b).

The last term in equation (14) is based on the amount of heat generated or absorbed per unit volume, with Q_0 being constant coefficient that may take either positive or negative values. The source term represents the heat generation that is distributed everywhere when Q_0 is positive ($Q_0 > 0$) and heat absorption when Q_0 is negative ($Q_0 < 0$); $Q_0 = 0$ is the case where there is no heat generation or absorption.

We present the non-dimensional variables as follows:

$$\left.\begin{aligned} t = \frac{t'\nu}{H^2}, \quad \vec{V} = \frac{H\vec{V}'}{\nu}, \quad (x,y,z) = \frac{1}{H}(x',y',z'), \quad \theta = \frac{T'-T_0}{T_1-T_2} \\ C = \frac{C'-C_0}{C_1-C_2}, \quad P = \frac{P'H^2}{\rho_0\nu^2}, \quad R = \frac{16\sigma*{T_0}^3}{3k\delta}, \quad \chi = \frac{H^2}{k} \\ M = \frac{\sigma B^2H^3}{\rho_0\nu}, \quad Q = \frac{H^2Q_0}{\rho_0C_P\nu}, \quad S = \frac{D^*C_0(T_1-T_2)}{D(C_1-C_2)} \end{aligned}\right\} \quad (16)$$

Under these non-dimensional variables, the governing equations take the form:

$$\frac{\partial V}{\partial t} + V.\nabla V = \nabla P + \nabla^2 V - \frac{R_T}{P_r}\theta + \frac{R_C}{P_r}C - \chi V - MV \quad (17)$$

$$\frac{\partial \theta}{\partial t} + V.\nabla\theta = \frac{1}{Pr}\nabla^2\theta + \frac{R}{Pr}\nabla^2\theta + Q\theta \quad (18)$$

$$\frac{\partial C}{\partial t} + V.\nabla C = \frac{1}{Le}(\nabla^2 C + S\nabla^2\theta) \quad (19)$$

Subsequently, non dimensionalizing the boundary conditions, Equations (6a) and (6b) with relevant variables from Equation (16), we obtain our dimensionless boundary conditions as:

$$V = 0, \quad \theta = 1, \quad C = 1 \qquad \text{at} \quad z = 0 \quad (20)$$

$$V = 0, \quad \theta = 0, \quad C = 0 \qquad \text{at} \quad z = 1 \quad (21)$$

3.2 Stability Analysis

3.2.1 Basic state and flow linearization

The basic state of the system is given by the static solution $V = 0$ and $\frac{\partial}{\partial t} = 0$ of equations (17) – (21). Thus, the static temperature, T_s , solutal mass concentration, C_s and pressure, P_s are obtained as:

$$P_s(z) = \frac{1}{P_r}\int (R_T T_s - R_C C_s)\, dz \quad (22)$$

$$T_s(z) = cos\left(\sqrt{\frac{PrQ}{(1+R)}}\right)z - cot\left(\sqrt{\frac{PrQ}{(1+R)}}\right)sin\left(\sqrt{\frac{PrQ}{(1+R)}}\right)z \quad (23)$$

$$C_s(z) = \frac{SPrQ_s}{(1+R)c}\left[cot\sqrt{c}sin\sqrt{c}z - cos\sqrt{c}z - z + 1\right] - z + 1 \quad (24)$$

3.2.2 Linear Stability Analysis

To access the stability of the state solutions, we let the initial solutions described by equations (22), (23) and (24) to be slightly perturbed. Thus, we define a perturbation of the form [23]:

$$V = 0 + (u, v, w)\,, \ \theta = T_s + \bar{\theta}\,, \quad C = C_s + \bar{C}\,, \quad P = P_s + \bar{P} \quad (25)$$

Upon substituting these perturbations into the non-dimensional Equations (17) – (19) and neglecting the products of disturbances, the linearized perturbation equations are obtained as:

$$\left(\frac{\partial}{\partial t} - \nabla^2 + \chi + M\right)(u, v, w) = \nabla P - \frac{1}{P_r}(R_T\bar{\theta} - R_C\bar{C})\hat{e_z} \quad (26)$$

$$\left(Pr\frac{\partial}{\partial t} - (1+R)\nabla^2 - PrQ\right)\bar{\theta} = Prw \quad (27)$$

$$\left(\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)\bar{C} = w + \frac{1}{Le}S\nabla^2\bar{\theta} \quad (28)$$

with the boundary condition given as:

$$w = 0, \quad \bar{\theta} = 1 \quad \bar{C} = 1 \qquad \text{at} \quad z = 0 \quad (29)$$

$$w = 0, \quad \bar{\theta} = 0 \quad \bar{C} = 0 \qquad \text{at} \quad z = 1 \quad (30)$$

Proceeding on the analysis, we reduce equation (26) to a scalar equation by taking the double curl of it, using the equation of continuity (∇.V=0) and keeping only the vertical component of the velocity yields:

$$\left(\frac{\partial}{\partial t} - \nabla^2 + \chi + M\right)\frac{\partial^2 w}{\partial z^2} = \frac{1}{P_r}(R_T\nabla_h^2\bar{\theta} - R_C\nabla_h^2\bar{C}) \quad (31)$$

Where $\nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian operator in the horizontal plate.

3.2.3 The dispersion Relation

We next examine the reaction of the system to all possible disturbances. This can be accomplished by expressing an arbitrary disturbance as a superposition of certain modes. Accordingly following [23], we apply the normal mode representation of the form

$$w = W(z)f(x,y)e^{\Omega t}, \quad \theta = \Theta(z)f(x,y)e^{\Omega t} \quad C = \phi(z)f(x,y)e^{\Omega t} \tag{32}$$

Where $\Omega = \Omega_R + i\Omega_C$ is complex and Ω_R, Ω_C are real numbers. Substituting equation (32) into equation (27), (28), (29), (30) and (31) we obtain the equations below:

$$(D^2 - a^2 + \propto Q - \propto \Omega)\Theta = -\propto W \tag{33}$$

$$(D^2 - a^2 - Le\Omega)\phi = -S(D^2 - a^2)\Theta - \frac{W}{Le} \tag{34}$$

$$(D^2 - a^2)(D^2 - a^2 - \chi - M - \Omega)W - \frac{1}{P_r}R_T a^2\Theta + \frac{1}{P_r}R_C a^2\phi = 0 \tag{35}$$

Note: $\nabla^2 = \frac{\partial^2}{\partial z^2} = D^2 - a^2$, $\propto = \frac{Pr}{(1+R)}$, $\Omega = \frac{\partial}{\partial t}$, $\nabla_h^2 = -a^2$, $\bar{C} = \phi$, $\bar{\theta} = \Theta$

subject to:

$$W = 0, \quad \Theta = 1 \quad \phi = 1 \qquad \text{at} \quad z = 0 \tag{36}$$

$$W = 0, \quad \Theta = 0 \quad \phi = 0 \qquad \text{at} \quad z = 1 \tag{37}$$

$D^2 = 0$ on a free surface. (38)

where a^2 is a wave number arising from the separation of variables.

Next, we reduce the system (33), (34) and (35) to a singular scalar equation by eliminating

Θ and ϕ through solving for the determinant of a matrix as seen below:

$$\begin{pmatrix} (D^2 - a^2)(D^2 - a^2 - \chi - M - \Omega) & -\dfrac{a^2R_T}{P_r} & \dfrac{a^2R_c}{P_r} \\ \propto & (D^2 - a^2 + \propto Q - \propto \Omega) & 0 \\ \dfrac{1}{Le} & -S(D^2 - a^2) & (D^2 - a^2 - Le\Omega) \end{pmatrix}$$

$$(D^2 - a^2)(D^2 - a^2 - \chi - M - \Omega)[(D^2 - a^2 + \propto Q - \propto \Omega)(D^2 - a^2 - Le\Omega)]W + \frac{a^2R_T}{P_r}[\propto (D^2 - a^2 - Le\Omega) - 0]W + \frac{a^2R_C}{P_r}\left[-\propto S(D^2 - a^2) - \frac{1}{Le}(D^2 - a^2 + \propto Q - \propto \Omega)\right]W = 0$$

$$(D^2 - a^2)(D^2 - a^2 - \chi - M - \Omega)(D^2 - a^2 + \propto Q - \propto \Omega)(D^2 - a^2 - Le\Omega)W + \frac{a^2\propto R_T}{P_r}(D^2 - a^2 - Le\Omega)W - \frac{a^2R_C}{P_r}\left[\frac{1}{Le}(D^2 - a^2 + \propto Q - \propto \Omega) + S \propto (D^2 - a^2)\right]W = 0 \tag{39}$$

now subject to:

$$W = D^2W = D^4W = \cdots = 0 \qquad \text{at } Z = 0 \text{ or } 1 \tag{40}$$

Where;

$\frac{d^2}{dx^2} + \frac{d^2}{dy^2} = -a^2$, $\propto = \frac{Pr}{(1+R)}$, $D = \frac{d}{dz}$

For the dispersion relation (39) in which the boundary condition (40) holds, we assume the solution of (40) for the lowest state in the form below [17]:

$$W = w_0 \sin \pi z \qquad w_0 \text{ is a constant} \tag{41}$$

Substituting equation (41) into (39) and simplifying for R_T we obtain:

$$\frac{a^2\propto R_T}{P_r}(D^2 - a^2 - Le\Omega) = -(D^2 - a^2)(D^2 - a^2 - \chi - M - \Omega)(D^2 - a^2 + \propto Q - \propto \Omega)(D^2 - a^2 - Le\Omega) + \frac{a^2R_c}{P_rLe}(D^2 - a^2 + \propto Q - \propto \Omega) + \frac{a^2SR_C\propto}{P_r}(D^2 - a^2)$$

Substituting $D^2 = \pi^2$ in the above equation we have:

$$R_T = \frac{(1+R)}{a^2(\pi^2+a^2+Le\Omega)}\Big[(\pi^2 + a^2)(\pi^2 + a^2 + \chi + M + \Omega)\left(\pi^2 + a^2 - \frac{PrQ}{(1+R)} + \frac{Pr\Omega}{(1+R)}\right)(\pi^2 + a^2 + Le\Omega) + \frac{a^2R_c}{P_rLe}\left(\pi^2 + a^2 - \frac{PrQ}{(1+R)} + \frac{Pr\Omega}{(1+R)}\right) + \frac{a^2SR_c}{(1+R)}(\pi^2 + a^2)\Big] \quad (42)$$

The transition from stability to instability occurs through a stationary state. Thus, to study the case of marginal stability which corresponds to stationary convection, we set $\Omega = 0$ in equation (42) to obtain

$$R_T = \frac{(1+R)}{a^2(\pi^2+a^2)}\Big[(\pi^2 + a^2)(\pi^2 + a^2 + \chi + M)\left(\pi^2 + a^2 - \frac{PrQ}{(1+R)}\right)(\pi^2 + a^2) + \frac{a^2R_c}{P_rLe}\left(\pi^2 + a^2 - \frac{PrQ}{(1+R)}\right) + \frac{a^2SR_c}{(1+R)}(\pi^2 + a^2)\Big]$$

Therefore,

$$R_T = \frac{(1+R)}{a^2}(\pi^2 + a^2)(\pi^2 + a^2 + \chi + M)\left(\pi^2 + a^2 - \frac{PrQ}{(1+R)}\right) + \frac{(1+R)R_c}{(\pi^2+a^2)P_rLe}\left(\pi^2 + a^2 - \frac{PrQ}{(1+R)}\right) + R_cS \quad (43)$$

Let $a = a_c$ and the corresponding thermal Rayleigh number be R_{Tcri} in equation (43) we have:

$$R_{Tcri} = \frac{(1+R)}{{a_c}^2}(\pi^2 + {a_c}^2)(\pi^2 + {a_c}^2 + \chi + M)\left(\pi^2 + {a_c}^2 - \frac{PrQ}{(1+R)}\right) + \frac{(1+R)R_c}{(\pi^2+{a_c}^2)P_rLe}\left(\pi^2 + {a_c}^2 - \frac{PrQ}{(1+R)}\right) + R_cS \quad (44)$$

where a_c is the critical wave number and R_{Tcri} is the critical thermal Rayleigh number. The critical wave number for the onset of instability is determined when:

$$\frac{\partial R_{Tcri}}{\partial a_c} = 0 \quad (45)$$

Using condition (45) we differentiate equation (44) in parts to obtain:

$$(1 + R)\left\{\left(\frac{\pi^2+{a_c}^2+\chi+M}{{a_c}^2}\right)\left[4a_c(\pi^2 + {a_c}^2) - 2\frac{PrQ}{(1+R)}a_c\right] + \left[(\pi^2 + {a_c}^2)^2 - \frac{PrQ}{(1+R)}(\pi^2 + {a_c}^2)\right]\left(\frac{-2\pi^2-2\chi-2M}{{a_c}^3}\right)\right\} + \frac{2a_c}{(\pi^2+{a_c}^2)^2}\frac{R_cQ}{Le} = 0 \quad (46)$$

Simplifying equation (46) we have:

$$Le(1 + R)[-2\pi^8{a_c}^2 + 2\pi^6{a_c}^4 - 2\pi^6\chi{a_c}^2 - 2\pi^6M{a_c}^2 + 8\pi^4{a_c}^6 + 2\pi^2\chi{a_c}^6 + 2\pi^2M{a_c}^6 + 7\pi^2{a_c}^8 + \chi{a_c}^8 + M{a_c}^8 + 2{a_c}^{10} - \pi^{10} - \pi^8\chi - \pi^8M] + PrLeQ[\pi^8 + \pi^6\chi + \pi^6M + 2\pi^6{a_c}^2 + 2\pi^4\chi{a_c}^2 + 2\pi^4M{a_c}^2 + \pi^2\chi{a_c}^4 + \pi^2M{a_c}^4 - 2\pi^2{a_c}^6 - {a_c}^8] + {a_c}^4R_cQ = 0 \quad (47)$$

Rearranging the terms in the order of a_c:

$$[2Le(1 + R)]{a_c}^{10} + [7Le(1 + R)\pi^2 + Le(1 + R)\chi + Le(1 + R)M - PrLeQ]{a_c}^8 + [8Le(1 + R)\pi^4 + 2Le(1 + R)\pi^2\chi + 2Le(1 + R)\pi^2M - 2PrLeQ\pi^2]{a_c}^6 + [2Le(1 + R)\pi^6 + PrLeQ\pi^2\chi + PrLeQ\pi^2M + R_cQ]{a_c}^4 + [2PrLeQ\pi^6 + 2PrLeQ\pi^4\chi + 2PrLeQ\pi^4M - 2Le(1 + R)\pi^8 - 2Le(1 + R)\pi^6\chi - 2Le(1 + R)\pi^6M]{a_c}^2 + [PrLeQ\pi^8 + PrLeQ\pi^6\chi + PrLeQ\pi^6M - Le(1 + R)\pi^{10} - Le(1 + R)\pi^8\chi - Le(1 + R)\pi^8M] = 0 \quad (48)$$

IV. RESULTS AND DISCUSSION

To be able to understand the physical implications of our model and compare our mathematical solutions to findings from existing literatures, profiles were developed using Mathematica software to test for the impact of various parameters such as Lewis number, Le, Soret parameter, S, Poroxity, χ, Heat Source, Q, Radiation, R, and Prandtl number, Pr, on the onset of instability for our system.

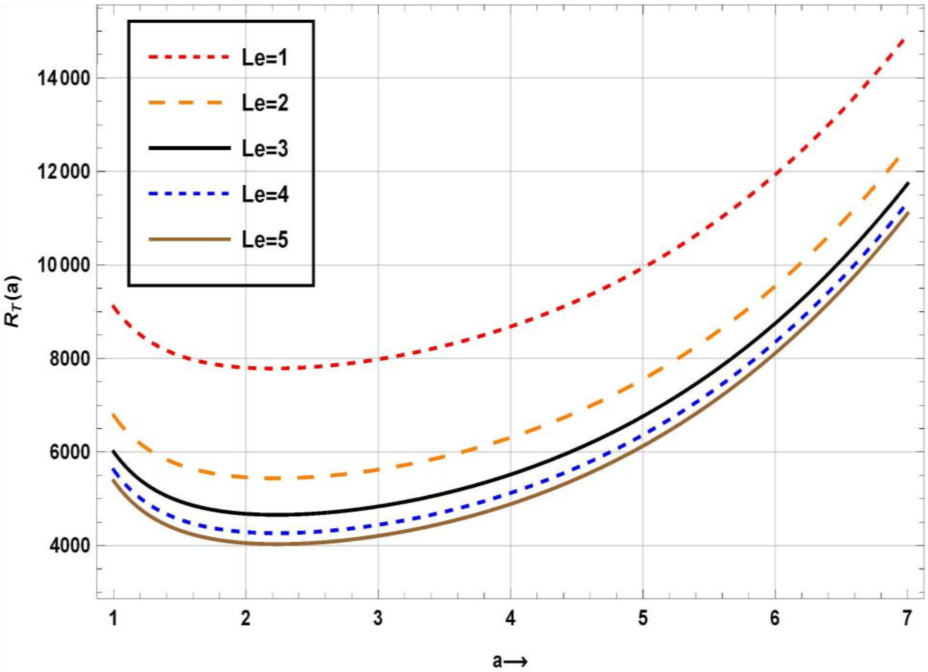

Figure: 2: Effect of Lewis number, Le, on the onset of instability for:

$Q = 1, R = 1, Rc = 1708, M = 1.0156, S = 1, \chi = 2$

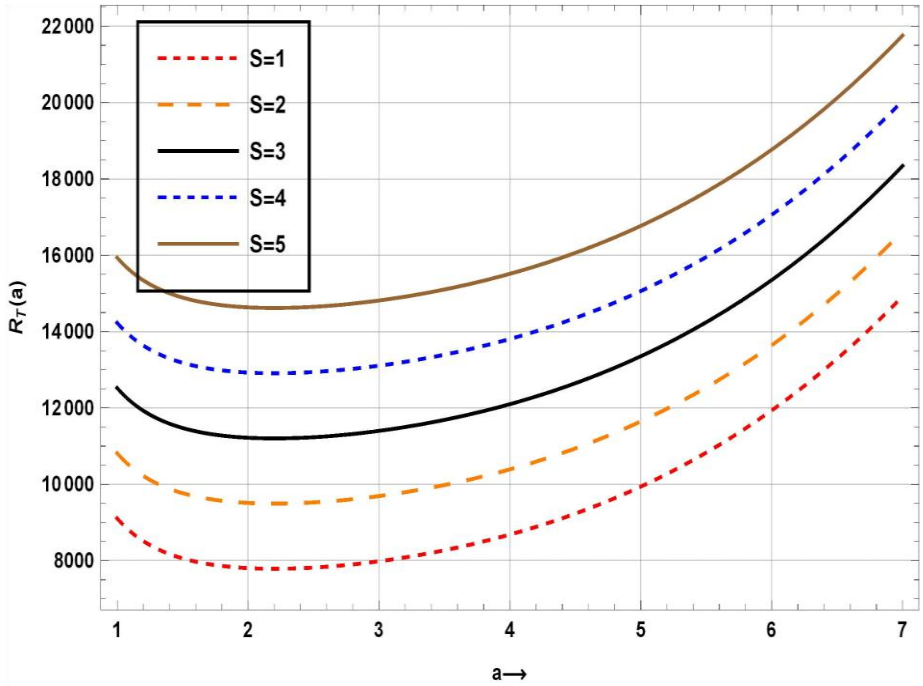

Figure: 3: Effect of Soret parameter, S, on the onset of instability for:

$Q = 1, R = 1, Rc = 1708, M = 1.0156, Le = 1, \chi = 2$

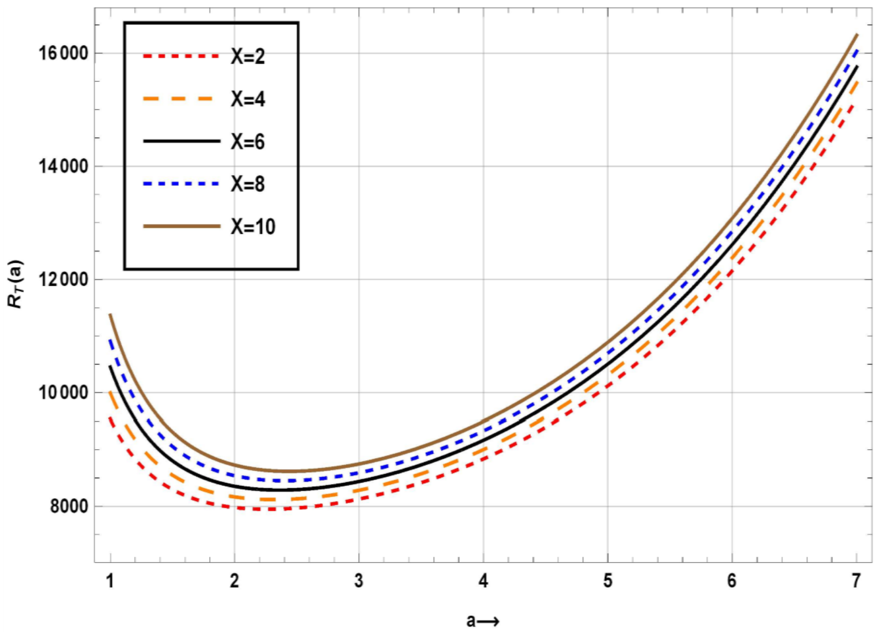

Figure: 4: Effect of Porosity, χ, on the onset of instability for:

$Le = 1, Q = 1, R = 1, Rc = 1708, M = 1.0156, S = 1$

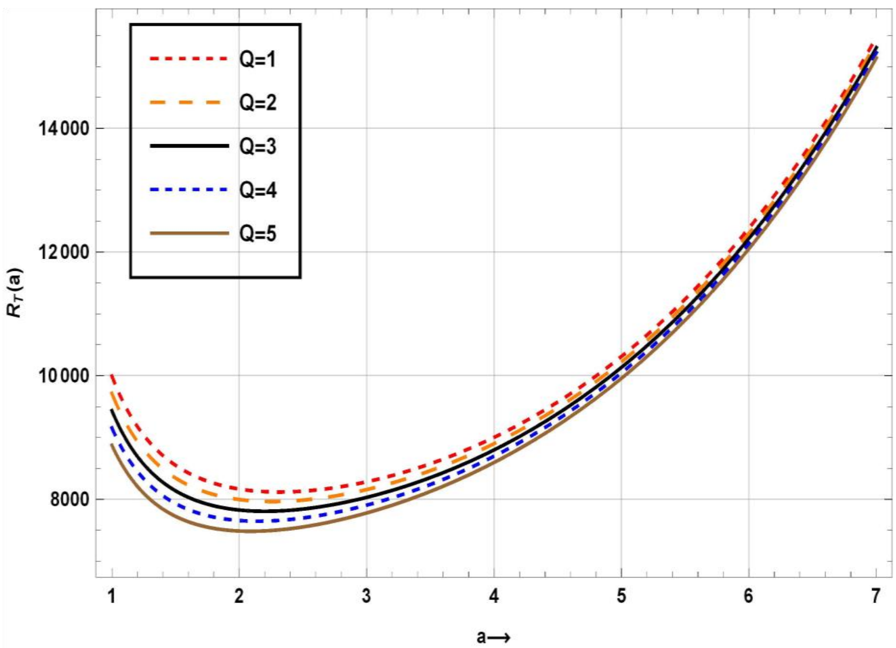

Figure: 5: Effect of internal heat parameter, Q, on the onset of instability for:

$Le = 1, \chi = 2, R = 1, Rc = 1708, M = 1.0156, S = 1$

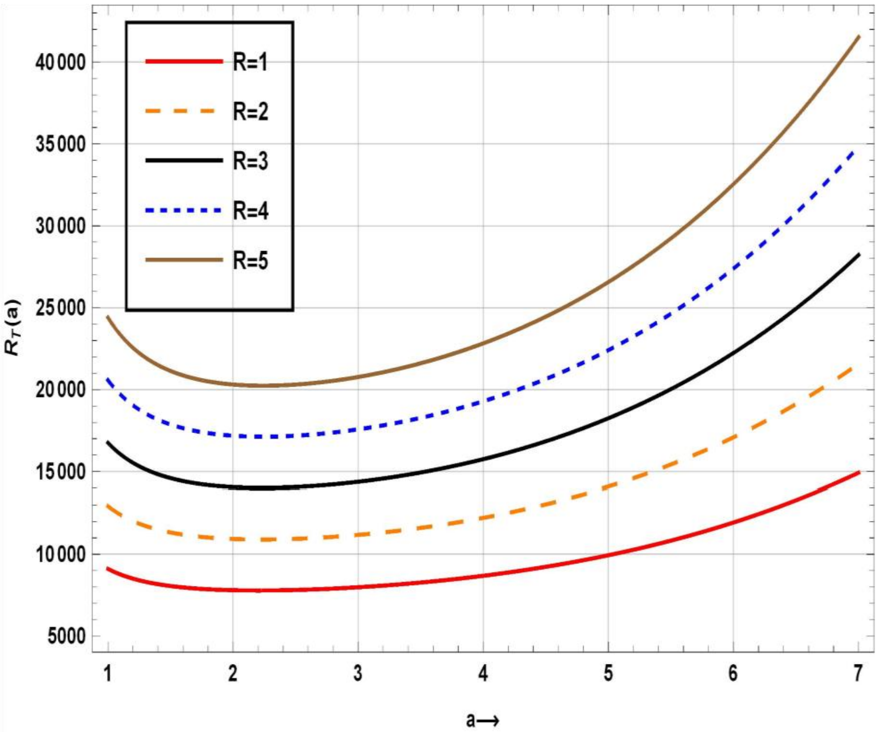

Figure: 6: Effect of Radiation, R, on the onset of instability for:

$Le = 1, \chi = 2, Q = 1, Rc = 1708, M = 1.0156, S = 1$

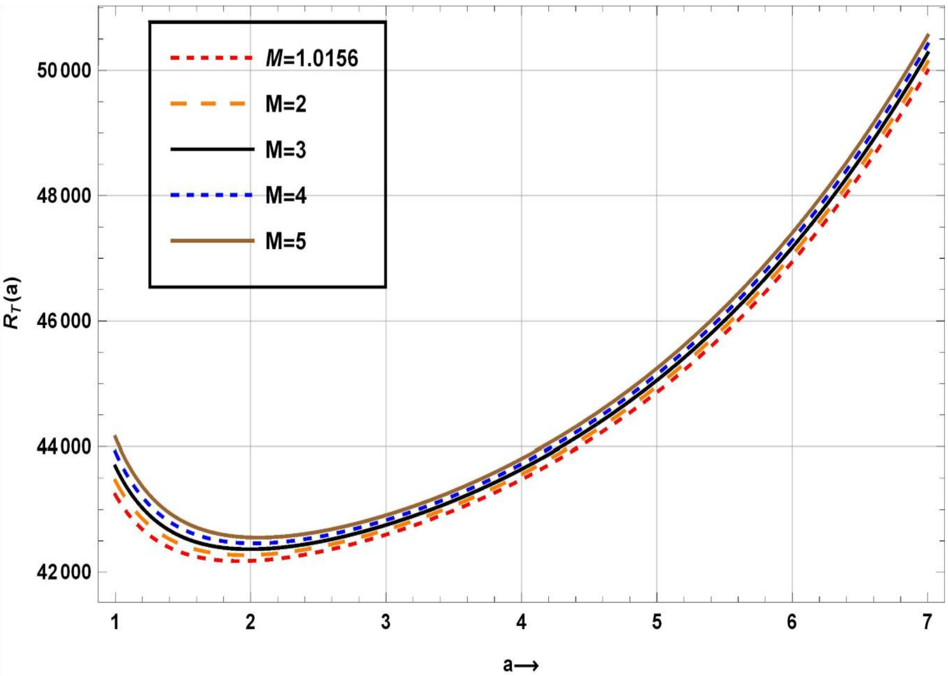

Figure: 7: Effect of Magnetic field, M, on the onset of instability for:

$Le = 1, \chi = 2, R = 1, Rc = 1708, R = 1, S = 1$

4.2 DISCUSSION

Figure 2: shows results for the effect of Lewis number, Le, on the onset of instability of our model. It is observed that as Le increases, thermal Rayleigh number R_T decreases, hence delaying the onset of instability and causing a stable system. An increase in Lewis number, represents an enhanced solute diffusivity, which results in a stronger stability of the solutal gradient that suppresses thermal convection. This leads to a reduction in thermal Rayleigh number, indicating increased stability. This result aligns with [24], who reported that high Le values suppress convection in porous media.

Figure 3: indicates the effect of Soret parameter, S, on the onset of instability. It is noted from our plot that as the Soret value increases, the thermal Rayleigh number R_T also increases. The increase in the thermal Rayleigh number R_T instigates the onset of instability thereby yielding a more unstable system. The Soret effect induces mass flux due to temperature gradients. A higher Soret number amplifies solutal buoyancy effects, which augment thermal buoyancy, destabilizing the system. This aligns with [25] and also [21].

Figure 4: shows the effect of Porosity, χ, on the onset of instability. It is observed from our plot that an increase in Porosity, χ, yields an increase in thermal Rayleigh number R_T, thereby facilitating the onset of instability. This implies that a higher Porosity, χ value translates into an unstable system. A higher porosity reduces resistance to fluid flow, promoting convection and increasing instability. [26] confirm that enhanced porosity increases Rayleigh numbers in porous media.

Figure 5: showcases the effect of Internal heat generation, Q, on the onset of instability. It is seen from our plot that as Internal heat generation, Q, increases, the thermal Rayleigh number R_T, reduces leading to a more stable system. An increase in Internal heat generation, Q, delays the onset of instability in the system. Internal heating tends to uniformly raise fluid temperature, reducing temperature gradients that drive convection. This stabilizes the system [27] discovered a similar trend in porous flows with internal heat generation.

It is shown in Figure 6 that an increase in Radiation, R, also yields an increase in thermal Rayleigh number R_T, facilitating the onset of instability. This implies that a higher Radiation, R, value translates into a destabilized system. Radiation enhances the thermal energy within the system, thereby increasing the temperature gradient and promoting instability. [28] noted this destabilizing influence of thermal radiation.

As shown in Figure 7, an increase in the Magnetic field yields a corresponding rise in the thermal Rayleigh number. This goes to mean that the system will need a stronger thermal Rayleigh gradient for convective motion to occur. It implies that the magnetic field has a high stabilizing effect on the system which suppresses fluid particle motion thereby requiring a higher thermal driving force to destabilize the system. Our result is consistent with the conclusion drawn by [29], who showed that magnetic field suppresses the onset of instability.

V. Conclusion

This study conducted a linear stability analysis on double diffusive convection in a fluid layer considering thermal diffusion and internal heat generation and absorption. In the course of this research work, a mathematical model was developed for the Newtonian fluid. The effects of Lewis number, Soret, Porosity, Internal heat source, Radiation, Prandtl number and other key factors on the onset of stationary instability were individually considered for our model, and a graphical representation provided for a better understanding of the behavior of the system under consideration.

By solving the perturbed equations and analyzing the behavior of thermal Rayleigh number as a function of various governing parameters, we draw the conclusion that: for a Newtonian fluid, the increase in Lewis number, Le, Internal heat generation, Q, and Magnetic field, M, parameters has a delaying effect on the onset of stationary instability, with higher values of these parameters resulting in a greater stabilization of the system, while an increase in the Soret, Porosity and Radiation parameters increases the onset of stationary instability, that is, higher Soret, Porosity and Radiation parameters destabilizes the system. Though our results were obtained from an idealized geometry, they are in agreement with existing literatures. Hence, they can be generalized and put into a wide range of applications as the need arises.

REFERENCE

[1] Radko, T. (2013). Double Diffusive Convection. Cambridge, UK: Cambridge Press.

[2] Xu, Z.X., Zhai, X.Z. & Yu, X. (2024). Three-Dimensional Double Diffusive Convection of Conducting Fluid Under a Magnetic Field. *Physics of Fluids,* 36(2).

[3] Saghir, M.Z, Jiang. C.G., Yan Y., Khawaja, M., Pan, S. & Chacha, M. (2005). Thermo-diffusion in Porous Media: Transport Phenomena in Porous Media III, 227-260.

[4] Reddy, R.M., Raju, M.C. S.& Raju G.S.S. (2016). Natural Convection Boundary Layer Flow of a Double Diffusive and Rotating Fluid Past a Vertical Porous Plate. *International Journal of Applied Computational Mathematics*. 3, 1251-1269.

[5] Crepeau, J. C. & Clarksean, R. (1997). Similarities Solutions of Natural Convection with Internal Heat Generation. *Journal of Heat Transfer*. 119, 183.

[6] Xia, Y. & Chen, J. (2023). Direct Numerical Simulation of Double Diffusive Natural Convection in a Closed Mixture Cavity Heated from below. *Thermal Science*, 27(5), Part B, 4261-4275.

[7] Isreal-Cookey, C., Ebiwareme, L. & Amos, E. (2017). Effect of Vertical Magnetic Field on the Onset of Double Diffusive Convection in a Horizontal Porous Layer with Concentration Based Internal Heat Source. *Asian Research Journal of Mathematics.* 7(1), 1-5.

[8] Isreal-Cookey, C., Amos, E. & Ebiwareme, L. & (2018). Soret and Magnetic Field Effects on Thermosolutal Convection in a Porous Medium with Concentration Based Internal Heat Source. *American Journal of Fluid Dynamics.* 8(1), 1-6.

[9] Noon N. J. & Haddad S. A. (2023). Stability Analysis of Double Diffusive Convection in Local Thermal Non equilibrium Porous Medium with Internal Heat Source and Reaction Effects. *Journal of Non-Equilibrium Thermodynamics.* 48(1) 25-39.

[10] Srinivasa, A. H. & Eswara, A.T. (2016). Effect of Internal Heat Generation/Absorption on MHD Free Convection from Isothermal Truncated Cone. *Alexandria Engineering Journal*, 55(2), 1367- 1373.

[11] Mahajan A. & Tripathi V. K., (2021). Stability of a Chemically Reacting Double Diffusive Fluid Layer in a Porous Medium. A Research Article, Heat Transfer, 50(6), 6148-6163.

[12] Isreal-Cookey, C., Ogulu, A., & Omubo-Pepple, V. B. (2003). Influence of Viscous Dissipation and Radiation on Unsteady MHD Free-convection Flow Past an Infinite Heated Vertical Plate in a Porous Medium with time-dependent Suction. *International Journal of Heat and Mass Transfer.* 46(13), 2305-2311.

[13] Sudha, G. & Sucharitha J. (2015). Thermal Radiation Effects on Oscillating Plate Temperature Flow of Micropolar Viscous Dissipative Fluid Past a Vertical Porous Plate in Presence of Couple Stresses. *International Journal of Mathematics Archive.* 6(6), 52-62.

[14] Umamaheswar, M., Varma, S. V. K., Raju, M. C. & Chamkha, A.J. (2016). Unsteady MHD Free Convection Double Diffusion Viscoelastic Fluid Flow Past an Inclined Permeable Plate in the Presence of Viscous Dissipation and Heat Absorption. *International Journal,* 6(4).

[15] Elgazery, N. S. (2008). Transient Analysis of Heat and Mass Transfer by Natural Convection in Power Law Fluid past a Vertical Plate Immersed in a Porous Medium (Numerical Study). *Application and Applied Mathematics,* 3(2).

[16] Rayleigh, L. (1916). On convection currents in a horizontal layer of fluid. *Philosophical Magazine*, 32(6), 529–546.

[17] Chandrasekhar, S. (1961). Hydrodynamic and Hydromagnetic Stability. Oxford University Press.

[18] Nield, D. A. (1968). Onset of convection in a fluid layer in a porous medium. *Journal of Fluid Mechanics,* 32(2), 393–398.

[19] Coli, A. N. & Bisang, J. M. (2023). Exploring the Impact of Concentration and Temoerature Variations on Transient Natural Convection in Metal Electrodeposition: A Finite Volume Method Analysis. *Journal of Electrochemical Society.* 170(8), 083505.

[20] Misra, J. C. & Sinha, A. (2013). Effect of Thermal Radiation on MHD Flow of Blood and Heat Transfer in a Permeable Capillary in Stretching Motion. *Heat and Mass Transfer.* 49(5), 617-628.

[21] Isreal-Cookey, C. & Amos, E. (2014). Soret and Radiation Absorption Effects on the Onset of Magneto-thermosolutal Convection in a Porous Medium. *Journal of Applied Mathematics and Bioinformatics.* 4(1), 71.

[22] Kaddeche, S., Henry, D. & BenHadid, H. (2003). Magnetic Stabilization of the Buoyant Convection between Infinite Horizontal Walls with a Horizontal Temperature Gradient. *Journal of Fluid Mechanics.* 480, 185–216.

[23] Drazin, P. G., & Reid, W. H. (2004). Hydrodynamic stability (2nd ed.). Cambridge University Press.

[24] El-Amin, M. F. (2006). The Effect of Kewis Number on Double Diffusive Convection in a Porous Medium. *Transport Porous Media,* 63(2), 301-317.

[25] Ganesan, P., & Loganathan, P. (2002). Soret and Dufour effects on unsteady free convection flow over a vertical surface. Heat and Mass Transfer, 38, 107–112.

[26] Nield, D. A., & Bejan, A. (2006). Convection in Porous Media (3rd ed.). Springer.

[27] Makinde, O. D. (2005). Free convection flow with thermal radiation and internal heat generation along a vertical porous plate. Applied Energy, 80(2), 183–190.

[28] El-Aziz, M. A. (2009). Thermal radiation effect on heat transfer over an unsteady stretching surface. *Arabian Journal for Science and Engineering,* 34(2B), 531–544.

[29] Malashetty, M.S, Umavathi, J.C., & Prakasha, D.G. (2006). Effects of Internal Heat Generation and Magnetic Field on the Onset of Convection in a Horizontal Porous Layer. *Heat and Mass Transfer,* 42(8), 665-673.