Quest Journals
Journal of Research in Applied Mathematics
Volume 12 ~ Issue 8 (August 2026) pp: 01-04
ISSN (Online): 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

N-Factor Satisfactory Marriage Problem with Varying Weight and Incomplete Preference List

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ABSTRACT: *Gale and Shapley proposed a GS algorithm to solve Marriage Problem. This algorithm results a man optimal or women optimal stable matching. In this paper a new algorithm called SMA is applied to N-Factor Marriage Problem with varying weight and Incomplete Preference list. The findings were discussed and illustrated with real life examples.*

KEYWORDS: *Optimal matching, Preference Value, Satisfactory value matrix, Satisfactory Level*

Received 05 Aug., 2026; Revised 14 Aug., 2026; Accepted 16 Aug., 2026 © The author(s) 2026.
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I. INTRODUCTION

The Stable Marriage Problem (SM) was first introduced by Gale and Shapley in 1962 [1]. An SM instance consists of two finite and equally sized sets of agents, traditionally referred to as men and women. Each man m_i ($1 \le i \le n$, n is the number of men) prefer women in strict order, forming his own preference list. Similarly, each woman w_j ($1 \le j \le n$) prefer men in strict order of preference, ranks all men in a strict order, forming her own preference list [2].

A stable marriage is defined as a one-to-one matching between men and women such that there exists no pair (m_i, w_j) who would both prefer each other over their assigned partners. Gale and Shapley proposed a fundamental algorithm, commonly known as the G.S. algorithm [3], which guarantees the construction of a stable matching for any given set of preference lists.

They further proved that at least one stable matching exists for every SM instance [4]. However, in general, multiple stable matching may exist for a single instance. The G.S. algorithm produces only one such matching, characterized by an optimality property. In particular, the man-proposing version yields a man-optimal stable matching, where each man is paired with his most preferred feasible partner, while each woman receives her least preferred partner among all stable matching. Conversely, reversing the roles results in a woman-optimal stable matching [5].

This observation motivates the exploration of stable matching that is not only stable but also optimal with respect to additional criteria. The Satisfactory Marriage Problem was addressed in [6] by incorporating a single factor into the preference structure. Subsequently, a multi-factor (N-factor) approach was introduced in [7], followed by the assignment of weights to these factors in [8]. Further extensions include N-factor models with varying weights [9], and models incorporating both varying weights and incomplete preference lists [10].

In this paper, we propose a solution to the marriage problem in which each participant evaluates multiple factors with varying importance, along with the consideration of incomplete preference lists.

II.N-FACTOR MARRIAGE PROBLEM WITH VARYING WEIGHT AND INCOMPLETE PREFERENCE LIST

A marriage problem involves two sets men and women each of size n. In this context, each man assigns different levels of importance to multiple factors within his preference list. Similarly, each woman considers multiple factors with varying weightages in her preference list. This leads to a matching problem characterized by two or more preference criteria. Therefore, it becomes essential to determine an appropriate matching between the members of the two groups by incorporating the weights of these factors into a preference value matrix, ensuring a mutually satisfactory outcome. The primary objective of this study is to identify a

satisfactory matching in an N-factor marriage problem, where each individual assigns different weightages to each factor. To determine the optimal matching between men and women while considering all preference factors, the SMA algorithm with a modification in Step 6 is applied. The satisfactory value matrices corresponding to each factor are combined according to their respective weights to form a resultant matrix. The Hungarian algorithm is then applied to this aggregated matrix to obtain the optimal satisfactory matching between men and women. An illustrative example of the N-factor Marriage Problem with varying weights is presented and discussed.

Example 1.1

Using modified SMA, determine satisfactory matching for an instance with four men m_1, m_2, m_3, m_4, three women w_1, w_2, w_3, w_4 and three factors F_1, F_2, F_3 with the preference list. The members of men and women group give weight for each factor for different proportion as given below.

Factor-F_1				Factor-F_2				Factor-F_3			
m_1	70%	w_1	35%	m_1	20%	w_1	35%	m_1	10%	w_1	30%
m_2	40%	w_2	25%	m_2	30%	w_2	30%	m_2	30%	w_2	45%
m_3	25%	w_3	10%	m_3	15%	w_3	15%	m_3	60%	w_3	75%
m_4	45%	w_4	50%	m_4	20%	w_4	30%	m_4	35%	w_4	20%

Preference list based on Factor F_1

m_1 : $w_1 w_3 w_2 w_4 w_1$: $m_4 m_2 m_3 m_1$
m_2 : $w_2 w_4 w_1 \quad w_2$: $m_3 m_1 m_2 m_4$
m_3 : $w_2 w_1 w_4 w_3 w_3$: $m_1 m_4 m_3$
m_4 : $w_4 w_2 w_3 w_1 w_4$: $m_2 m_3 m_4 m_1$

Preference list based on Factor F_2

m_1 : $w_3 w_4 w_2 \quad w_1$: $m_2 m_3 m_4$
m_2 : $w_2 w_3 w_4 w_1 w_2$: $m_3 m_4 m_1 m_2$
m_3 : $w_1 w_2 w_3 w_4 w_3$: $m_4 m_1 m_2 m_3$
m_4 : $w_4 w_3 w_1 w_2 w_4$: $m_1 m_2 m_4 m_3$

Preference list based on FactorF_3

m_1 : $w_2 w_3 w_4 w_1 w_1$: $m_3 m_4 m_1 m_2$
m_2 : $w_3 w_4 w_1 w_2 w_2$: $m_4 m_1 m_2 m_3$
m_3 : $w_4 w_1 w_2 \quad w_3$: $m_2 m_4 m_1$
m_4 : $w_1 w_2 w_3 w_4 w_4$: $m_1 m_2 m_4 m_3$

Solution:

The SMA is applied to the above instance. For each factor of preference lists, preference value for each member is assigned based on their position in the preference list. The preference values are tabulated in the form of Men's Preference value matrix (PM_m) and Women's Preference value matrix(PM_w). The sum of PM_m and transpose of PM_w gives satisfactory value matrix for each factor which are shown below.

The Satisfactory Value Matrix for Factor F_1 with weightage is

$$SM_{m\,(F_1)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \\ m_4 \end{array} \begin{array}{c} \begin{array}{cccc} w_1 & w_2 & w_3 & w_4 \end{array} \\ \begin{pmatrix} 0.7875 & 0.5375 & 0.625 & 0.3 \\ 0.4625 & 0.525 & 0 & 0.8 \\ 0.3625 & 0.5 & 0.1125 & 0.5 \\ 0.4625 & 0.4 & 0.3 & 0.7 \end{pmatrix} \end{array}$$

The Satisfactory Value Matrix for Factor F_2 with weightage is

$$SM_{m\,(F_2)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \\ m_4 \end{array} \begin{array}{c} \begin{array}{cccc} w_1 & w_2 & w_3 & w_4 \end{array} \\ \begin{pmatrix} 0 & 0.125 & 0.3125 & 0.45 \\ 0.425 & 0.375 & 0.3 & 0.375 \\ 0.4125 & 0.4125 & 0.1125 & 0.1125 \\ 0.275 & 0.275 & 0.3 & 0.2 \end{pmatrix} \end{array}$$

The Satisfactory Value Matrix for Factor F_3 with weightage is

$$SM_{m\,(F_3)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \\ m_4 \end{array} \begin{array}{c} \begin{array}{cccc} w_1 & w_2 & w_3 & w_4 \end{array} \\ \begin{pmatrix} 0.175 & 0.4375 & 0.45 & 0.25 \\ 0.225 & 0.3 & 1.05 & 0.375 \\ 0.75 & 0.4125 & 0 & 0.65 \\ 0.575 & 0.7125 & 0.7375 & 0.1875 \end{pmatrix} \end{array}$$

In this instance, three factors are considered with different proportions by different members in preference list. So, the resultant satisfactory value matrix is formed by considering all weightage of factors and the satisfactory value matrix for each factor, which is given below.

The resultant Satisfactory Value Matrix is

$$SM_{m\,(R)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \\ m_4 \end{array} \begin{array}{c} \begin{array}{cccc} w_1 & w_2 & w_3 & w_4 \end{array} \\ \begin{pmatrix} 0.9625 & 1.225 & 1.3875 & 1 \\ 1.1125 & 1.2 & 1.35 & 1.55 \\ 1.525 & 1.325 & 0.225 & 1.2625 \\ 1.3125 & 1.3875 & 1.3375 & 1.0875 \end{pmatrix} \end{array}$$

The Hungarian algorithm of assignment technique is applied to the above matrix and matching are found. The satisfactory matching obtained on applying SMA is $(m_1, w_3), (m_2, w_4), (m_3, w_1)$ *and* $(m_4, w_2)$and satisfactory value for each pair is given in the following table.

Satisfactory matching	Satisfactory value	Satisfactory value of men	Satisfactory value of women
(m_1, w_3)	1.3875	0.8	0.5875
(m_2, w_4)	1.55	0.675	0.875
(m_3, w_1)	1.525	0.7875	0.7375
(m_4, w_2)	1.3875	0.65	0.7375

Table 1.1: The preference value table of variants of Weighted N-factor Marriage problem

The above result shows that the satisfactory value of any member of matching in both groups on applying SMA algorithm is minimum of 0.5875.The obtained solution is the optimum matching for both the groups and satisfactory level of men and women are 49.78% and 50.22% respectively.

III. CONCLUSION

This study presents an N-Factor Marriage Problem that incorporates varying weights and incomplete preference lists. The SMA algorithm is used to find best matches between men and women by taking in to account various factors of differing importance.Each factor is shown with real life-examples. The results indicate that both men and women achieve a high level of satisfaction, leading to optimal pairings. A well-established assignment technique is employed to solve the problem involving multiple factors. This approach provides a practical framework for addressing real-life matching problems and supports individuals in making well-informed and effective decisions.

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