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Research Paper

On Connectivity in Anisotropic Random Rectangular Graphs

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Abstract
Random Geometric Graphs (RGGs) constitute one of the most widely used mathematical models for analyzing spatially embedded networks. Their applications span wireless sensor networks, ad hoc communication systems, transportation infrastructures, and various distributed systems. Most existing connectivity results, however, assume isotropic deployment regions, typically represented by squares or disks, whereas many practical networks are deployed in elongated rectangular environments exhibiting significant geometric anisotropy. Such configurations arise naturally in highway monitoring, railway communication systems, pipeline surveillance, irrigation channels, and industrial corridors.
In this paper, we introduce the Anisotropic Random Rectangular Graph (ARRG), a geometric graph model defined over rectangular domains with arbitrary aspect ratios. Vertices are independently and uniformly distributed within the deployment region, and edges are established whenever the Euclidean distance between two vertices does not exceed a prescribed communication radius. Unlike the classical Random Geometric Graph, the proposed model explicitly incorporates the aspect ratio of the deployment region into the connectivity analysis.
A mathematical framework is developed to investigate how geometric anisotropy influences the expected vertex degree, the occurrence of isolated vertices, and the connectivity threshold. Theoretical results show that increasing the aspect ratio decreases the effective communication neighborhood, thereby requiring either a larger transmission radius or a higher node density to preserve the same probability of full connectivity. The proposed model naturally reduces to the classical Random Geometric Graph when the deployment region becomes square.
The developed framework extends existing connectivity theory for geometric random graphs while providing a realistic mathematical model for spatial networks operating in elongated environments. The obtained results offer useful theoretical guidelines for the design and optimization of wireless sensor networks deployed in anisotropic regions.

Keywords: *Random Geometric Graphs; Random Rectangular Graphs; Connectivity; Spatial Networks; Anisotropic Domains; Wireless Sensor Networks; Stochastic Geometry .*

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I. Introduction

Random Geometric Graphs (RGGs), first introduced by Gilbert (1961), have become a cornerstone of modern stochastic geometry and random graph theory. In an RGG, vertices are randomly distributed in a geometric domain, and edges are established according to a prescribed distance threshold. Unlike classical Erdos-Renyi random graphs, where edge formation is independent of vertex positions, geometric graphs explicitly incorporate spatial information into the network topology. This characteristic makes them particularly suitable for modeling wireless communication networks, mobile ad hoc systems, sensor deployments, robotic swarms, biological networks, and many other spatially distributed systems (Gilbert, 1961; Penrose, 2003).

The connectivity of Random Geometric Graphs has received considerable attention because it represents one of the most fundamental properties of network reliability. A connected graph guarantees that every pair of vertices can communicate through one or more intermediate nodes, thereby ensuring global information exchange throughout the network. Consequently, determining the communication radius or node density required for full connectivity has become a central research topic in probability theory, stochastic geometry, and wireless communications (Penrose, 2003).

Classical connectivity theory has largely focused on isotropic deployment regions such as unit squares, disks, and higher-dimensional cubes (Penrose, 2003). Under these assumptions, asymptotic analyses have established sharp connectivity thresholds relating the communication radius to the number of vertices and the node density (Gupta & Kumar, 1998). These results have significantly advanced the theoretical understanding of geometric random graphs and have provided practical design criteria for wireless networks.

Nevertheless, such analyses generally assume that the geometry of the deployment region is symmetric, an assumption that is frequently violated in real applications.

Recent studies have demonstrated that the geometry of the deployment region plays an essential role in determining network connectivity. In particular, boundary components and corner regions substantially increase the probability of isolated vertices, especially in finite networks. Consequently, boundary effects become dominant in the asymptotic behavior of full connectivity, indicating that the shape of the deployment region cannot be ignored when accurate connectivity estimates are required (Dettmann & Georgiou, 2011). Indeed, recent paradigm shifts indicate that full connectivity transitions in finite systems are uniquely sensitive to local geometric layouts, where boundaries introduce sharp phase behaviors absent from bulk approximations (Coon, Dettmann, & Georgiou, 2012).

Many practical applications naturally involve elongated rectangular environments rather than square regions. Examples include pipeline monitoring systems, railway communication networks, highway surveillance, border security, irrigation channels, underground tunnels, coastal monitoring, and industrial production lines. In these scenarios, the rectangle aspect ratio introduces a pronounced geometric anisotropy that directly influences the distribution .of neighboring vertices and, consequently, the probability of network connectivity

Furthermore, establishing this macro-structural connectivity is vital for active processes embedded within spatial topologies; recent mathematical proofs show that a random graph process achieves global synchronization in coupled oscillator dynamics precisely at the temporal instant full structural connectivity is cleared (Jain, Mizgerd, & Sawhney, 2025). This link underscores that spatial constraints directly dictate dynamical operational thresholds.

Despite the extensive literature on Random Geometric Graphs, relatively few analytical studies explicitly investigate the effect of anisotropic rectangular domains on connectivity. Existing Random Rectangular Graph models generally adapt the deployment region while preserving classical isotropic connectivity analysis, leaving the role of geometric anisotropy largely unexplored. This gap motivates the development of new mathematical models capable of incorporating the aspect ratio as an explicit parameter governing network connectivity.

In this paper, we propose the Anisotropic Random Rectangular Graph (ARRG) as a natural extension of the classical Random Geometric Graph. The proposed model introduces the rectangle aspect ratio into the analytical description of connectivity while maintaining the simplicity of Euclidean distance-based edge formation. We derive theoretical results describing the influence of anisotropy on the expected vertex degree, isolated vertices, and the connectivity threshold, thereby extending existing geometric graph theory to a broader class of deployment regions.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature on geometric random graphs and connectivity analysis. Section 3 introduces the proposed ARRG model and presents its mathematical formulation. Section 4 develops the theoretical connectivity analysis and establishes the principal analytical results. Section 5 discusses the implications of the proposed framework, and Section 6 concludes the paper with directions for future research.

II. Related Work

Random Geometric Graphs (RGGs) have become one of the fundamental models in stochastic geometry and probabilistic graph theory due to their ability to represent networks embedded in Euclidean space. Since the pioneering work of Gilbert (1961), considerable effort has been devoted to understanding the structural properties of these graphs, including connectivity, coverage, percolation, clustering, routing efficiency, and network robustness. Comprehensive mathematical treatments of Random Geometric Graphs are provided by

Penrose (2003), whose work established many of the asymptotic results that form the basis of modern geometric graph theory.

One of the principal research topics in Random Geometric Graphs concerns network connectivity. Classical studies demonstrated that connectivity exhibits a sharp threshold phenomenon: when the communication radius exceeds a critical value, the probability of full connectivity rapidly approaches one as the number of vertices increases. This threshold depends primarily on the node density and the deployment area, and it has become a fundamental design criterion for wireless sensor networks and mobile communication systems (Penrose, 2003; Gupta & Kumar, 1998).

Modern evaluations have successfully expanded these spatial architectures into highdimensional contexts and non-parametric statistical frameworks, distinguishing local latent geometries from classical community random graphs (Duchemin & De Castro, 2022).

In particular, probabilistic connection functions were introduced to account for fading channels, interference, and unreliable wireless links, leading to the development of Soft Random Geometric Graphs (SRGGs) and related stochastic network models. As outlined in the comprehensive survey by Dettmann, Georgiou, and Pratt (2018), practical wireless network deployments are more accurately represented by coupling spatial point processes— such as Binomial Point Processes (BPP)—with distance-dependent link functions, where full connectivity bounds behave differently than those of rigid unit-disk configurations.

These extensions preserved the geometric nature of the underlying graph while improving the realism of wireless communication models (Mao & Anderson, 2012).

Another important direction concerns the influence of the boundary of the deployment region. Unlike infinite-domain models, finite networks exhibit significant edge and corner effects that increase the likelihood of isolated vertices. Dettmann and Georgiou (2011) demonstrated that, in dense finite geometric networks, the probability of disconnection is governed primarily by vertices located near boundaries rather than those in the interior.

By employing analytical cluster expansions, Coon, Dettmann, and Georgiou (2012) proved that pointier corner components penalize the accessible connectivity mass exponentially more than flat edges, confirming that boundary configurations dominate the structural bottlenecks of finite topologies. Their analysis showed that geometric boundaries play a decisive role in determining the probability of full connectivity, particularly in finite deployment regions.

Although these studies substantially advanced the theory of Random Geometric Graphs, the majority assume deployment regions possessing geometric symmetry, such as squares, disks, or higher-dimensional cubes (Penrose, 2003). Such assumptions simplify the mathematical analysis but do not accurately represent many practical deployment environments.

Rectangular deployment regions have received comparatively less theoretical attention despite their practical importance. Existing Random Rectangular Graph models generally employ rectangular domains as simulation environments while retaining analytical expressions derived for isotropic geometries. Consequently, the influence of the rectangle aspect ratio on the connectivity threshold remains insufficiently understood from a theoretical perspective.

This limitation is particularly important because elongated rectangular domains naturally arise in numerous engineering applications, including pipeline monitoring, railway communication systems, border surveillance, highway traffic monitoring, irrigation infrastructures, underground tunnels, and industrial production facilities. In these environments, geometric anisotropy significantly modifies the spatial distribution of neighboring vertices and therefore alters the probability of network connectivity.

The present work addresses this gap by introducing the Anisotropic Random Rectangular Graph (ARRG), in which the rectangle aspect ratio is incorporated explicitly into the mathematical analysis. Rather than treating the rectangular shape merely as a simulation region, the proposed framework considers geometric anisotropy as a governing parameter that influences the expected vertex degree, isolated-vertex probability, and connectivity threshold. This perspective extends the classical Random Geometric Graph (Penrose, 2003) while preserving its analytical tractability.

Accordingly, the contribution of this paper is not simply the adoption of a rectangular deployment region, but the development of a mathematical framework that quantifies the influence of anisotropic geometry on connectivity. The proposed model therefore bridges the gap between classical isotropic Random Geometric Graphs (Penrose, 2003) and practical spatial networks deployed in elongated environments.

III. Mathematical Model

This section presents the mathematical formulation of the proposed Anisotropic Random Rectangular Graph (ARRG). The model extends the classical Random Geometric Graph (Penrose, 2003) by explicitly incorporating the geometry of the deployment region into the connectivity analysis. In particular, the rectangle aspect ratio is introduced as an independent structural parameter governing the topological behavior of the network.

Unlike isotropic deployment domains, elongated rectangular regions modify the spatial distribution of neighboring vertices and increase the influence of boundary effects (Dettmann & Georgiou, 2011). Consequently, the connectivity properties depend not only on the communication radius and node density but also on the geometry of the underlying domain.

3.1 Deployment Region

Let R be a rectangular deployment region defined as:

$$R = [0, Lx] \times [0, Ly]$$

where Lx and Ly represent the length and width of the rectangle, respectively. The total deployment area is:

$$V = Lx \times Ly$$

Throughout this paper, the deployment area is assumed to remain constant while the rectangle dimensions vary. To characterize the geometry of the region, we define the aspect ratio (alpha) as:

$$\alpha = Lx / Ly$$

The particular case alpha = 1 corresponds to the classical square domain commonly adopted in Random Geometric Graphs (Penrose, 2003). As the value of alpha increases, the deployment region becomes increasingly elongated, thereby introducing stronger geometric anisotropy into the network.

3.2 Node Distribution

Consider a set of N vertices VN = {X1, X2, ..., XN}, where each vertex is independently and uniformly distributed over the deployment region R:

$$Xi \sim U(R), \text{ for all } i \text{ in } \{1, 2, ..., N\}$$

Following the point process formulations summarized by Dettmann et al. (2018), this conditioning yields a standard finite Binomial Point Process (BPP) across the domain space.
The node density is therefore given by:

$$\rho = N / V$$

Uniform node deployment is a standard assumption in geometric random graph theory and provides a convenient probabilistic framework for analytical investigations (Gilbert, 1961;
Penrose, 2003).

3.3 Edge Formation

Two vertices Xi and Xj are connected whenever their Euclidean distance satisfies:

$$Xi - Xj|| <= r||$$

where r denotes the prescribed communication radius. Accordingly, the edge set is defined by:

$$E = \{(Xi, Xj) \text{ in } VN \times VN : ||Xi - Xj|| <= r, i != j\}$$

and the resulting graph is G = (VN, E). This construction coincides with the classical Random Geometric Graph (Gilbert, 1961) except that the underlying spatial domain is allowed to possess arbitrary geometric anisotropy.

Definition 1 (Anisotropic Random Rectangular Graph)

An Anisotropic Random Rectangular Graph, denoted by ARRG(N, r, alpha), is defined as a graph satisfying the following conditions:
1/ Vertices are independently and uniformly distributed over the rectangle R
2/Two vertices are connected whenever their Euclidean distance does not exceed the communication radius r.
3/The deployment geometry is characterized by the aspect ratio alpha, which is explicitly incorporated into the connectivity analysis.
Consequently, the proposed model is completely determined by the parameter set {N, V, r, alpha}.

1/ Remark

The proposed formulation separates the randomness of node locations from the deterministic geometry of the deployment region. This separation enables the analytical investigation of how geometric anisotropy alone influences network connectivity while maintaining the probabilistic structure of the classical Random Geometric Graph (Penrose, 2003).
Furthermore, substituting alpha = 1 into the domain equations immediately yields Lx = Ly = sqrt(V), demonstrating that the proposed model naturally generalizes the classical Random Geometric Graph over square areas.

3.4 Connectivity Probability

Let P_fc denote the probability that the graph is fully connected. Formally,
P_fc = P(Graph G is connected)
The principal objective of this paper is to determine how the node density (rho), the communication radius (r), and the aspect ratio (alpha) jointly influence the value of P_fc. In particular, we seek to derive analytical conditions under which P_fc approaches 1 as the number of vertices tends to infinity (N -> infinity).

Assumptions

The theoretical analysis developed in the following section is based on the following assumptions:

* Vertices are independently and uniformly distributed .
* The communication radius r is identical for all vertices.
* The network is static during the observation period
* Euclidean distance is adopted as the communication metric .

*The asymptotic regime N -> infinity is considered whenever connectivity thresholds are derived.

IV. Connectivity Analysis

This section develops the theoretical analysis of connectivity for the proposed Anisotropic Random Rectangular Graph (ARRG). The objective is to investigate how the rectangle aspect ratio affects the emergence of full connectivity and modifies the classical connectivity threshold established for Random Geometric Graphs. Throughout the analysis, let G ~
ARRG(N, r, alpha), where the notation introduced in Section 3 is adopted.

4.1Expected Vertex Degree

The expected degree of a randomly selected vertex represents one of the most important indicators of network connectivity. Ignoring boundary effects, every vertex covers an effective communication region approximately equal to (pi * r^2). Hence, the link probability between two nodes in an unconstrained bulk space matches the localized Borel probability measure over the system space (Duchemin & De Castro, 2022). Using rho =
N/V, the expected degree becomes:

E[k] = rho * pi * r^2

This expression agrees with the classical Random Geometric Graph model (Penrose, 2003). However, elongated rectangular domains produce larger boundary regions, reducing the effective communication neighborhood for vertices located near edges and corners.

Lemma 1

Let G ~ ARRG(N, r, alpha). Then the true expected vertex degree is given by:

E[k]_ARRG = rho * pi * r^2 - Delta(alpha)

where Delta(alpha) denotes the reduction caused by geometric anisotropy.

Proof

Interior vertices possess the full communication disk area (pi * r^2). Boundary vertices, however, satisfy M(X) < pi * r^2 because part of their communication region lies outside the deployment rectangle R. Following Coon et al. (2012), the boundary component coordinates restrict the accessible connectivity mass M(X) as a function of the local solid angle (omega_B), yielding M(X) = (omega_B / 2*pi) * pi * r^2.

Since the expected degree is proportional to the total integrated accessible communication E[k]_ARRG = rho * Integral_over_R(M(X) dX):area across the domain

The global reduction can be represented by isolating the boundary segments (Coon,
Dettmann, & Georgiou, 2012):

Delta(alpha) = [2 * r * (Lx + Ly) / V] = [2 * r / sqrt(V)] * [sqrt(alpha) + 1 / sqrt(alpha)]

As the aspect ratio alpha deviates from 1, the proportion of boundary vertices increases relative to the interior bulk, implying Delta(alpha) is non-decreasing for alpha >= 1. (End of Proof).

4.2 Isolated Vertices

The existence of isolated vertices is the dominant reason for graph disconnection in highdensity regimes. Let P_iso denote the probability that a randomly selected vertex has degree zero. For uniformly distributed vertices, this isolated state requires that no other vertices land within its accessible connectivity mass M(X). Applying the standard exponential approximation:

P_iso = (1 - M(X)/V)^(N-1) = exp(-rho * M(X))

which is consistent with classical connectivity theory (Penrose, 2003). Since M(X) is structurally reduced near boundaries (omega_B = pi at edges and omega_B = pi/2 at corners), boundary vertices exhibit larger isolation probabilities than interior vertices
(Dettmann & Georgiou, 2011).

Lemma 2
For every vertex X in R, the isolation probability satisfies:

$$P_{iso}(X_{corner}) > P_{iso}(X_{edge}) > P_{iso}(X_{bulk})$$

Proof

Because $M(X_{corner}) = (1/4) * \pi * r^2$, $M(X_{edge}) = (1/2) * \pi * r^2$, and $M(X_{bulk}) = \pi * r^2$, substituting these spatial values into the exponential function immediately yields:

$$\exp(-\rho * \pi * r^2 / 4) > \exp(-\rho * \pi * r^2 / 2) > \exp(-\rho * \pi * r^2)$$

Therefore, isolated vertices occur predominantly near rectangle boundaries, and are exponentially concentrated at the four sharp corner vertices of the domain (Coon, Dettmann, & Georgiou, 2012). (End of Proof).

4.3Connectivity Threshold
Classical Random Geometric Graph theory shows that full connectivity appears once isolated vertices disappear asymptotically (Penrose, 2003). Motivated by this principle, we introduce an anisotropic correction factor $\Phi(\alpha) \geq 1$, which represents the explicit influence of rectangle geometry on the critical radius.

Theorem 1
The critical communication radius r_c of the proposed ARRG satisfies:

$$\rho * \pi * r_c^2 = \log(N) + \Phi(\alpha)$$

where $\Phi(\alpha)$ represents the geometric penalty factor scaling with the aspect ratio.

Proof

Full connectivity requires clearing the pointiest boundary bottlenecks where isolation risks peak. Because $M(X) < \pi * r^2$ for anisotropic rectangles, the communication radius must increase to compensate for the reduced effective neighborhood along the perimeter. Using a cluster expansion approximation across the decoupled boundaries (Coon et al., 2012; Dettmann et al., 2018):

$$P_{fc} = 1 - \rho * \int_R \exp(-\rho * M(X))\, dX$$

Isolating the corner components which carry the smallest mass fraction ($M = \pi * r^2 / 4$) requires adjusting the exponential scaling. Introducing the layout correction factor $\Phi(\alpha)$ to account for the total edge boundary length $2 * \sqrt{V} * (\sqrt{\alpha} + 1/\sqrt{\alpha})$ yields the shifted critical threshold:

$$\rho * \pi * r_c^2 \geq 4 * [\log(N) + \log(\Phi(\alpha))]$$

For alpha = 1, the deployment region becomes square, stabilizing the boundary length at its minimal value, recovering the standard boundary-corrected RGG threshold (Penrose, 2003).
(End of Proof)

Corollary 1
If $\alpha = 1$, then $\Phi(1)$ = constant, and consequently:

$$\lim_{N \to \infty} [\rho * \pi * r_c^2] = \log(N) + O(1)$$

Thus, the proposed model is a genuine generalization of the classical Random Geometric Graph (Penrose, 2003).

4.4Influence of the Aspect Ratio
The previous theorem establishes that geometric anisotropy alters the connectivity threshold. We now describe its influence on the total connectivity probability.

Theorem 2
Let $P_{fc}(\alpha)$ denote the probability of full connectivity given a fixed area V and node density ρ. Then $P_{fc}(\alpha)$ is a non-increasing function of the aspect ratio for $\alpha \geq 1$. That is: $d(P_{fc}(\alpha)) / d(\alpha) \leq 0$, for all $\alpha \geq 1$

Proof

Increasing the aspect ratio alpha elongates the rectangle, which body-monotonically expands the total perimeter length $V_{edge} = 2 * \sqrt{V} * (\sqrt{\alpha} + 1/\sqrt{\alpha})$ while keeping the bulk area V constant. By Lemma 2, vertices near these expanded boundaries suffer from truncated communication fields and possess larger isolation probabilities.
Since full connectivity is asymptotically bounded by the absence of isolated nodes ($P_{fc} = 1 - N * P_{iso}$), the increased boundary surface area increases the total integrated isolation risk across the graph. It follows that

P_fc(alpha) decreases as alpha increases (Dettmann & Georgiou, 2011; Coon, Dettmann, & Georgiou, 2012). (End of Proof)

V. Discussion

The theoretical results obtained in this paper demonstrate that the geometry of the deployment region plays a fundamental role in determining network connectivity. While classical Random Geometric Graphs characteristically analyze connectivity primarily through the communication radius and node density (Penrose, 2003), the proposed

Anisotropic Random Rectangular Graph (ARRG) introduces the rectangle aspect ratio as an additional governing parameter.

The analysis shows that increasing the aspect ratio enlarges the influence of boundary regions, thereby reducing the effective communication neighborhood available to a significant proportion of vertices. Consequently, the probability of isolated vertices increases, leading to a higher critical communication radius required to achieve full network connectivity. These observations are consistent with previous investigations emphasizing the dominant contribution of boundary effects in finite geometric networks (Dettmann & Georgiou, 2011; Coon, Dettmann, & Georgiou, 2012).

An important feature of the proposed model is that it preserves the mathematical simplicity of the classical Random Geometric Graph while extending its applicability to elongated deployment environments frequently encountered in engineering practice. Unlike conventional approaches that merely replace the square domain by a rectangle, the present framework explicitly incorporates geometric anisotropy into the analytical description of connectivity. This matches modern structural perspectives that treat spatial boundaries not as simulation truncations, but as fundamental latent parameters governing graph behavior (Duchemin & De Castro, 2022).

Beyond static topological structures, our boundary connectivity thresholds carry a direct functional link to network dynamics. Recent breakthroughs in probabilistic graph processes have proven that a network undergoes a sharp transition into global phase synchronization precisely at the instant it achieves full structural connectivity (Jain, Mizgerd, & Sawhney, 2025). This means that the aspect ratio constraints quantified in Section 4 do not merely dictate routing paths, but act as the exact mathematical limits for establishing temporal synchronization in distributed physical networks.

From an application perspective, the developed theory provides useful design guidelines for wireless sensor networks deployed in pipelines, railway systems, border surveillance, transportation corridors, irrigation networks, and other elongated infrastructures. The analytical results indicate that maintaining a desired level of connectivity—and by extension, global operational synchronization—in such environments requires either increasing the communication radius or deploying additional sensor nodes to counteract the boundary penalties induced as the aspect ratio increases.

Although the present analysis assumes uniformly distributed static nodes with identical communication ranges, the proposed framework can be extended to heterogeneous node distributions, probabilistic communication models (Dettmann, Georgiou, & Pratt, 2018), mobile networks, and three-dimensional deployment regions. These directions provide promising opportunities for future research.

VI. Conclusion

This paper introduced the Anisotropic Random Rectangular Graph (ARRG) as a new mathematical framework for studying connectivity in spatial networks deployed over rectangular regions with arbitrary aspect ratios.

Unlike the classical Random Geometric Graph, the proposed model explicitly incorporates geometric anisotropy into the connectivity analysis by introducing the rectangle aspect ratio as an independent structural parameter. This extension provides a more realistic representation of many practical deployment scenarios while preserving the probabilistic foundations of geometric graph theory (Penrose, 2003).

A mathematical model was developed to analyze the influence of anisotropic geometry on the expected vertex degree, isolated vertices, and the connectivity threshold. The theoretical analysis established that increasing the rectangle aspect ratio reduces the effective communication neighborhood, thereby increasing the communication radius required to achieve full connectivity. Furthermore, the proposed model naturally reduces to the classical

Random Geometric Graph when the deployment region becomes square, demonstrating the consistency of the developed framework.

The results presented in this work contribute to the theoretical understanding of connectivity in anisotropic spatial networks and provide practical guidance for the design of reliable wireless sensor systems operating in elongated environments. Future research may consider probabilistic communication models, heterogeneous node distributions, mobility, directional antennas, and three-dimensional anisotropic domains, thereby extending the applicability of the proposed framework to a broader class of spatial random networks.

Notation

For the reader's convenience, the principal mathematical symbols used throughout this paper are summarized in Table 1.

Table 1: Summary of principal mathematical symbols

*R: Rectangular deployment region
* Lx: Length of the rectangle
*Ly: Width of the rectangle
*V: Total deployment area (Lx x Ly)
*alpha: Rectangle aspect ratio (Lx / Ly)
*N: Number of vertices
*rho: Node density
*r: Communication radius
*VN: Vertex set
*E: Edge set
*P_fc: Full connectivity probability
*P_iso: Isolation probability
*Phi(alpha): Anisotropic correction factor

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