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Research Paper

The Behavior of Prime Factors and Their Impact on the Decreasing Density of Prime Numbers Toward Infinity

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ABSTRACT: *This The decreasing density of prime numbers is one of the fundamental phenomena observed in number theory. While prime numbers continue to exist infinitely, their frequency becomes progressively lower as the numerical range increases. This paper proposes a structural explanation for this behavior based on the influence of prime factors on the positional distribution of prime numbers. The study examines groups generated from the products of consecutive prime numbers, namely: (P_7=210), (P_{11}=2310), (P_{13}=30030), and (P_{17}=510510). The results indicate that each additional prime factor creates new composite numbers and eliminates an increasing number of candidate prime positions. Consequently, the proportion of positions occupied by prime numbers decreases as the number system expands. The analysis suggests that the reduction in prime density is a direct consequence of the cumulative influence of prime factors. Although prime numbers remain infinite in quantity, their occurrence becomes increasingly sparse because larger prime factors occupy a growing number of available positions within the numerical structure.*

KEYWORDS: *Prime Numbers, Prime Factors, Prime Density, Prime Distribution, Composite Numbers, Number Theory.*

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I. INTRODUCTION

Prime numbers are the fundamental building blocks of arithmetic and number theory. Every composite number can be expressed as a product of prime factors, making prime numbers the foundation upon which the structure of the natural numbers is built [1-2]. Despite centuries of research, the distribution of prime numbers remains one of the most important and challenging topics in mathematics.

One of the most remarkable properties of prime numbers is that they become less frequent as the numerical range increases [3-4]. Although it has been proven that there are infinitely many prime numbers, their density gradually decreases toward infinity [5]. This phenomenon has traditionally been studied through analytical methods, probability models, and asymptotic formulas such as the prime number theorem [6]. However, these approaches primarily describe the behavior of prime numbers rather than explain the structural mechanism responsible for their decreasing frequency.

The present study proposes an alternative perspective based on the influence of prime factors. The central hypothesis is that the decreasing density of prime numbers is a direct consequence of the cumulative effect of prime factors on the numerical system. As each new prime factor is introduced, additional composite numbers are generated, reducing the number of positions available for prime numbers. Consequently, while prime numbers continue to occur infinitely often, their proportion relative to the total set of natural numbers becomes progressively smaller.

To investigate this phenomenon, the study examines numerical structures generated from the products of consecutive prime numbers. Particular attention is given to the products of prime numbers:

$P_7=7*5*3*2=210$,
$P_{11}=11*7*5*3*2=2310$,
$P_{13}=13*11*7*5*3*2=30030$,
$P_{17}=17*13*11*7*5*3*2=510510$

The distribution of prime numbers within the corresponding generated groups is analyzed to determine how the inclusion of larger prime factors influences the number of surviving prime positions. The results suggest that the reduction in prime density is not merely a statistical observation but may be interpreted as the natural outcome of an expanding system of prime factors. Each newly introduced prime factor contributes additional composite numbers, progressively eliminating candidate prime positions and producing a lower concentration of primes within larger numerical intervals.

The objective of this paper is to demonstrate the relationship between prime-factor growth and the decreasing density of prime numbers and to provide a structural explanation for why prime numbers become increasingly sparse as the number system approaches infinity.

II. METHODOLOGY BASED ON THE INFLUENCE OF PRIME FACTORS

Let: $P_m=P_n*P_{n-1}*P_{n-2}*\ldots*3*2.$
$P_r=P_k*P_{k-1}*P_{k-2}*\ldots*P_n*P_{n-1}*P_{n-2}*\ldots*3*2.$
P_r =Product of prime numbers greater than P_m.
Composite numbers are formed by multiplying two prime numbers together.
Prime factorization: The process of breaking down a composite number into prime factor P_f:
Let $P_f=P_n$.
Composite numbers=$P_f*(P_n, Pn_{+1}, P_{n+2}, P_{n+3},\ldots,\infty)$.
Composite numbers=$P_n*(P_n, Pn_{+1}, P_{n+2}, P_{n+3},\ldots,\infty)$.

2.1. If $P_k \le P_n$ and $P_r \le P_{m\}}$ Ri=Prime or non-prime number.

2.2. $P_r=P_{n-1}*P_{n-2}*P_k*P_{k-1}*P_{k-2}*\ldots*\ldots*3*2$

2.3. $R_i=((P_n*(P_n, Pn_{+1},\ldots,\infty)+P_r)/P_n$.
$R_i = ((P_n*(P_n, Pn_{+1},\ldots,\infty)+P_{n-1}*P_{n-2}*P_k*P_{k-1}*P_{k-2}*\ldots*\ldots*3*2)/P_n$.
Refer to Table 1:
Columns from (1 to 6 <7 and 8) indicate prime numbers and non-prime numbers.

2.4. If $P_k \ge P_n$ and $P_r \ge P_{m\}}$ Ri=Non-prime number.

2.5. Let $P_m=P_n*P_{n-1}*P_{n-2}*\ldots*3*2.$
$P_r= P_k*P_{k-1}*P_{k-2}*\ldots*P_n*P_{n-1}*P_{n-2}*\ldots*3*2.$
$R_i=((P_n*(P_n, P_{n+1}, P_{n+2}, P_{n+3},\ldots,\infty)+P_r)/P_n$.
$R_1=(P_n*P_{n+1})+(P_k*P_{k-1}*P_{k-2}*\ldots*P_n*P_{n-1}*P_{n-2}*\ldots*3*2)/P_n,\ i=1$
$R_1=(P_n*P_{n+1})+(P_k*\ P_{k-1}*P_{k-2}*\ldots*\ P_n*\ P_{n-1}*\ P_{n-2}*\ldots*3*2)/P_n.$
(P_n*P_{n+1})=non-prime number.
$R_1=(P_n*P_{n+1})+(P_k*P_{k-1}*P_{k-2}*\ldots*P_n*P_{n-1}*P_{n-2}*\ldots*3*2)/P_n.$
R_1=integer number, non-prime number.

III. EXAMPLE

3.1. Consider the following prime products:
$P_7=7*5*3*2=210,$
$P_{11}=11*7*5*3*2=2310,$
$P_{13}=13*11*7*5*3*2=30030,$
$P_{17}=17*13*11*7*5*3*2=510510.$

3.2. Refer to Table 1 Columns 1.
$P_n=17.$
$P_r= P_7=7*5*3*2=210.$
Composite numbers=$17*(17, 19, 23,\ldots,\infty)$
$R_i=17*(17, 19, 23,\ldots,\infty)+P_7.$
$R_1=17*17+210=499$ prime number.
$R_2=17*19+210=533$ Non-prime number.

3.3. Refer to table 1 Columns 4.
Composite numbers=$17*(17, 19, 23,\ldots,\infty).$
$P_n=17.$
$P_r=P_{11}=11*7*5*3*2=2310.$

$R_i=17*(17, 19, 23,\ldots,\infty)+P_{11}$.
$R_1=17*19+2310=2{,}633$ prime number.
$R_2=17*23+2310=2{,}701$ Non-prime number.

3.4. Refer to Table 1. Columns 7& 8.
Let $P_m=P_n*P_{n-1}*P_{n-2}*\ldots*3*2$.
$P_{17}=17*13*11*7*5*3*2=510510$.
$P_r=P_m=510510$.
$P_n=17$.
Composite numbers$=P_n*(P_n, P_{n+1}, P_{n+2}, P_{n+3},\ldots,\infty)$.
Composite numbers$=17*(17, 19, 23,\ldots,\infty)$
$P_{n+1}=19$,
$P_n*P_{n+1}=17*19=323$
$P_r=P_m=510510$.
$R_i=(P_n*(P_n, P_{n+1}, P_{n+2}, P_{n+3},\ldots,\infty)+P_r)/P_n$
$R_1=(P_n*P_{n+1})+(P_k*P_{k-1}*P_{k-2}*\ldots*P_n*P_{n-1}*P_{n-2}*\ldots*3*2)/P_n$.
$R_1=((17*19)+17*13*11*7*5*3*2)/17=510{,}833/17=30049$, integer number, $510{,}833$ non-prime.
$P_{n+2}=23$,
$R_2=(P_n*P_{n+2}+P_k*P_{k-1}*P_{k-2}*\ldots*P_n*P_{n-1}*P_{n-2}*\ldots*3*2)/P_n$.
$R_2=(17*23+17*13*11*7*5*3*2)/17=510{,}901/17=30053$, integer number. $510{,}901$ non-prime.
$P_{n+3}=29$,
$R_3=(P_n*P_{n+2}+P_k*P_{k-1}*P_{k-2}*\ldots*P_n*P_{n-1}*P_{n-2}*\ldots*3*2)/P_n$.
$R_3=(17*29+17*13*11*7*5*3*2)/17=511{,}003/17=30{,}059$,integer number. $511{,}003$ non-prime.

3.5. Let $P_{n+3}=29$.
$P_r>P_m$
$P_r=19*17*13*11*7*5*3*2=9699690$
$R_1=(P_n*P_{n+3}+P_k*P_{k-1}*P_{k-2}*\ldots*P_n*P_{n-1}*P_{n-2}*\ldots3*2)/P_n$, $i=1$
$R_1=(17*29+19*17*13*11*7*5*3*2)/17=(493+9699690)/17$
$R_1=9{,}700{,}183/17=570{,}599$ integer number.
The number $9{,}700{,}183$ non-prime number.

Table 1: Indicate the decreasing of prime numbers to word infinity.

				1	2	3	4	5	6	7	8
	Table (1)			Pr7<P17	Pr7<P17	Pr11<P17	Pr11<P17	Pr13<P17	Pr13<P17	P_{17}+B	4xP_{17}+B
	A	C	B=AxC	210+B	210x2+B	2310+B	2310x8+B	30030+B	30030x6+B	510510+B	510510x4+B
1	1	17	17	227	437	2327	18497	30047	180197	510527	2042057
2	17	17	289	499	709	2599	18769	30319	180469	510799	2042329
3	19	17	323	533	743	2633	18803	30353	180503	510833	2042363
4	23	17	391	601	811	2701	18871	30421	180571	510901	2042431
5	29	17	493	703	913	2803	18973	30523	180673	511003	2042533
6	31	17	527	737	947	2837	19007	30557	180707	511037	2042567
7	37	17	629	839	1049	2939	19109	30659	180809	511139	2042669
8	41	17	697	907	1117	3007	19177	30727	180877	511207	2042737
9	43	17	731	941	1151	3041	19211	30761	180911	511241	2042771
10	47	17	799	1009	1219	3109	19279	30829	180979	511309	2042839
11	53	17	901	1111	1321	3211	19381	30931	181081	511411	2042941
12	59	17	1003	1213	1423	3313	19483	31033	181183	511513	2043043
13	61	17	1037	1247	1457	3347	19517	31067	181217	511547	2043077
14	67	17	1139	1349	1559	3449	19619	31169	181319	511649	2043179
15	71	17	1207	1417	1627	3517	19687	31237	181387	511717	2043247
16	73	17	1241	1451	1661	3551	19721	31271	181421	511751	2043281
17	79	17	1343	1553	1763	3653	19823	31373	181523	511853	2043383
18	83	17	1411	1621	1831	3721	19891	31441	181591	511921	2043451
19	89	17	1513	1723	1933	3823	19993	31543	181693	512023	2043553
20	97	17	1649	1859	2069	3959	20129	31679	181829	512159	2043689
21	101	17	1717	1927	2137	4027	20197	31747	181897	512227	2043757
22	103	17	1751	1961	2171	4061	20231	31781	181931	512261	2043791
23	107	17	1819	2029	2239	4129	20299	31849	181999	512329	2043859
24	109	17	1853	2063	2273	4163	20333	31883	182033	512363	2043893
25	113	17	1921	2131	2341	4231	20401	31951	182101	512431	2043961
	P_{17}=17x13x11x7x5x3x2			15	18	14	10	11	8	76	
	P7=7x5x3x2		P_{11}=11x7x5x3x2		P_{13}=13x11x7x5x3x2					Non-prime	prime

Refer to Table (1).

Using the prime positions identified in the original group, new groups are generated by adding multiples of these prime products.

The numerical results show that many positions that remain prime for smaller values of (P_m) become composite when additional prime factors are introduced. Each newly added prime factor creates additional divisibility conditions and therefore eliminates more candidate prime locations.

For example, Table (1) demonstrates that the number of surviving prime numbers decreases as the prime product increases. This behavior indicates that larger prime factors contribute to the formation of more composite numbers and reduce the proportion of positions available for primes.

Consequently, the distribution of prime numbers becomes increasingly sparse as the numerical range expands. It follows that every number generated by this expression is divisible by (P_n) and is therefore composite whenever its value exceeds (P_n) is divisible by 17 and consequently cannot be prime.

This simple observation illustrates how the introduction of additional prime factors automatically creates new composite numbers and eliminates candidate prime positions. As larger prime factors become incorporated into the numerical structure, the number of composite numbers increases, contributing to the progressive reduction in the density of prime numbers.

IV. CONCLUSION

The results presented in this study suggest that the decreasing density of prime numbers can be interpreted as a direct consequence of the cumulative influence of prime factors. As new prime factors are introduced into the numerical structure, additional composite numbers are generated, eliminating an increasing number of candidate prime positions. Although prime numbers continue to occur infinitely often, their relative frequency decreases because the number of composite numbers grows more rapidly.

The analysis supports the view that prime-number distribution is governed by structural relationships among prime factors rather than by purely random behavior. The progressive reduction in prime density toward infinity may therefore be understood as the natural outcome of the increasing influence of prime factors on the organization of the number system.

This framework provides an alternative perspective on prime-number distribution and may contribute to future investigations concerning prime density, prime gaps, and the positional structure of prime numbers.

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