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Research Paper

Tulgeity of Restricted Superline Graph of $C_4^{(n)}$

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***Abstract**: Tulgeity $\tau(G)$ is the maximum number of disjoint, point induced, non-acyclic subgraphs contained in G. In this paper we find the tulgeity of the restricted super line graph of $C_4^{(n)}$.*

Key words: *Tulgeity, Restricted Super line graph.*

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I. Introduction:

Point partition number (Gray Chartrand) of a graph G is the minimum number of subsets into which the point- set of G can be partitioned so that the sub graph induced by each subset has the property P. Dual to this concept of point partition number of a graph is the maximum number of subsets into which the point set of G can be partitioned such that the subgraph induced by each subset does not have the property P. Define the property P such that a graph G has the property P if G contains no subgraph that is homeomorphic from the complete graph K_3.This point partition number, and dual point partition number for the property P is referred as point arboricity and tulgeity of G respectively. Equivalently the tulgeity is the maximum number of vertex disjoint cycles in G so that each subgraph is not acyclic. The formula for tulgeity of complete bipartite graph was given in Gray Chartrand.,1968[3]. One point union of n cycles of length 4 is $C_4^{(n)}$

All graphs considered in this paper are simple graphs. Restricted Super line graph of index r of a graph G, denoted by $RL_r(G)$, is a modification of the concept of the super line graph $L(G)$ introduced by Bagga. The vertices of $RL_r(G)$ are the r-element subsets of $E(G)$ and two vertices S and T are adjacent if there exists exactly one pair of edges, one from each of the sets S and T, which are adjacent in G[5]

II. Main Results:

To avoid the complexity in listing the vertices of restricted super line graph , in this paper we represent the vertex induced by the edges e_i,e_j in G as v_{ij} in $RL_2(G)$.

***Outline of the proof* :** Here we derive the formula for tulgeity of restricted superline graph of index 2 in six cases. We covered all the vertices of $RL_2(G)$ with C_3s whenever $\left|\binom{E(G)}{2}\right|$ is a multiple of 3. If $\left|\binom{E(G)}{2}\right|$ is not a multiple of 3, then we cover $\left|\binom{E(G)}{2}-4\right|$ vertices with C_3s and the remaining 4 vertices with C_4 . Thus we obtain maximum number of induced cyclic subgraphs

***Theorem* 2.1**: For $n > 5$, the tulgeity of restricted superline graph of index 2 of one point union of n cycles of order 4 is $\tau\left(RL_2\left(C_4^{(n)}\right)\right) = \frac{\binom{4n}{2}}{3} = \frac{4n^2-n}{6}$.

Proof: Let v_1 be the root vertex and the edges of C_4s are labeled as $i,(i+1),i',(i+1)',i = 1,3,5\ldots n-1$ & the edges $i,(i+1)$ are incident to the vertex v_1. Then $C_4^{(n)}$ has $4n$ edges. Thus $\left|V\left(RL_2\left(C_4^{(n)}\right)\right)\right| = \binom{4n}{2}$ that are partitioned as given below.

$$V_1 = \{v_{i,j} / i \neq j, 1 \leq i, j \leq n\}$$

$$V_2 = \{v_{i',j'} /, 1 \leq i', j' \leq n\}$$

$$V_3 = \{v_{i,j'} / i \neq j, 1 \leq i, j' \leq n\}$$

Clearly $|V_1| = \binom{2n}{2}$, $|V_2| = \binom{2n}{2}$ & $|V_3| = 4n^2$

$$\Rightarrow |V_1| + |V_2| + |V_3| = 2\binom{2n}{2} + 4n^2 = 8n^2 - 2n = \frac{4n(4n-1)}{2} = \binom{4n}{2}.$$

Further we prove this result in six cases when $n \equiv 0,1,2,3,4,5 (\text{mod } 6)$

Clearly the vertices of V_1 induces a null graph as these are the vertices formed with the edges of star of $C_4^{(n)}$. Here first we construct the C_3s between the vertices of V_3, and between the vertices of V1,V2 .With the remaining vertices of V_3 and V_2 we form triangles with the vertices of V_1.Thus we see that all vertices are exhausted.

By division algorithm, $n = 6Q + R, 0 \leq R \leq 5$. Here n stands for number of C_4s in S_{nC4}. In all cases we consider odd i, since the edges in each cycle are $e_i, e_{i+1}, e_{i'}, e_{(i1)}$

$e_i, e_{i+1}, e_{i+1'}, e_{i'}, e_i$ where e_i, e_{i+1} are incident with the central vertex v_0.

Outline of the proof: in each case we partition the vertex set into the subsets of distinct C_3s as S_i, S_j, S_k, S_l and we prove that sum of the cardinalities of all these sets is equal to $|V(RL_2(S_nC_4)|$.

Case1: $n \equiv 0 (\text{mod } 6), n = 6q$,:

The vertices of V_1, V_2, V_3 are partitioned as S_i, S_j, S_k and S_l having disjoint C_3s as follows.

$$S_i = \left\{\begin{matrix} v_{i',j'}, v_{i,k}, v_{i,j'}, 1 \leq odd\ i \leq 2n-2 \\ j = 2n, 2n-1, 2n-2, \ldots, i+1 \\ k = i+1, i+2, \ldots 2n-1, 2n \\ j \neq i+1, i+2 \end{matrix}\right\} \cup \left\{\begin{matrix} v_{i,2n}, v_{i',j'}, v_{i,(j+1)'} \\ j = i+1,\ for\ odd\ i \\ 1 \leq i \leq 2n-3 \end{matrix}\right\} \cup \left\{\begin{matrix} v_{i',j'}, v_{i,(2n-1)}, v_{i,(j-1)'} \\ j = i+2,\ for\ odd\ i \\ 1 \leq i \leq 2n-3 \end{matrix}\right\}.$$

Thus $\left|S_i\right| = 2n - i$.

$$\sum_{i=1}^{2n-2}\left|S_i\right| = \sum_{i=1}^{2n-2}(2n-i) = (2n-1)+(2n-2)+.....+2 = n(2n-1)-1 = (n-1)(2n+1)$$

$$S_j = \begin{Bmatrix} v_{i,i'}, v_{i+2,(i+2)'}, v_{i+5,i+4'} \ \ for\ i = 1,5,9...2n-7, \\ v_{i,i'}, v_{i+2,(i+2)'}, v_{i+5,i+4'} \ \ for\ \ i = 2,6,10...2n-6, \end{Bmatrix} \cup \begin{Bmatrix} v_{2n-3,(2n-3)'}, v_{2n-1,(2n-1)'}, v_{2,1'}; \\ v_{2n-2,(2n-2)'}, v_{2n,2n'}, v_{4,3'} \end{Bmatrix}$$

Thus $\left|S_j\right| = (\frac{2n-7-1}{4}+1)+(\frac{2n-6-2}{4}+1)+2 = \frac{2n-4}{4}+\frac{2n-4}{4}+2 = n$

For each l,

$$S_l = \begin{Bmatrix} v_{i,(3l-2)'}, v_{i,(3l-1)'}, v_{i-1,(3l)'} \ \ for\ odd\ l, 1 \le l \le 4Q-1;\ 3l+3 \le i \le 2n \\ v_{(i-1),(3l-2+i)'}, v_{i,(3l-1+i)'}, v_{i,(3l)'} \ \ for\ even\ l, 2 \le l \le 4Q-2;\ 3l+1 \le i \le 2n \end{Bmatrix}$$

Thus $\left|S_l\right| = \begin{pmatrix} (2n-3l-3)+1 \\ for\ odd\ l \\ 1 \le l \le 4Q-1 \end{pmatrix} + \begin{pmatrix} (2n-3l-1)+1 \\ for\ even\ l \\ 2 \le l \le 4Q-2 \end{pmatrix}$.

$$\sum_{i=1}^{2n-2}\left|S_l\right| = \sum_{l=1}^{4q-1}\begin{pmatrix} 2n-3l-2 \\ for\ odd\ l \end{pmatrix} + \sum_{l=2}^{4q-2}\begin{pmatrix} 2n-3l \\ for\ even\ l \end{pmatrix}$$

Thus there are $((2n-5)+(2n-11)+(2n-17)+....+7+1)+((2n-6)+(2n-12)+(2n-18)+...+12+.6)$

$$= \frac{n/3}{2}((2n-5)+1) + \frac{n-3/3}{2}((2n-6)+6)$$

$$= \frac{n}{6}(2n-4) + \frac{n-3}{6}(2n) = \frac{n(2n-5)}{3}$$

The remaining vertices form C_3s as follows.

$$S_p = \begin{Bmatrix} v_{3i,(3i-2)'}, v_{3i,(3i-1)'}, v_{3i+2,(3i+1)'};\ v_{(3i+1),(3i-2)'}, v_{2n,(3i+1)'}, v_{3i+2,(3i-2)'}; \\ v_{(3i+2),(3i-2)'}, v_{(3i+2),(3i-1)'}, v_{2n,(3i)'} \\ 1 \le odd\ i \le 4Q-1 \end{Bmatrix} \cup \left\{ v_{2n,(2n-1)'}, v_{2n,(2n-2)'}, v_{(2n)',(2n-1)'} \right\}$$

$$\Rightarrow \left|S_p\right| = 3\left(\frac{((2Q-1)-1)}{2}+1\right)+1 = 6Q+1 = n+1$$

Thus number of disjoint $c_3s = \left|S_i\right| + \left|S_j\right| + \left|S_l\right| + \left|S_p\right| = (n(2n-1)-1) + n + \frac{n(2n-5)}{3} + n + 1 = \frac{2n(4n-1)}{3}$

Clearly all cycles are disjoint . Hence

$$\tau\left(RL_2\left(C_4{}^{(n)}\right)\right) \geq \frac{2n(4n-1)}{3}$$

Case2: $n \equiv 1(\text{mod}\, 6), n = 6Q+1$: The set S_i is same as $n \equiv 0(\text{mod}\, 6)$.

So, $|Si| = (n-1)(2n+1)$

Remaining vertices of V_3 form C_3 as the sets S_j, S_l as follows.

$$S_j = \begin{Bmatrix} v_{i,i'}, v_{i+2,(i+2)'}, v_{i+5,i+4'} \ \ for\ i = 1,5,9...2n-5, \\ v_{i,i'}, v_{i+2,(i+2)'}, v_{i+5,i+4'} \ \ for\ \ i = 2,6,10...2n-8, \end{Bmatrix} \cup \begin{Bmatrix} v_{2n-1,(2n-1)'}, v_{2n-3,(2n-3)'}, v_{2,1'}; \\ v_{2n-4,(2n-4)'}, v_{(2n-2),(2n-2)'}, v_{4,3'} \end{Bmatrix}$$

Thus $\left|S_j\right| = \left(\frac{2n-5-1}{4}+1\right)+\left(\frac{2n-8-2}{4}+1\right)+2 = n$

$$S_l = \begin{Bmatrix} v_{i,(3l-2)'}, v_{i,(3l-1)'}, v_{i-1,(3l)'} \ \ for\ odd\ l, 1 \leq l \leq 4q-1;\ 3l+3 \leq i \leq 2n \\ v_{(i-1),(3l-2+i)'}, v_{i,(3l-1+i)'}, v_{i,(3l)'} \ \ for\ even\ l, 2 \leq l \leq 4q;\ 3l+1 \leq i \leq 2n \end{Bmatrix}$$

Thus $\left|S_l\right| = \begin{pmatrix} (2n-3l-3)+1 \\ for\ odd\ l \\ 1 \leq l \leq 4q-1 \end{pmatrix} + \begin{pmatrix} (2n-3l-1)+1 \\ for\ even\ l \\ 2 \leq l \leq 4q-2 \end{pmatrix}.$

$$\sum_{i=1}^{4q-2}\left|S_l\right| = \sum_{l=1}^{4q-1}\begin{pmatrix} 2n-3l-2 \\ for\ odd\ l \end{pmatrix} + \sum_{l=2}^{4q}\begin{pmatrix} 2n-3l \\ for\ even\ l \end{pmatrix}$$

Thus there are $\left((2n-5)+(2n-11)+(2n-17)+....+3\right)+\left((2n-6)+(2n-12)+(2n-18)+....+2\right)$
$= \frac{n-1}{6}(2n-2)+\frac{n-1}{6}(2n-4) = \frac{(n-1)(2n-3)}{3}$

The remaining vertices form C_3s as follows.

$$S_p = \begin{Bmatrix} v_{3,1'}, v_{3,2'}, v_{5,4'}; v_{4,1'}, v_{4,2'}, v_{2n,4'}; \\ v_{5,1'}, v_{5,2'}, v_{2n,3'}; v_{9,7'}, v_{9,8'}, v_{11,10'}; \\ v_{10,7'}, v_{10,8'}, v_{2n,9'}; v_{11,7'}, v_{11,8'}, v_{2n,10'}; \end{Bmatrix} \cup \begin{Bmatrix} v_{3i,(3i-2)'}, v_{3i,(3i-1)'}, v_{3i+2,(3i+1)'}; \\ v_{(3i+1),(3i-2)'}, v_{2n,(3i+1)'}, v_{3i+2,(3i-2)'}; \\ v_{(3i+2),(3i-2)'}, v_{(3i+2),(3i-1)'}, v_{2n,(3i)'} \\ 5 \leq odd\ i \leq \dfrac{2n-5}{3} \end{Bmatrix} \cup \begin{Bmatrix} v_{2n,(2n-1)'}, v_{2n,3'}, v_{(2n)4'} \\ v_{(2n)',(2n-1)'}, v_{2n,9'}, v_{(2n),(10)'} \end{Bmatrix}$$

$$\Rightarrow \left|S_p\right| = 8 + 3\left(\frac{\left(\frac{(2n-5)}{3} - 5\right)}{2} + 1\right) = n + 1$$

Thus number of disjoint c3s $= \left|S_i\right| + \left|S_j\right| + \left|S_l\right| + \left|S_p\right| =$

$$\left(n(2n-1)-1\right) + \left(n-1\right) + \frac{(n-1)(2n-3)}{3} + \left(n+1\right) = \frac{2n(4n-1)}{3}$$

Thus number of disjoint cycles are

$$\tau\left(RL_2\left(C_4^{(n)}\right)\right) \geq \frac{2n(4n-1)}{3}$$

Case 3: $n \equiv 2(\bmod 6)$: The set Si is same as $n \equiv 0(\bmod 6)$.

So, $|Si| = (n-1)(2n+1)$

$$S_j = \begin{Bmatrix} v_{i,i'}, v_{i+2,(i+2)'}, v_{i+5,i+4'} & for\ i = 1,5,9...2n-7, \\ v_{i,i'}, v_{i+2,(i+2)'}, v_{i+5,i+4'} & for\ i = 2,6,10...2n-6, \end{Bmatrix} \cup \begin{Bmatrix} v_{2n-1,(2n-1)'}, v_{2n-3,(2n-3)'}, v_{2,1'}; \\ v_{2n,(2n)'}, v_{(2n-2),(2n-2)'}, v_{4,3'} \end{Bmatrix}$$

Thus $\left|S_j\right| = \left(\frac{2n-7-1}{4} + 1\right) + \left(\frac{2n-6-2}{4} + 1\right) + 2 = n$

Remaining vertices of V_3 form C_3 as the set S_l as follows.

$$S_l = \begin{Bmatrix} v_{i,(3l-2)'}, v_{i,(3l-1)'}, v_{i-1,(3l)'} & for\ odd\ l,\ 1 \leq l \leq 4q-1;\ 3l+3 \leq i \leq 2n \\ v_{(i-1),(3l-2+i)'}, v_{i,(3l-1+i)'}, v_{i,(3l)'} & for\ even\ l, 2 \leq l \leq 4q;\ 3l+1 \leq i \leq 2n \end{Bmatrix} \cup \begin{Bmatrix} v_{2n-1,(2n-3)'}, v_{2n-1,(2n-2)'}, v_{2n-1,(2n)'}; \\ v_{2n,(2n-3)'}, v_{2n,(2n-2)'}, v_{(2n-1)',(2n)'} \end{Bmatrix}$$

Thus $\left|S_l\right| = \begin{pmatrix} (2n-3l-3)+1 \\ for\ odd\ l \\ 1 \leq l \leq 4q-1 \end{pmatrix} + \begin{pmatrix} (2n-3l-1)+1 \\ for\ even\ l \\ 2 \leq l \leq 4q \end{pmatrix} + 2$.

$$\sum\left|S_l\right| = \sum_{l=1}^{4q-1}\begin{pmatrix} 2n-3l-2 \\ for\ odd\ l \end{pmatrix} + \sum_{l=2}^{4q}\begin{pmatrix} 2n-3l \\ for\ even\ l \end{pmatrix} + 2$$

$$\text{Thus there are}\left((2n-5)+(2n-11)+(2n-17)+....5\right)+\left((2n-6)+(2n-12)++....10+4\right)+2$$

$$=\left(\frac{(2n-4)\big/3}{2}\left((2n-5)+5\right)\right)+\left(\frac{(2n-4)\big/3}{2}\left((2n-6)+4\right)\right)+2$$

$$=\left(\frac{(n-2)(2n-1)}{3}+2\right)C_3s$$

The remaining vertices form C_3s as follows.

$$S_p=\begin{cases} v_{3i,(3i-2)'}, v_{3i,(3i-1)'}, v_{3i+2,(3i+1)'};\ v_{(3i+1),(3i-2)'}, v_{2n,(3i+1)'}, v_{3i+2,(3i-2)'}; \\ v_{(3i+2),(3i-2)'}, v_{(3i+2),(3i-1)'}, v_{2n,(3i)'} \\ 1\le odd\ i\le 4Q-1 \end{cases}$$

$$\Rightarrow |S_p| = 3\left(\frac{((4Q-1)-1)}{2}+1\right)=6Q=n-2$$

Thus number of disjoint c_3s $=|S_i|+|S_j|+|S_l|+|S_p|=$

$$\left(n(2n-1)-1\right)+n+\left(\frac{(2n-1)(n-2)}{3}+2\right)+n-2=\frac{2n(4n-1)}{3}-\frac{1}{3}$$

Hence the minimum number of cycles of RL2(Snc4) is

$$\tau\left(RL_2\left(C_4^{(n)}\right)\right)\ge\frac{2n(4n-1)}{3}\ \text{.---------------------(i)}$$

Case4: $n\equiv 3(mod\ 6)$: In this case the sets si,Sj are same as $n\equiv 0(\mathrm{mod}\,6)$

$$S_l=\begin{cases} v_{i,(3l-2)'}, v_{i,(3l-1)'}, v_{i-1,(3l)'}\ \ for\ odd\ l, 1\le l\le 4Q+1;\ 3l+3\le i\le 2n \\ v_{(i-1),(3l-2+i)'}, v_{i,(3l-1+i)'}, v_{i,(3l)'}\ \ for\ even\ l, 2\le l\le 4Q;\ 3l+1\le i\le 2n \end{cases}$$

Thus $|S_l|=\begin{pmatrix}(2n-3l-3)+1\\ for\ odd\ l\\ 1\le l\le 4Q+1\end{pmatrix}+\begin{pmatrix}(2n-3l-1)+1\\ for\ even\ l\\ 2\le l\le 4Q\end{pmatrix}$.

$$\sum|S_l|=\sum_{l=1}^{4Q+1}\begin{pmatrix}2n-3l-2\\ for\ odd\ l\end{pmatrix}+\sum_{l=2}^{4Q}\begin{pmatrix}2n-3l\\ for\ even\ l\end{pmatrix}$$

$$\text{Thus there are}\left((2n-5)+(2n-11)+(2n-17)+....13+7\right)+\left((2n-6)+(2n-12)+(2n-18)+....6\right)$$

$$=\frac{n-3\big/3}{2}\left((2n-5)+7\right)+\frac{n-3\big/3}{2}\left((2n-6)+6\right)$$

$$=\frac{n-3}{6}(2n+2)+\frac{n-3}{6}(2n)=\frac{(n-3)(2n+1)}{3}$$

The remaining vertices form C_3s as follows.

$$S_p = \begin{Bmatrix} v_{3i,(3i-2)'}, v_{3i,(3i-1)'}, v_{3i+2,(3i+1)'}; v_{(3i+1),(3i-2)'}, v_{2n,(3i+1)'}, v_{3i+2,(3i-2)'}; \\ v_{(3i+2),(3i-2)'}, v_{(3i+2),(3i-1)'}, v_{2n,(3i)'} \\ 1 \le odd\ i \le 4q-1 \end{Bmatrix} \cup \begin{Bmatrix} v_{2n-3,(2n-5)'}, v_{2n-3,(2n-4)'}, v_{n-1,n'}; \\ v_{2n-2,(2n-5)'}, v_{2n-2,(2n-4)'}, v_{n,(n-1)'}; \\ v_{2n-1,(2n-5)'}, v_{2n-1,(2n-4)'}, v_{n-1,(n-1)'}; \\ v_{2n,(2n-5)'}, v_{2n,(2n-4)'}, v_{2n,2n'}; \\ v_{2n,(2n-3)'}, v_{2n,(2n-2)'}, v_{2n-1,(2n)'}; \\ v_{2n-1,(2n-3)'}, v_{2n-1,(2n-2)'}, v_{2n-1,(2n)'} \end{Bmatrix}$$

$$\Rightarrow |S_p| = 3\left(\frac{((4q-1)-1)}{2}+1\right)+6 = 6q+6 = n+3$$

Thus number of disjoint c_3s $= |S_i| + |S_j| + |S_l| + |S_p| =$

$$\left(\binom{2n}{2}-1\right)+(n-1)+(n+3)+\frac{(2n+1)(n-3)}{3} = \frac{8n^2-2n}{6}$$

Thus number of disjoint cycles is

$$\tau\left(RL_2\left(C_4^{(n)}\right)\right) \ge \frac{2n(4n-1)}{3}$$

Case5: $n \equiv 4(\text{mod}\,6)$: The sets si,Sj are Same as $n \equiv 0(\text{mod}\,6)$

$$S_l = \begin{Bmatrix} v_{i,(3l-2)'}, v_{i,(3l-1)'}, v_{i-1,(3l)'}\ for\ odd\ l, 1 \le l \le 4q+1;\ 3l+3 \le i \le 2n \\ v_{(i-1),(3l-2+i)'}, v_{i,(3l-1+i)'}, v_{i,(3l)'}\ for\ even\ l, 2 \le l \le 4q;\ 3l+1 \le i \le 2n \end{Bmatrix} \cup \begin{Bmatrix} v_{2n-3,(2n-5)'}, v_{2n-3,(2n-4)'}, v_{n-1,n'}; \\ v_{2n-2,(2n-5)'}, v_{2n-2,(2n-4)'}, v_{n,(n-1)'}; \\ v_{2n-1,,(2n-5)'}, v_{2n-1,(2n-4)'}, v_{n-1,(n-1)'} \end{Bmatrix}$$

Thus $|S_l| = \begin{pmatrix} (2n-3l-3)+1 \\ for\ odd\ l \\ 1 \le l \le 4Q+1 \end{pmatrix} + \begin{pmatrix} (2n-3l-1)+1 \\ for\ even\ l \\ 2 \le l \le 4Q \end{pmatrix}$.

$$\sum |S_l| = \sum_{l=1}^{4Q+1} \begin{pmatrix} 2n-3l-2 \\ for\ odd\ l \end{pmatrix} + \sum_{l=2}^{4Q} \begin{pmatrix} 2n-3l \\ for\ even\ l \end{pmatrix}$$

Thus there are $((2n-5)+(2n-11)+(2n-17)+....9+3)+((2n-6)+(2n-12)+(2n-18)+....+8)+3$

$$= \frac{n-1/3}{2}((2n-5)+3) + \frac{n-4/3}{2}((2n-6)+8)+3$$

$$= \frac{n-1}{6}(2n-2) + \frac{n-4}{6}(2n+2)+3 = \frac{2n^2-5n+6}{3}$$

The remaining vertices form C_3s as follows.

$$S_p = \left\{ \begin{array}{l} v_{3i,(3i-2)'}, v_{3i,(3i-1)'}, v_{3i+2,(3i+1)'}; v_{(3i+1),(3i-2)'}, v_{2n,(3i+1)'}, v_{3i+2,(3i-2)'}; \\ v_{(3i+2),(3i-2)'}, v_{(3i+2),(3i-1)'}, v_{2n,(3i)'} \\ 1 \le odd\ i \le 4Q+1 \end{array} \right\}$$

$$\Rightarrow |S_p| = 3\left(\frac{((4Q-1)-1)}{2}+1\right) = 6Q+3 = n-1$$

Thus number of disjoint c_3s $= |S_i| + |S_j| + |S_l| + |S_p| =$

$$\left(\binom{2n}{2}-1\right) + (n-1) + \frac{2n^2-5n+6}{3} + (n-1) = \frac{8n^2-2n}{6}$$

Thus the number of disjoint cycles are -

$$\tau\left(RL_2\left(C_4^{(n)}\right)\right) \ge \frac{2n(4n-1)}{3} \quad \text{--------------------(i)}$$

Case 6: $n \equiv 5(\mathrm{mod}\, 6)$:

The set Si is Same as $n \equiv 1(\mathrm{mod}\, 6)$

Remaining vertices of V_3 form C_3 as the sets S_j, S_l as follows.

$$S_j = \left\{ \begin{array}{ll} v_{i,i'}, v_{i+2,(i+2)'}, v_{i+5,i+4'} & for\ i = 1,5,9 \ldots 2n-5, \\ v_{i,i'}, v_{i+2,(i+2)'}, v_{i+5,i+4'} & for\ i = 2,6,10 \ldots 2n-8, \end{array} \right\} \cup \left\{ v_{2n-4,(2n-4)'}, v_{(2n-2),(2n-2)'}, v_{4,3'} \right\}$$

Thus $|S_j| = \left(\frac{2n-5-1}{4}+1\right) + \left(\frac{2n-8-2}{4}+1\right) + 1 = n-1$

$$S_l = \left\{ \begin{array}{ll} v_{i,(3l-2)'}, v_{i,(3l-1)'}, v_{i-1,(3l)'} & for\ odd\ l, 1 \le l \le 4Q+1;\ 3l+3 \le i \le 2n \\ v_{(i-1),(3l-2+i)'}, v_{i,(3l-1+i)'}, v_{i,(3l)'} & for\ even\ l, 2 \le l \le 4Q+2;\ 3l+1 \le i \le 2n \end{array} \right\}$$

Thus $|S_l| = \begin{pmatrix} (2n-3l-3)+1 \\ for\ odd\ l \\ 1 \le l \le 4Q+1 \end{pmatrix} + \begin{pmatrix} (2n-3l-1)+1 \\ for\ even\ l \\ 2 \le l \le 4Q+2 \end{pmatrix}.$

$$\sum |S_l| = \sum_{l=1}^{4Q+1} \begin{pmatrix} 2n-3l-2 \\ for\ odd\ l \end{pmatrix} + \sum_{l=2}^{4Q+2} \begin{pmatrix} 2n-3l \\ for\ even\ l \end{pmatrix}$$

Thus there are $\left((2n-5)+(2n-11)+(2n-17)+....+11+5\right)+\left((2n-6)+(2n-12)+(2n-18)+....+10+4\right)$

$$=\frac{2n-4/6}{2}\left((2n-5)+5\right)+\frac{2n-4/6}{2}\left((2n-6)+4\right)$$

$$=\frac{2n-4}{12}(2n)+\frac{2n-4}{12}(2n-2)=\frac{(2n-1)(n-2)}{3}$$

The remaining vertices form C_3s as follows.

$$S_p=\begin{cases} v_{3,1'},v_{3,2'},v_{2n-1,(2n-1)'}; \\ v_{4,1'},v_{4,2'},v_{2n-1,(2n)'}; \\ v_{5,1'},v_{5,2'},v_{2n,(2n-3)'}; \\ v_{9,7'},v_{9,8'},v_{2n,2n'}; \\ v_{10,7'},v_{10,8'},v_{5,4'}; \\ v_{11,7'},v_{11,8'},v_{2,1'}; \end{cases} \cup \begin{cases} v_{3i,(3i-2)'},v_{3i,(3i-1)'},v_{3i+2,(3i+1)'}; \\ v_{(3i+1),(3i-2)'},v_{2n,(3i+1)'},v_{3i+2,(3i-2)'}; \\ v_{(3i+2),(3i-2)'},v_{(3i+2),(3i-1)'},v_{2n,(3i)'} \\ 5\le odd\ i\le 4Q+1 \end{cases} \cup \begin{cases} v_{2n,3'},v_{2n,4'},v_{2n-1,2n}; \\ v_{2n,9'},v_{2n,10'},v_{(2n-1)',2n'}; \\ v_{2n-1,(2n-3)'},v_{2n-1,(2n-2)'},v_{2n,9'}; \end{cases}$$

$$\Rightarrow |S_p|=6+3\left(\frac{((4q+1)-5)}{2}+1\right)+3=6Q+6=n+1$$

Thus number of disjoint c_3s $=|S_i|+|S_j|+|S_l|+|S_p|=$

$$\left(\binom{2n}{2}-1\right)+(n-1)+\frac{(n-2)(2n-1)}{3}+n+1=\frac{8n^2-2n-1}{3}=\left(\frac{8n^2-2n}{3}-\frac{1}{3}\right)C$$

Hence minimum number of cycles of RL2(SnC4)) is

$$\tau\left(RL_2\left(C_4^{(n)}\right)\right)\ge\frac{8n^2-2n}{3}=\frac{2n(4n-1)}{3}$$ ----------------------------------(i)

But $\left|E\left(S_{nC_4}\right)\right|=4n$ .So number of C_3s on these vertices $\le\frac{\binom{4n}{2}}{3}$ ………………………..(2)

So, from (i),(ii) , we get $\tau\left(RL_2\left(S_{nc_4}\right)\right)=\frac{\binom{4n}{2}}{3}$

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