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Research Paper

Exponential demand-based inventory model for non-instantaneous deteriorating items

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Abstract
This paper examines an inventory model for non-instantaneously deteriorating items wherein demand varies with time. In contrast to many existing studies that assume a constant deterioration rate, the proposed work is based on the practical observation that numerous products begin to deteriorate progressively as they near the end of their effective lifespan. The aim of the model is to incorporate realistic inventory characteristics, including time-dependent demand and variable sales revenue over time. These considerations provide a more accurate representation of inventory dynamics under fluctuating market and product conditions. To determine optimum order quantity and maximum profit, numerical example is presented to prove the applicability and effectiveness of the proposed model.

Keywords: *Inventory, exponential demand, Maximum life time, Sales revenue cost, deterioration.*

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I. INTRODUCTION

Latest decades, inventory models involving deteriorating items have attracted significant attention in the fields of inventory, supply chain, and operations research because of their relevance to products with limited shelf lives. The main foundation of inventory theory was established by Harris (1915), who introduced the Economic Order Quantity (EOQ) concept. This research work was later refined by Wilson (1934), who formulated the classical EOQ model for determining the optimal replenishment quantities. Later, Whitin (1957) expanded inventory analysis by the examining products whose value and usability decline toward the end of their storage period.A major improvement in the deteriorating inventory research was made by Ghare and Schrader (1963), who developed an EOQ inventory model for items subject to the exponential decay under the constant deterioration rate without shortages. Their contribution provided on the foundation for the extensive ensuing investigations. Covert and Philip (1973) prolonged this research work adopting a new two-parameter Weibull distribution to represent the variable deterioration behaviour.

Philip (1974) more enhanced the model by employing a three-parameter Weibull distribution, thereby improving the flexibility of the deterioration representation. Misra (1975) suggested an EPQ model integrating the both fixed and variable deterioration rates. Later, Shah and Jaiswal (1977), Aggarwal (1978), Dave and Patel (1981), and Chowdhury and Choudhuri (1983) developed various inventory policies for the deteriorating products under different operational assumptions, contributing substantially to evolute of the inventory control theory.

Further the developments were reported by Kang and Kim (1983), who investigated the pricing and production decisions for deteriorating products. Dave (1986) examined the replenishment policies for such items, while Aggarwal and Jaggi (1989) proposed the ordering strategies specifically tailored to deteriorating inventory systems. Raafat (1991) provided a comprehensive review of literature on continuously deteriorating inventory models which highlighting the key trends and research directions. Additional contributions from Chung and Ting (1993) and Goyal and Gunasekaran (1995) broadened the applicability of deteriorating inventory systems to more realistic operating environments. Bhunia and Maiti (1998) introduced the models incorporating time-dependent deterioration patterns, including linear deterioration behaviour.As research progressed, greater emphasis was placed on the interaction amongstthe demand, time, and deterioration. Chang and Dye (1999) developed an inventory model with partial backlogging under the varying demand conditions, while Wee (1999) investigated inventory systems considering quantity discounts, and pricing decisions.

Goyal and Giri (2001) revised emerging developments in deteriorating inventory modelling. Abad (2001) and Mukhopadhyay et al. (2004) further enriched the inventory literature by examining alternative forms of time-dependent deterioration.In the practical situations, many products retain their original quality for a certain period, after the procurement before deterioration starts. This phenomenon is commonly mentioned to non-instantaneous deterioration. Ouyang et al. (2006) formally introduced the concept of non-instantaneous deterioration while studying inventory systems with permissible delays in payment. Next, Zhou and Gu (2007), Malik et al. (2008), Liao (2008), Lee and Hsu (2009), and Singh and Malik (2009, 2010) extended the inventory theory by incorporating trade credit, variable demand, production considerations, stock-dependent demand, and partial backlogging into the non-instantaneous deterioration models.Subsequent studies by Singh and Malik (2010, 2011), Malik and Kumar (2011), Gupta et al. (2013), Sarkar and Sarkar (2013a, 2013b), Ghoreishi et al. (2014), Sheng et al. (2015), Sarkar et al. (2015), and Malik et al. (2016) introduced the numerous practical considerations as inflation, stock-dependent demand, probabilistic deterioration, quality improvement, trade credit financing, and controllable lead times.

Recent research has increased thefocused-on sustainability and environmentally responsible inventory management practices. Malik et al. (2017) investigated the twowarehouses inventory model usinglifetime constraints under inventory level-dependent demand. Kumar et al. (2017) incorporated variable holding and sales revenue costs into the inventory decision-making. Later studies by Mathur et al. (2019), Kumar et al. (2019), Saxena et al. (2020), Malik and Garg (2021), Bansal et al. (2021), Kumar et al. (2022), Beniwal et al. (2022), and Yadav et al. (2022) explored topics including fuzzy optimization with sustainability, inflation, trade credit, ramp-type demand, and uncertainty management.More recently, Sharma et al. (2023) examined part of advertising efforts withstimulating demand for deteriorating products, while Tyagi et al. (2023) reviewed the fuzzy inventory approaches for pharmaceutical and cosmetic industries. Malik et al. (2024) developed an EOQ model integrating the trade credit under preservation technology. Similarly, Singh and Goel (2024) emphasized the circular supply chains and reverse logistics, whereas Jayaswal et al. (2024) investigated possessions of carbon emissions, shortages, trade credit, and learning. Recent contributions by Rani et al. (2025) Mehta et al. (2025) and Verma et al. (2026) further progressive sustainable inventory modelling through the incorporation of the preservation technology, green investments, two-warehouse inventory system, climate-resilient supply-chain practices, carbon tax policies, inflation, and the learning effects.Mehta et al. (2026) develops a sustainable two-warehouse inventory model for perishable goods that minimizes total cost while considering learning effects, carbon emissions, and increasing demand.

Although substantial inventory literature on deteriorating items, limited attention has been devoted to models that simultaneously consider non-instantaneous deterioration and time-varying deterioration rates. Since these characteristics commonly arise with the real-world inventory. Numerical problemisused to reveal the suitability of planned model which shows its performance under different operating conditions.

II. NOTATIONS AND ASSUMPTIONS

To frame the proposed inventory model in a very clear and structured manner, ensuing notations and assumptions are defined accordingly:

Notations

ue^{vt}	Time dependent demand rate, assumed to follow an exponential form with respect to time t.
C_0	Fixed ordering cost incurred for each order placed in the inventory system
C_h	Holding cost/carrying cost per unit item/unit time.
C_p	Purchasing cost/ unit item paid to acquire goods.
C_d	Deterioration cost per unit item due to spoilage, decay, or loss in quality over time.
$S_1 - S_2(t)$	Time dependent selling price per unit item.
$\theta(t)$	Time-based deterioration rate at time t, given by $\theta(t) = \frac{1}{1+R-t}$where $0 \le \theta(t) \le 1$.
L	Maximum storage capacity.
R	Maximum life time of the product before complete deterioration occurs.
$Y_1(t)$	Inventory level during [0, t_1], when items remain fresh with no deterioration.
$Y_2(t)$	Inventory Level during [t_1,T], when deterioration starts and continues.
t_1	Time point separating fresh and deteriorating phases of inventory.
T	Total length of inventory replenishment cycle.
TP	Total profit of inventory system, used as objective function.

Assumption

- The Demand is assuming to kept an exponential growth pattern over time, expressed as ue^{vt}, where u represent initial demand level and v denotes growth parameter leading the rate of increase.
- The proposed inventory modeldrives over a finite planning horizon divided into two distinct phases. In the interval $[0, t_1]$, all items are considered fresh and no deterioration takes place. After t_1, deterioration begins in the on-hand inventory. Once deterioration starts, no repair, replenishment is permitted. The proposed deterioration rate is time-dependent and defined as

$$\theta(t) = \frac{1}{1 + R - t}$$

Where R represented the maximum usable lifetime of the item.
- The system is designed such that shortages are not allowed under any circumstances, ensuring full demand fulfilment throughout the cycle.

MATHEMATICAL MODEL

The model presents the dynamic performance of the inventory level $Y(t)$ over the planning horizon, beginning with the initial stock L at time $t = 0$. It is observed that the inventory initially declines smoothly as a result of effect of buyer demand, which is assumed to follow an exponential pattern, while deterioration is absent during this period. However, after the transition time t_1, deterioration of items commences in addition to the ongoing exponential demand, where deterioration is considered the time-dependent, thereby accelerating the rate of depletion of inventory. As a result, the inventory level decreases more rapidly in this region and eventually reaches zero at the terminal time T, ensuring complete exhaustion of stock within the cycle.

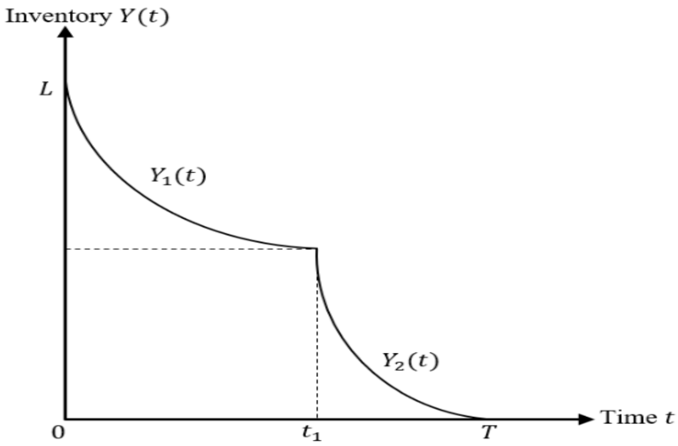

Fig.1. Graphically presentation of the proposed model

During time interval $[0, t_1]$, the inventory level Y_1decreases only because of customer demand, and no deterioration occurs in this period. After time t_1, both demand and deterioration begin simultaneously and continue until the inventory level reaches zero (Fig. 1). Therefore, the inventory levels at any time t over the interval $[0, T]$ are symbolised by the following differential equations.

The differential equation:

$$\frac{dY_1(t)}{dt} = -ue^{vt}, 0 \leq t \leq t_1 \qquad \dots(1)$$

and for another interval:

$$\frac{dY_2(t)}{dt} + \theta(t)I_2(\mathrm{t}) = -ue^{vt}, t_1 \leq t \leq T \qquad \dots(2)$$

To determine the inventory level function corresponding to the two phases of the inventory cycle, the governing differential equations are solved under the boundary conditions $Y_1(0) = L$ and $Y_2(T) = 0$, respectively. Here, L means the initial stock available at the beginning of the cycle, while the condition $Y_2(T) = 0$ indicates that the inventory becomes completely depleted at the terminal time T. By incorporating these boundary conditions into the solution procedure, the analytical expressions representing the inventory levels over the planning horizon are derived in this manner:

Solution ofequation (1):

$$\frac{dY_1(t)}{dt} = -ue^{vt}, \qquad 0 \leq t \leq t_1$$

Separating the variables, and integrating both sides, we get

$$\int d\,Y_1(t)dt = -\int u\,e^{vt}dt$$

$$Y_1(t) = -\frac{u}{v}e^{vt} + C \qquad \dots(3)$$

To find the unknown value of the constant C; boundary condition applies as$Y_1(0) = L$:

$$Y_1(t) = -\frac{u}{v}e^{vt} + L + \frac{u}{v} \qquad \dots(4)$$

Now, the solution of the equation (2) becomes

$$\frac{dY_2(t)}{dt}+\frac{1}{1+R-t}Y_2(t)=-ue^{vt}\ ,\ \ t_1\leq t\leq T$$

Solution using integrating factor:

$$\frac{Y_2(t)}{1+R-t}=\int\frac{-ue^{vt}}{1+R-t}dt+C$$

Let us substitute:

$$\frac{Y_2(t)}{(1+R-t)}=u\int\frac{e^{v(1+R-p)}}{p}dp+C$$

Using the series expansion:

$$\frac{Y_2(t)}{1+R-t}=ue^{v(1+R)}\left[\log p-vp+\frac{v^2p^2}{4}-\frac{v^3p^3}{18}+\cdots\right]+C$$

Applying the boundary condition $Y_2(T)=0$:

$$\frac{Y_2(t)}{1+R-t}=ue^{v(1+R)}\begin{bmatrix}\log(1+R-t)-v(1+R-t)\\ +\dfrac{v^2(1+R-t)^2}{4}\\ -\dfrac{v^3(1+R-t)^3}{18}+\cdots\end{bmatrix}+C \qquad \ldots(5)$$

Incorporating the given boundary condition $Y_2(T)=0$ with the derived inventory model, the unknown constant C. These conditions show the depletion of inventory stock at replenishment cycle;thus,it ensures the validity and completeness of the solution.Now$Y_2(t)$, after putting 1+R=K, we get

$$Y_2(t)=(K-t)ue^{vK}\begin{bmatrix}(\log\dfrac{(K-t)}{(K-T)})+(t-T)\{v+\dfrac{v^2}{4}\big((T+t)+2K\big)\\ +\dfrac{v^3}{18}(T^2+Tt+t^2)+\dfrac{v^3}{6}(K^2+K(T+t))\}\end{bmatrix} \ldots(6)$$

The inventory level function $Y(t)$fulfils the continuity with critical time point $t=t_1$. The inventory expressions corresponding to the two phases of the proposed model which become identical. Accordingly, with the equations (4) and (6), the analytical expression maximizes inventory level L is derived as follows.

$$L=\frac{u}{v}e^{vt}-\frac{u}{v}+(K-t)ue^{vK}\begin{bmatrix}\left(log\frac{(K-t)}{(K-T)}\right)\\ +(t-T)\{v+\frac{v^2}{4}\big((T+t)+2(1+R)\big)\\ +\frac{v^3}{18}(T^2+Tt+t^2)\ +\ \frac{v^3}{6}(K^2+K(T+t))\}\end{bmatrix}\ldots(7)$$

The optimum profit per cycle consists of the following cost components. The ordering Cost incurred in each cycle is given by

$$OC=C_o \qquad \ldots(8)$$

The deterioration cost represents the loss incurred due to the spoilage or decay of items during the inventory cycle. Hence, the total deterioration cost per cycle is given by

$$DC = \int_{t_1}^{T} C_d\, \theta(t)\, Y_2(t)dt$$

$$DC = C_d\, u\, e^{vK} \begin{bmatrix} (t_1 - T) + (K - t_1)\left(\log\dfrac{(K-t_1)}{(K-T)}\right) \\ +\left(\dfrac{v^3}{6}K^2 + \dfrac{v^2}{2}K + v\right)\left(\dfrac{T^2}{2} - tt_1 + \dfrac{t_1^2}{2}\right) \\ +\left(\dfrac{v^2}{4} + \dfrac{v^3}{6}K\right)\left(\dfrac{2T^3}{3} - tt_1^2 + \dfrac{t_1^3}{3}\right) \\ +\dfrac{v^3}{18}\left(\dfrac{3T^4}{4} - T^3 t_1 + \dfrac{t_1^4}{4}\right) \end{bmatrix} \quad \ldots(9)$$

The purchasing cost associated with acquiring inventory items during a single cycle is expressed as follows:

$$PC = C_p * L$$

By putting the value of L from equation (7):

$$PC = C_p * \left\{ \begin{matrix} \frac{u}{v}e^{vt} - \frac{u}{v} + (K-t)ue^{vK} \\ \left[\begin{matrix} \left(log\frac{(K-t)}{(K-T)}\right) + (t-T)\{v + \frac{v^2}{4}\left((T+t) + 2(1+R)\right) \\ +\frac{v^3}{18}(T^2 + Tt + t^2) + \frac{v^3}{6}(K^2 + K(T+t))\} \end{matrix} \right] \end{matrix} \right\} \ldots (10)$$

The total sales revenue generated during each inventory cycle is given by

$$SRC = C_s \int_0^T u\, e^{vt} dt$$

$$\therefore C_s = (S_1 - S_2 t)$$

$$SRC = uS_1\left(\frac{e^{vt} - 1}{v}\right) - uS_2\left(\frac{Tve^{vt} - e^{vt} + 1}{v^2}\right) \quad \ldots(11)$$

The holding cost associated with storing inventory items over a single cycle is expressed as follows:

$$HC = C_h * \left(\int_0^{t_1} Y_1(t)dt + \int_{t_1}^{T} Y_2(t)dt\right)$$

$$HC = C_h \begin{bmatrix} \left(L + \frac{u}{v}\right)t_1 - \frac{u}{v^2}(e^{vt_1} - 1) \\ +\left(\frac{3}{2}K^2 + \frac{t_1^2}{2} - Kt_1\right)\log\left(\frac{K-t_1}{K-T}\right) + (T - t_1)\left\{\begin{pmatrix} -2K - KT \\ V + 2K + \frac{v^2 T}{4} \\ +\frac{v^3 KT^2}{18} + \frac{v^3 K^2}{6} + \frac{v^3 Kt}{6} \end{pmatrix}\right\} \\ +(T^2 - t_1^2)\left\{(v + 2K)\left(\frac{K+T}{2}\right) + \frac{v^2 T^2}{8} + \frac{v^3 T^3}{36} + \frac{v^3 K^3 (K+2T)}{12} + \frac{v^3 KT^2}{12}\right\} \\ +(T^3 - t_1^3)\left\{\frac{-(v+2K)}{3}\right\} + (T^4 - t_1^4)\left\{\frac{v^2(K-1)}{16} - \frac{v^3 K}{36}\right\} + (T^5 - t_1^5)\left\{\frac{-v^3}{90}\right\} \end{bmatrix} \ldots(12)$$

Thus, the optimum inventory profit (TP)per cycle as per unit time is given by

$$TP = \frac{1}{T}[SRC - OC - HC - DC - PC] \quad \ldots (13)$$

To determine the optimal cycle time *T**, our aim to optimize the total present value of profit (*TP*). This is achieved by differentiating the *TP*function with respect to T, and we have

$$\frac{dTP}{dT} = 0 \ldots (14)$$

To ensure that the solution corresponds to a maximum, the second derivative must satisfy.

$$\frac{d^2TP}{dT^2} < 0 \quad \ldots (15)$$

Since the profit function is highly non-linear, an analytical solution may not be feasible. Therefore, the optimal inventory system is determined using the solution procedure.

NUMERICAL EXAMPLE

Exemplify the solution procedure,consider:

$C_o = 1200,\ S_1 = 220,\ S_2 = 1.5,\ C_p = 110,\ C_h = 0.10,\ C_d = 0.06,\ R = 30,\ t_1 = 0.5,\ u = 1000,$ and $v = 50$. Applying the optimization technique, the optimal ordering cycle is determined as $T = 3.75$. The corresponding optimal profit and inventory level are calculated as $TP^* = 78{,}357$ and$L^* = 4{,}250$ respectively. The following graph represents the relation between Total profit and time cycle (Fig.2).

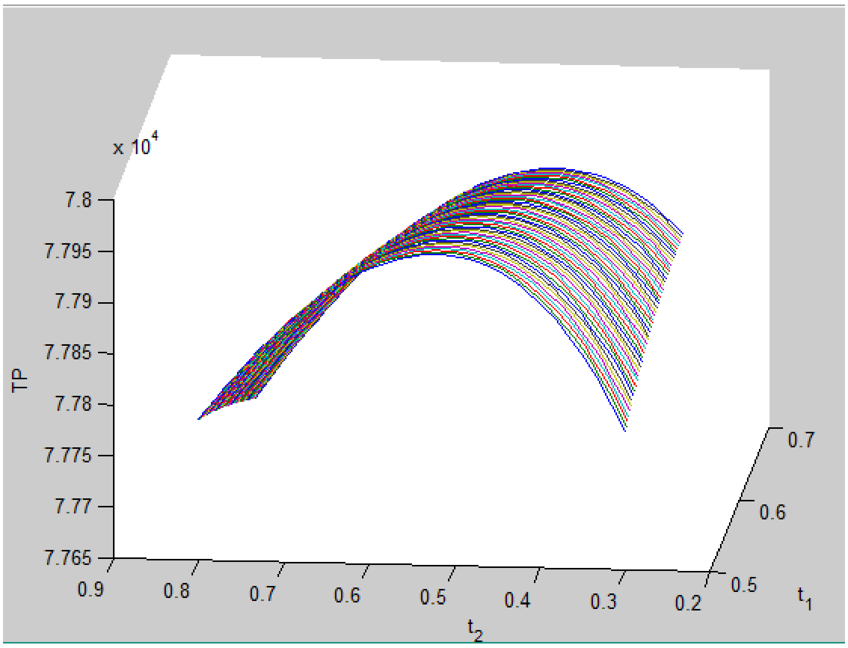

Fig. 2 Graphical representation of TP with t_1 and t_2

III. CONCLUSION

The most existing inventory models assumes constant deteriorating rates. However, in practical situations, many products do not begin to deteriorate immediately and their deterioration rate changes over time due to limited shelf life and aging effects. To address this limitation, the study proposes amathematical model with non-instantaneous deteriorating items. Also included a time-dependent deterioration rate and an exponential demand pattern that varies with time. The proposed model captures the livelyadvancement of both demand and deterioration, thereby providing a more realistic representation of actual inventory systems. Numerical illustration is examining the impression of significant model constraints on the optimum replenishment policy and overall system performance. The outcomesspecify that the proposed structure can effectively care the inventory decision-making in uncertainty and time-sensitive environments. Furthermore, the model may be prolonged by incorporating for additional practical considerations asprice-dependent demand, inflation, probabilistic deterioration, two-stage inventory systems, and the trade credit policies, thereby enhancesthe applicability for solving the complex supply chain and inventory management problems.

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