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Research Paper

Mathematical Modelling of the Prices of Two Competing Commodities in a Stock Market

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Abstract
Financial and commodity markets play a significant role in economic growth, capital formation, and risk management. Prices of competing commodities often influence one another due to substitution effects, shared demand factors, and global economic conditions. Understanding an interaction under uncertainty is important for investors. This study develops a mathematical model to describe the stock price behavior of competing commodities in a stochastic environment. The model assumes that stock price is proportional to available stock, and that the rate of return (drift) depends on the interaction between competing commodities. Market volatility and future uncertainty are incorporated to reflect real market fluctuations. A stochastic differential equation based on Brownian motion is formulated to represent price dynamics. Ito's calculus is applied to liberalize and solve the equation, leading to an explicit exponential solution for the stock price process. The results show that stock prices follow a stochastic exponential growth model influenced by both drift and volatility components. The model provides an effective framework for analyzing price dynamics, forecasting future movements, and supporting risk management decisions in competitive commodity markets.

Keywords**: **Financial Market, Commodities, Stochastic, Volatility, Brownian motion, Ito's calculus

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I. Introduction

The origins of financial markets date back to ancient civilizations, but modern financial markets took shape in the 17th century with the establishment of stock exchanges like the Amsterdam Stock Exchange. The 20th century saw rapid growth and globalization of financial markets, facilitated by technological advancements and deregulation (**Kindleberger & Aliber, 2011**). Financial markets are platforms where financial securities such as stocks, bonds, derivatives, and currencies are traded. They are generally categorized into: Capital markets (e.g., stock and bond markets), Money markets (short-term debt instruments), Derivatives markets, and Foreign exchange markets. These markets support economic growth by enabling capital raising, investment, and risk distribution **(Mishkin & Eakins, 2018**). Commodity markets deal with the trading of raw or primary products. These are typically divided into; Hard commodities (e.g., oil, gold, metals) and Soft commodities (e.g., agricultural products like coffee, wheat, and cotton), these markets are essential for price discovery and provide a mechanism for producers and consumers to hedge against price volatility (German, 2005). Derivatives like futures, options, and swaps play a crucial role in both markets. They allow participants to hedge against risks such as interest rate fluctuations, commodity price changes, and currency exchange volatility (Hull, 2017). The stock market is a key component of the financial system, facilitating the issuance, buying, and selling of company shares. It acts as a barometer of economic health and a mechanism for capital formation. This market provides liquidity, enabling investors to buy and sell shares easily (Mishkin & Eakins, 2018). Financial and commodity markets play a vital role in the global economy by enabling efficient resource allocation, price discovery, risk management, and liquidity. A clear understanding of their structure and functioning is essential for investors, policymakers, and researchers (Hull, 2018; Mishkin & Eakins, 2018). The prices of commodities frequently interact, especially when the goods are substitutes, complements, or direct competitors. For example, commodities such as crude oil and natural gas in the energy sector, or gold and silver in the precious metals market, often exhibit interconnected price movements due to their similar uses and overlapping demand bases (Zhang & Wei, 2010). These interactions can be effectively captured and analyzed through mathematical models, which provide a structured framework for interpreting market dynamics and forecasting price behavior (Hamilton, 2009).

Smith et al., (2018) conducted a studied on Game Theory-based Competitive Market Model. Smith and his team explored how two crucial commodities—gold and oil—compete in global markets. Using a game theory-based model, they identified a Nash equilibrium framework that accounted for the strategic interactions between these two commodities. Their research highlighted that oil prices, being highly volatile due to geopolitical factors, often drive changes in gold prices. Gold, on the other hand, acts as a safe-haven asset in times of economic uncertainty, making its price movements a key factor in market predictions. By applying this model to real market data, the study provided actionable insights into the potential behavior of these commodities under varying global economic scenarios. **Johnson and Lee, (2020**) carried a research on Multi-Agent Simulation of Commodity Markets. In this study they created a multi-agent simulation to model the competitive dynamics between natural gas and coal markets. They designed agents to represent various market participants, such as producers, consumers, and traders. By allowing these agents to interact within the simulated environment, they observed how each agent's decision-making process influenced market outcomes. The study found that coal and natural gas markets often exhibited a substitutive relationship, with prices of one commodity affecting demand for the other. The model was particularly effective in demonstrating how sudden policy changes or technological advancements could alter the balance of competition between these two energy sources. **Patel, (2017**) studied Stochastic Differential Equations (SDEs) for Commodity Price Dynamics. He applied stochastic differential equations to model the price dynamics of crude oil and natural gas. The model was designed to account for the random nature of market shocks, including geopolitical events, technological breakthroughs, and regulatory changes. By capturing the volatility of these commodities, the study demonstrated that oil and gas prices often follow correlated stochastic processes, making them interdependent in the financial markets. The research also showed how this model could be used for hedging and risk management, as well as for understanding long-term market behavior under uncertainty.

Zhang and Wong (2019) conducted a research on Econometric Model for Commodity Price Interactions. In their research they developed an econometric model based on vector autoregression (VAR) to study the interactions between wheat and corn prices. They explored the economic factors that influence the demand for these two agricultural commodities, including weather patterns, policy changes, and international trade dynamics. **Nguyen and Tran, (2022)** conducted a study on Portfolio Optimization with Competing Commodities. They examined how portfolio optimization could be applied to investing in competing commodities like crude oil and natural gas. They focused on creating an optimal mix of these two commodities within a portfolio, based on their price correlations and volatility. By incorporating both short-term and long-term risk measures, they demonstrated that a dynamic portfolio strategy could significantly reduce risk while maintaining competitive returns. Their findings were particularly relevant for institutional investors and hedge funds looking to balance risk and reward in the volatile energy markets. Wu (2015) conducted research on the Artificial Neural Networks (ANN) for Forecasting Commodity Prices. In his research he employed artificial neural networks (ANN) to forecast the prices of gold and silver. By training the network on historical data and allowing it to detect patterns, the study showed that ANN outperformed traditional time series models, particularly when it came to forecasting price movements during times of high volatility. The neural network's ability to account for complex, non-linear relationships in the data made it a powerful tool for commodity traders who needed to adapt quickly to market changes.

Chen et al., (2020) conducted a research on Dynamic Stochastic General Equilibrium (DSGE) Model. They applied a Dynamic Stochastic General Equilibrium (DSGE) model to examine the relationship between oil and natural gas prices. By integrating macroeconomic factors such as GDP growth, inflation, and interest rates, they showed how global economic conditions influence the dynamics between these two energy commodities. Ahmed and Garcia (2019) Studied Continuous-Time Stochastic Model for Commodity Hedging. This study introduced a continuous-time stochastic model to help commodity producers and traders hedge against price risks in coffee and cocoa markets. By using Monte Carlo simulations, they assessed the effectiveness of different hedging strategies, such as forward contracts and options. Their results suggested that hedging strategies could reduce exposure to price volatility, but the choice of strategy depended heavily on the nature of the market and the trader's risk tolerance.

Liu, (2017) Studied Hybrid Model of GARCH and Copula for Volatility Forecasting. Liu combined Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models with Copula functions to analyze the volatility of agricultural commodities, specifically corn and soybeans. GARCH models are used to capture time-varying volatility, while Copulas were employed to model the dependencies between the two commodities. This hybrid model allowed Liu to predict how shocks in one market, such as changes in demand or unexpected supply disruptions, could affect the volatility in the other market. By using this model, traders could better forecast extreme price movements, improving risk management strategies in agricultural markets. Kumar et al., (2021) conducted a research on Reinforcement Learning for Commodity Market Strategy; they applied reinforcement learning (RL) algorithms to develop adaptive trading strategies for competing commodities such as oil and gold. They designed an RL model that learned from historical price data and market conditions, optimizing its decision-making process for entering and exiting positions.

Jackson et al., (2019) conducted a research on Monte Carlo Simulation for Price Forecasting. Jackson and his team applied Monte Carlo simulations to model the future price movements of copper and aluminum. The method used random sampling to simulate a wide range of possible outcomes, incorporating stochastic volatility and external market factors. Their research revealed that the distribution of possible future prices was highly skewed, meaning that while the mean predicted price was useful, and the true potential range of prices was much wider than conventional models suggested. This finding is particularly important for investors and risk managers who need to account for extreme price movements in their forecasts.

Singh and Choi, (2022) carried a researched on the Cross-Commodity Regression Analysis, They applied cross-commodity regression analysis to investigate how price fluctuations in corn and wheat were correlated. Their analysis revealed that external factors, such as changes in international trade agreements and agricultural subsidies, were key drivers of price movements in both markets. The study found that while the two commodities generally moved in tandem, there were significant periods where one outperformed the other, depending on regional production conditions. This model provided actionable insights for traders who could use this knowledge to hedge risks and make more informed investment decisions in the agricultural sector.

Zhao et al., (2018) conducted a Dynamic Factor Models for Multi-Commodity Market Analysis. Zhao and colleagues introduced a dynamic factor model (DFM) to analyze the co-movements between several commodities, including oil, gold, and silver. Their approach identified key macroeconomic factors that drove the price movements of these commodities, such as inflation rates, interest rates, and geopolitical events. By isolating these factors, the model was able to predict how changes in one commodity's price would likely affect the others. The study found that gold and oil, in particular, shared a strong co-movement pattern, with oil price shocks often leading to significant shifts in gold prices, making this model a powerful tool for commodity market forecasting.

Miller and Wright (2023) researched on Hybrid Agent-Based Model and Econometric Forecasting. They combined agent-based modeling with econometric forecasting to examine how commodity markets respond to competitive forces between oil and natural gas. They created agents to represent market participants such as producers, traders, and consumers, each with their own strategies and risk preferences. Martinez and Zhang, (2021) conducted a study on Supply-Demand Equilibrium Model for Competing Commodities. In this study they examined the supply-demand dynamics between competing agricultural commodities like corn and wheat. They developed a model that incorporated factors such as crop yields, weather patterns, and global trade to determine how shifts in supply and demand affected the price equilibrium. Zhao and Chen, (2022) conducted a researched on Bayesian Network Model for Multi-Commodity Market Interaction. Zhao and Chen utilized a Bayesian network model to investigate the probabilistic interactions between competing commodities, particularly coffee and cocoa. The model allowed them to represent the uncertainty and conditional dependencies between various market factors, such as weather conditions, global demand, and policy changes.

Williams and Evans, (2021) researched on Agent-Based Model for Price Impact Analysis Williams and Evans used an agent-based model to simulate the price impacts of strategic decisions in the natural gas and coal markets. They created agents with different strategies for production, investment, and consumption, and simulated their interactions over time. Their model showed that when one commodity (coal) experienced a significant price increase, it could lead to shifts in the demand for natural gas as power producers switched to cheaper alternatives. Roberts et al., (2020) conducted a research on Game-Theoretic Model for Competitive Commodities. Roberts and his colleagues applied a game-theoretic approach to model competition between oil and natural gas markets. They built a sequential game model in which players (producers) made decisions about how much to supply in response to price changes and the actions of competitors. Gupta and Sharma, (2018) studied Dynamic Equilibrium Model for Commodities with Long-Term Trends. In this study they developed a dynamic equilibrium model to study the long-term price trends of competing agricultural commodities such as rice and wheat.

Zhang, (2022) conducted a research on Hybrid Time-Series and Neural Network Model for Commodity Forecasting. In this study they combined time-series forecasting methods with neural network models to predict the future prices of copper and aluminum. By using historical price data alongside machine learning techniques, they were able to model both linear and non-linear relationships between commodity prices and external market factors. Roy and Ghosh, (2019) studied Structural Vector Autoregressive (SVAR) Model for Commodity Price Spillovers. They utilized a Structural Vector Autoregressive (SVAR) model to investigate the spillover effects between the prices of oil and gold. They modeled how economic and geopolitical shocks in one market could affect price movements in the other. Lee et al., (2023) studied Hybrid Computational Model for Competitive Commodities in Financial Markets. Lee and his team developed a hybrid computational model that integrated Monte Carlo simulations with agent-based modeling to study how competing commodities, such as natural gas and oil, behave in financial markets. The model allowed for the incorporation of both stochastic processes (random fluctuations) and agent behavior, simulating how market participants react to price changes, geopolitical events, and other economic factors.

Anderson and Mitchell (2022) studied Cross-Commodity Risk Management Framework, where they developed a model that allowed traders to identify and manage the risks associated with price fluctuations across multiple markets simultaneously. Their framework used Monte Carlo simulations to assess the potential outcomes of various hedging strategies and provided recommendations for optimal risk mitigation.

Assumptions

Before we formulate mathematical model to understand the stock prices of various commodities at the stock market, let's first and foremost consider the following assumptions:

(i.) The stock price is proportional to the stock available

(ii.) The rate of stock return of drift is proportional to the square root of the sum and product of the competing commodities at the stock exchange

(iii.) Ito's calculus is considered to linearized the system representing the stock price

(iv.) The stock price could be affected by the volatility of prices due to the uncertainty of the stock market.

(v.) The stock price could be affected by the future of uncertainty of the stock market.

Mathematical Formulation

Following the aforementioned assumptions, we derived the basic differential equation representing the stock price as:

$$dS_t = \mu S_t dt + \sigma S_t dw_t \tag{3.1}$$

Equation (3.1) is called the equation of Brownian motion

Ito's calculus

$$X_t = \ln\left(S_t\right) \tag{3.2}$$

Taking the exponential of equation (3.2), we have,

$$S_t = e^{X_t} \tag{3.3}$$

Differentiating equation (3.2) partially, we have:

$$\frac{\partial X_t}{\partial S_t} = \frac{1}{S_t} \tag{3.4}$$

Differentiating equation (3.4) partially, we have:

$$\frac{\partial^2 X_t}{\partial S_t^2} = -\frac{1}{S_t^2} \tag{3.5}$$

Differentiating equation (3.2) partially, we have:

$$\frac{\partial X_t}{\partial t} = 0 \text{ and } \frac{\partial^2 X_t}{\partial t^2} = 0 \tag{3.6}$$

Squaring equation (3.1), we have:

$$\left(dS_t\right)^2 = \left(\mu S_t dt + \sigma S_t dw_t\right)^2 = \mu^2 S_t^2 \left(dt\right)^2 + 2\mu\sigma S_t^2 dt dw_t + \sigma^2 S_t^2 \left(dw_t\right)^2 \tag{3.7}$$

Expanding equation (3.2) using the Taylor series expansion, that is:

$$dX_t = \frac{\partial X_t}{\partial S_t} dS_t + \frac{1}{2}\frac{\partial^2 X_t}{\partial S_t^2}\left(dS_t\right)^2 + \frac{\partial X_t}{\partial t} dt + \frac{1}{2}\frac{\partial^2 X_t}{\partial t^2}\left(dt\right)^2 + \ldots \tag{3.8}$$

$$dX_t = \frac{\partial X_t}{\partial S_t} dS_t + \frac{1}{2}\frac{\partial^2 X_t}{\partial S_t^2}\left(dS_t\right)^2 \tag{3.9}$$

$$dX_t = \frac{\partial X_t}{\partial S_t}\left(\mu S_t dt + \sigma S_t dw_t\right) + \frac{1}{2}\frac{\partial^2 X_t}{\partial S_t^2}\left(\mu^2 S_t^2 \left(dt\right)^2 + 2\mu\sigma S_t^2 dt dw_t + \sigma^2 S_t^2 \left(dw_t\right)^2\right) \tag{3.10}$$

$$dX_t = \left(\mu S_t \frac{\partial X_t}{\partial S_t} dt + \sigma S_t \frac{\partial X_t}{\partial S_t} dw_t \right) + \left(\mu^2 S_t^2 \frac{1}{2} \frac{\partial^2 X_t}{\partial S_t^2} (dt)^2 + 2\mu\sigma S_t^2 \frac{1}{2} \frac{\partial^2 X_t}{\partial S_t^2} dt dw_t + \sigma^2 S_t^2 \frac{1}{2} \frac{\partial^2 X_t}{\partial S_t^2} (dw_t)^2 \right) \tag{3.11}$$

$dt \to 0, (dt)^2 \to 0$, and $(dw_t)^2 \to dt$, so that equation (3.11) can be reduced to

$$dX_t = \mu S_t \frac{\partial X_t}{\partial S_t} dt + \sigma^2 S_t^2 \frac{1}{2} \frac{\partial^2 X_t}{\partial S_t^2} dt + \sigma S_t \frac{\partial X_t}{\partial S_t} dw_t \tag{3.12}$$

Substituting equations (3.4)-(3.5) into equation (3.12), we have:

$$dX_t = \mu S_t \frac{1}{S} dt - \sigma^2 S_t^2 \frac{1}{2S_t^2} dt + \sigma S_t \frac{1}{S} dw_t \tag{3.13}$$

$$dX_t = \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma dw_t \tag{3.14}$$

Integrating equation (3.14), we have;

$$\int dX_t = \int \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma \int dw_t \tag{3.15}$$

$$X_t(t) = \left(\mu - \frac{\sigma^2}{2} \right) t + \sigma w_t + A \tag{3.16}$$

Substituting equation (3.2) into equation (3.16), we have:

$$\ln(S_t) = \left(\mu - \frac{\sigma^2}{2} \right) t + \sigma w_t + A \tag{3.17}$$

Taking exponent of equation (3.17), we have:

$$S_t = \exp\left(\left(\mu - \frac{\sigma^2}{2} \right) t + \sigma w_t + A \right) \tag{3.18}$$

$$S_t(t) = B \exp\left(\left(\mu - \frac{\sigma^2}{2} \right) t + \sigma w_t \right) \tag{3.19}$$

$$B = S_0 e^{-\sigma w_t(0)} \tag{3.20}$$

Substituting equation (3.20) into equation (3.19), we obtain the solution representing stock price as:

$$S_t(t) = S_0 e^{-\sigma w_t(0)} \exp\left(\left(\mu - \frac{\sigma^2}{2} \right) t + \sigma w_t \right) \tag{3.21}$$

$$S_t(t) = S_0 e^{-\sigma w_t(0)} \exp\left(\left((\mu + R_\tau) - \frac{\sigma^2}{2} \right) t + \sigma w_t \right) \tag{3.22}$$

$$S_t(t) = S_0 e^{-\sigma w_t(0)} \exp\left(\left(\lambda_1 \lambda_2 - \frac{\sigma^2}{2} \right) t + \sigma w_t \right) \tag{3.23}$$

$$S_t(t) = S_0 e^{-\sigma w_t(0)} \exp\left(\left(\lambda_1 + \lambda_2 - \frac{\sigma^2}{2} \right) t + \sigma w_t \right) \tag{3.24}$$

$$S_t(t) = S_0 e^{-\sigma w_t(0)} \exp\left(\left((\lambda_1 \lambda_2)^2 - \frac{\sigma^2}{2} \right) t + \sigma w_t \right) \tag{3.25}$$

$$S_t(t) = S_0 \exp\left(\left((\lambda_1 + \lambda_2)^2 - \frac{\sigma^2}{2}\right)t + \sigma w_t\right) \tag{3.26}$$

$$S_t(t) = S_0 \exp\left(\left(\sqrt{(\lambda_1 + \lambda_2)} - \frac{\sigma^2}{2}\right)t + \sigma w_t\right) \tag{3.27}$$

RESULTS

We performed numerical simulation using Wolfram Mathematica, version 12, where the pertinent parameters were varied and investigated to see the extent to which the can affect the stock prices. The results are presented below:

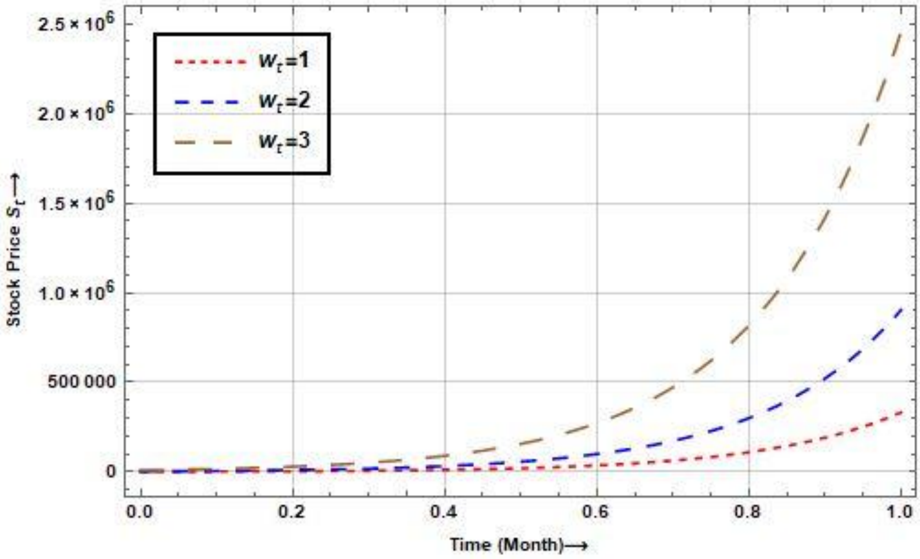

Figure 1: Impact of variation of uncertainty on the Stock Price for both competing commodities holding other contributing factors fixed

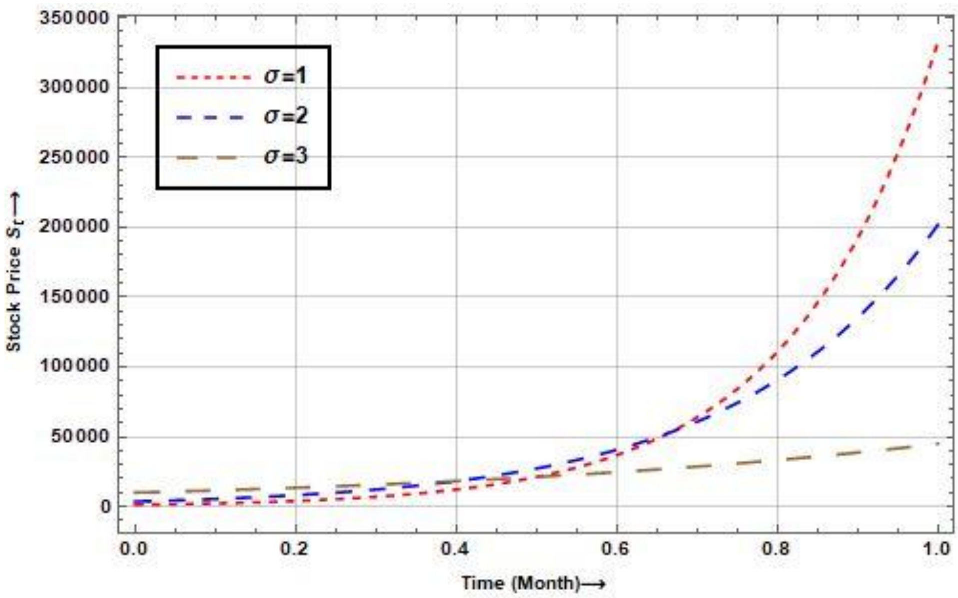

Figure 2: Impact of variation of volatility on the Stock Price for both competing commodities holding other contributing factors fixed

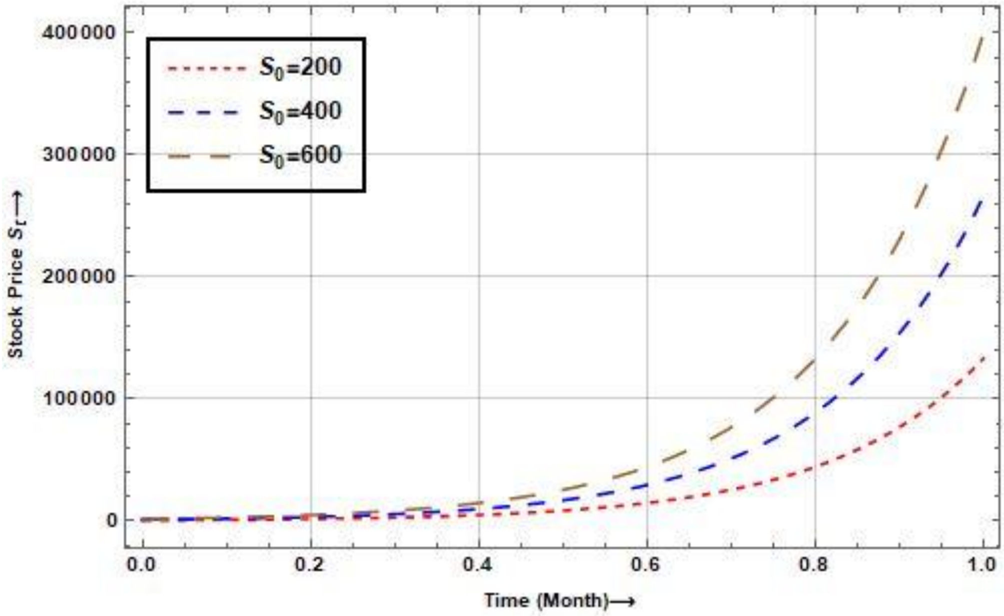

Figure 3: Impact of variation of initial Stock level on the Stock Price for both competing commodities holding other contributing factors fixed

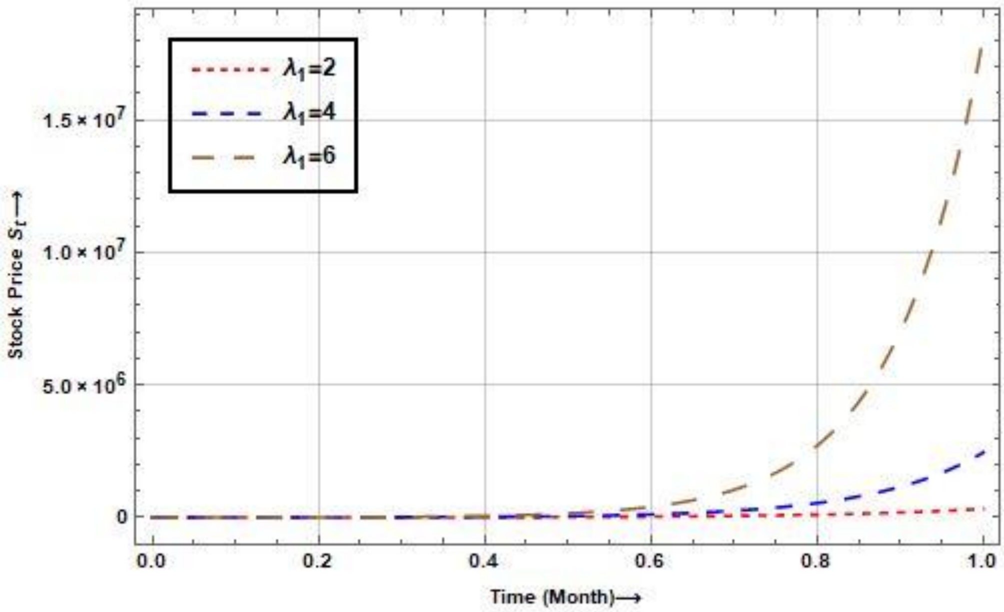

Figure 4: Impact of the variation of the first commodity on Stock Price holding the second commodity and other contributing factors fixed

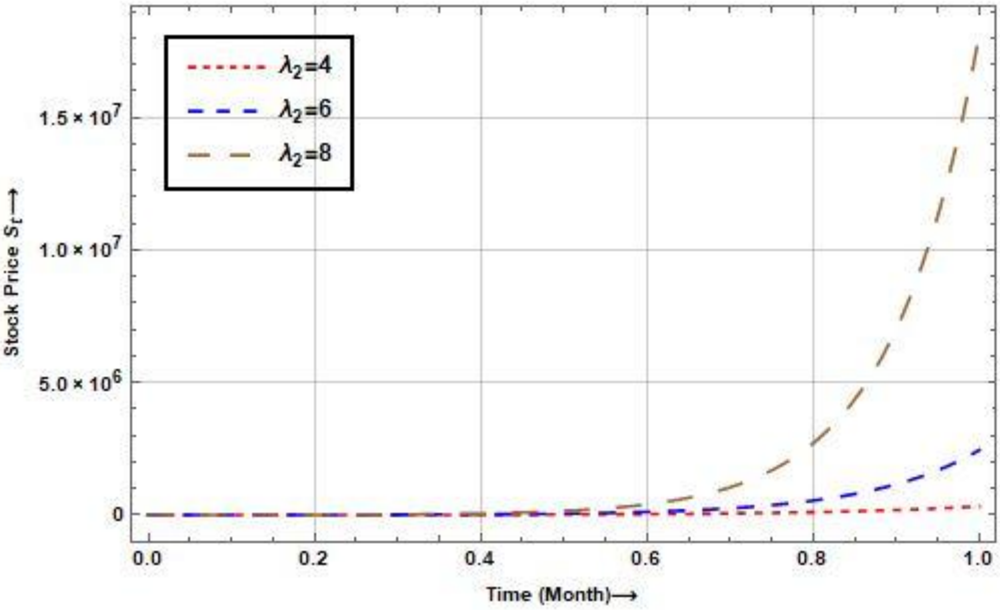

Figure 5: Impact of the variation of the second commodity on Stock Price holding the first commodity and other contributing factors fixed

II. Discussion of Result

Figure 1 invested the effect of uncertainty on stock price; the increase in uncertainty significantly alters stock price trajectories. The higher uncertainty tends to produce stronger fluctuations and reduced stability in stock prices. This indicates that market unpredictability amplifies risk and causes price dispersion between the competing commodities. In essence, the uncertainty acts as a destabilizing factor, increasing investor hesitation and speculative behavior, which leads to irregular stock movements. In real markets, this can correspond to economic shocks, policy ambiguity, or incomplete information. Figure 2 depicts effect of volatility on stock price, and the revealed that the rising volatility increases the amplitude of stock price changes. The system exhibits sharper peaks and troughs as volatility grows. One commodity may gain temporary dominance depending on its sensitivity to volatility changes. It simply means that the volatility serves as a volatility amplifier, enhancing short-term gains and losses. This suggests that speculative conditions can dramatically reshape competitive advantage between commodities. The effect of initial stock level was investigated and results shown using Figure 3. It showed that different initial stock levels lead to different equilibrium paths. The higher initial levels generally sustain stronger price responses over time. The long-term dynamics depend heavily on starting conditions. However, it shows that initial market positioning matters greatly; early stock availability influences future price evolution. This reflects path dependence, where early market conditions shape long-term outcomes. Figure 4 illustrates the variation of the first commodity on the stock price while other variables are fixed. It can be seen that increasing the influence or quantity of the first commodity directly impacts overall stock price. The response suggests competitive pressure on the opposing commodity. Depending on parameter strength, dominance or suppression effects emerge. The above result means that the first commodity can act as a market driver, influencing price stability and potentially shifting equilibrium toward its own dominance when strengthened. Finally, the change in the second commodity likewise affects stock prices but may exhibit different sensitivity compared to the first. Competitive interaction creates compensatory or opposing price movements. The balance between both commodities determines system stability. It simply means that, the second commodity plays a regulating role, and its variation highlights the interdependence inherent in multi-commodity markets, as it can be seen in Figure 5. In conclusion, we have been able to formulate the mathematical model which represents the prices of two competition commodities in a stock market, solved the model and obtained the stock price function, performed numerical simulation, and discussed the results appropriately.

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