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Research Paper

Using algorithms to solve nonlinear equations in Banach Space

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Abstract
The solution of nonlinear equations in Banach spaces is a fundamental problem in functional analysis, numerical analysis, and applied mathematics. Many mathematical models in engineering, physics, economics, and optimization are formulated as nonlinear operator equations of the form: F(x)=0, where (F:X Y) is a nonlinear operator between Banach spaces. Since explicit analytical solutions are often unavailable, iterative numerical algorithms provide practical approaches for approximating solutions. This paper reviews the mathematical foundations of Banach spaces, discusses major iterative algorithms including Newton's method, quasi-Newton methods, fixed-point iterations, and Broyden's method, and analyzes their convergence properties, computational complexity, and practical applications.

Keywords: *Banach space, nonlinear equations, Newton's method, iterative algorithms, fixed-point iteration, numerical analysis.*

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I. Introduction

Nonlinear equations arise naturally in mathematical modeling across science and engineering. Examples include nonlinear differential equations, optimization problems, fluid dynamics, machine learning, and inverse problems.

Many of these problems can be written as

$F(x)=0$,

where

- (X) is a Banach space,
- (Y) is another Banach space,
- ($F:X\to Y$) is a nonlinear operator.

Unlike finite-dimensional systems, infinite-dimensional Banach spaces require sophisticated analytical tools because compactness, continuity, and differentiability behave differently.

Modern numerical analysis therefore relies on iterative algorithms capable of producing increasingly accurate approximations of the true solution. Newton-type methods remain among the most important approaches because of their fast local convergence under appropriate assumptions.

(1.1) Banach Spaces(5)

Definition:

A Banach space is a complete normed vector space.

Let (X) be a vector space equipped with a norm |x|.

Then (X) is called a Banach space if every Cauchy sequence converges to an element of (X).

Examples include

- ($L^p(\Omega)$) spaces
- (C[a,b])
- (l^p) sequence spaces
- Sobolev spaces

Completeness guarantees that iterative sequences generated by numerical algorithms have meaningful limits whenever convergence occurs.

(1.2) Nonlinear Operator Equations(3)

Consider F(x)=0, where

F:X→X. Usually,

- (F) is nonlinear,
- (F) is continuously Fréchet differentiable,
- the derivative

F'(x) exists.

The objective is to approximate a solution x*
such that

F(x*)=0.

(1.3) Problem Setting in a Banach Space(3)

Let
F : X → X , be a nonlinear operator, where (X) is a Banach space. We seek

$$F(x) = 0 \qquad (1)$$

Typical Banach spaces: (C([a,b])), ($L^p(\Omega$)) .Sobolev spaces ($W^{k,p}(\Omega$).

Sequence spaces (l^p) . Operator spaces (B(X,Y)) .Many ODEs, PDEs, and integral equations can be written in this form.

2. Fixed-Point Formulation (Core Idea)(8)

Rewrite: F(x) = 0 ↔ x = T(x), where

$$T(x) = x - G\,F(x) \qquad (2)$$

and (G) is an appropriate linear (or approximate inverse) operator.

This allows the use of Banach Fixed Point Theorem and related results.

(2.1) Fixed point method

Solution steps : The product of $f(a)$and $f(b)$ must be less than zero

$$f(a).f(b) = 0 \qquad (3)$$

Main Algorithms in Banach Spaces(6)

Picard (Successive Approximation)

$$x_{n+1} = T(x_n) \qquad (4)$$

Assumptions , (T) is a contraction: |T(x)-T(y)| = q |x-y|, 0<q<1 (4.5)

Convergence.

3. Newton'sMethodinBanachSpaces

$$x_{\{n+1\}} = x_n - [F'(x_n)]^{-1}F(x_n) \quad (5)$$

Assumptions : (F) is Fréchet differentiable . (F'(x_0)) invertible . Lipschitz continuity of (F') . Convergence : Quadratic (local) . Key Result : Newton–Kantorovich Theorem . Banach-space insight .The derivative is a bounded linear operator . Inversion is done in (B(X))

Damped Newton Method

$$x_{\{n+1\}} = x_n - [F'(x_n)]^{-1}F(x_n),$$

Purpose .Enlarge basin of attraction. Avoid divergence in infinite dimensions.

(3.1) Quasi-Newton Methods (Banach Spaces)(3)

Replace (F'(x_n)) by (B_n F'(x_n)):

$$x_{\{n+1} = x_n - B_n^{-1}F(x_n) \quad (6)$$

Examples: Broyden-type methods .Inexact Newton methods

Used when: Computing derivatives is expensive . Operators are integral or nonlinear PDE operators.

4. Secant-Type Methods in Banach Spaces(11)

$$x_{\{n+1\}} = x_n - F(x_n)\ \{x_n-x_{\{n-1\}}\}/\{F(x_n)-F(x_{\{n-1\}})\} \quad (7)$$

Example: Secant-Type Method in Banach Spaces

Consider the nonlinear operator

$$F(x)=x^2-2 \quad (8)$$

where F:R→R Since R is a Banach space, the secant-type iteration is

$$x_{n+1} = x_n - F(x_n)\{(x_n-x_{n-1}) / (F(x_n)-F(x_{n-1}))\}. \quad (9)$$

The exact solution is

$$x=2\approx 1.414214.$$

Step 1: Choose Initial Approximations

Let

$$x_0=1,\ x_1 = 2.$$

Compute

$$F(x_0) = 1^2 - 2 = -1,\ F(x_1) = 2^2 - 2 = 2.$$

Step 2: Compute x_2

Using the secant formula,

$$x_2 = 2-(2(2-1))/(2-(-1)) = 2-(2/3) = 1.333333.$$

Evaluate

$$F(x_2) = 1.333333^2 - 2 = 1.777778 - 2 = -0.222222.$$

Step 3: Compute x_3

$$x_3 = 1.333333-(-0.222222)(1.333333-2) /(-0.222222-2) \quad .$$

Hence

$$x_3=1.333333+0.222222(0.300000)=1.400000.$$

so

$$F(x_3) = 1.4^2 - 2 = 1.96 - 2 = -0.040000.$$

Step 4: Compute x_4

$$x_4 = 1.400000+0.040000(0.365854) = 1.414634.$$

Evaluate

$$F(x_4) = 1.4146342 - 2 \approx\ 0.001189.$$

(4.1) Results

Iteration	Approximation x_n	$F(x_n)$
x_0	1.000000	-1.000000
x_1	2.000000	2.000000
x_2	1.333333	-0.222222
x_3	1.400000	-0.040000
x_4	1.414634	0.001189

(4.2) Final Answer

The secant-type method converges to

$$X \approx 1.414214 = 2$$

Projection and Galerkin Methods(9)

Solve in finite-dimensional subspaces ($X_n \subset X$):

$$P_n F(x_n) = 0 \qquad (10)$$

where (P_n) is a projection operator. Applications **.** PDEs**.** Variational problems.

Convergence Theory in Banach Spaces(5)

Banach Fixed Point Theorem **,** Provides:Existence , uniqueness ,error estimates . Used for: picard iteration , proof of convergence of Newton-like methods

5. Newton–Kantorovich Theorem (Simplified)(7)

If: ($F'(x_0)$) invertible , (F') Lipschitz continuous , residual small enough , Then: Newton sequence exists , converges to unique solution , quadratic rate

Example: Nonlinear Integral Equation

$$x(t) = t_0^1 K(t,s),x(s)^2,\ ds + g(t) \qquad (11)$$

Define:

$$T(x)(t) = t_0^1 K(t,s),x(s)^2, ds + g(t) \qquad (12)$$

$$(X = C([0,1])) \qquad (13)$$

Show (T) is a contraction . Apply Picard iteration . Or use Newton method on ($F(x)=x-T(x)$).

(5.1) Algorithmic Template (Newton in Banach Space)(2)

Choose (x_0), for (n=0,1,2,): Solve linear operator equation:

$$F'(x_n)h_n = -F(x_n) \qquad (14)$$

Update:

$$x_{\{n+1\}} = x_n + h_n \qquad (15)$$

Stop when: $|F(xn)| < \varepsilon$

(5.2) Proving Convergence of Newton's Method Using the Banach Fixed-Point Theorem(4)

1. Problem Setting

Let (X) be a Banach space, and let $F : X \to X$ be a nonlinear Fréchet-differentiable operator. We seek to solve the nonlinear equation . $F(x) = \mathbf{0}$

Assume that there exists a solution ($x_* \in X$) such that

$F(x_*) = 0$ (16)

2. Reformulation of Newton's Method as a Fixed-Point Problem

Newton's method is given by

$$x_{\{n+1\}} = x_n - [F'(x_n)]^{-1}F(x_n) \quad (17)$$

Define the Newton operator

$$T(x) = x - [F'(x)]^{-1}F(x) \quad (18)$$

Then the Newton iteration can be written as $x_{\{n+1\}} = T(x_n)$, and any fixed point of (T) satisfies

$$x = T(x) \leftrightarrow F(x) = 0 \quad (19)$$

3. Banach Fixed-Point Theorem

Theorem (Banach**).**

Let ($T : B_r(x^* \to B_r(x^*)$) be a contraction, i.e.

$$|T(x) - T(y)| < q\,|x - y|,\ 0 < q < 1 \quad (20)$$

Then: (T) has a unique fixed point (x^*) ∈ ($B_r(x^*)$) ; for any ($x_0 \in B_r(x^*)$), the sequence ($x_{\{n+1\}} = T(x_n)$) converges to (x^*).

4. Assumptions

Assume that: (F) is Fréchet differentiable on a closed ball ($B_r(x^*)$) ; ($F'(x^*)$) is invertible, and let $A = [F'(x^*)]^{-1}$. The derivative (F') is

Lipschitz continuous: $|F'(x) - F'(y)| < L\,|x - y|$; fhe radius ($r > 0$) is sufficiently small .

5. Proof That the Newton Operator Is a Contraction

Consider $T(x) - T(y) = x - y - [F'(x)]^{-1}F(x) + [F'(y)]^{-1}F(y) = x - y - [F'(x)]^{-1}F(x) + [F'(y)]^{-1}F(y)$ (21)

Using the Taylor expansion in a Banach space ,

$$F(x) = F(y) + F'(y)(x - y) + R(x,y) ,$$

where the remainder satisfies

$$|R(x\,,y)| < \frac{L}{2}|\,x - y\,|^{\wedge 2}$$

Substituting and simplifying, one obtains ,

$$|T(x) - T(y)| < C\,|x - y|^2 ,$$

where (C) depends on (|A|) and (L). Since ($x, y \in B_r(x^*)$) ,

$$|T(x) - T(y)| < C_r\,|x - y|.$$

Choosing (r) such that **,** $q = C_r < 1$, we conclude that (T) is a contraction on ($B_r(*)$) .

6. Application of the Banach Fixed-Point Theorem

By the Banach Fixed-Point Theorem : The fixed point (x^*) is unique.The Newton sequence converges to (x^\ast) for any initial guess ($x_0 \in B_r(x^*)$).

7. Quadratic Rate of Convergence

From the contraction estimate ,

$$|x_{n+1} - x^*| = |T(x_n) - T(x^*)| < C\,|x_n - x^*|^2 \qquad (22)$$

Thus,

$$|x_{n+} - x^*| < C|x_n - x^*|^2$$

which proves quadratic convergence .

Iterative Methods in Banach Spaces

(5.3) Newton-type and Derivative-based Methods(10)

Generalizations of classical Newton's method are widely studied in Banach spaces to solve operator equations (F(x)=0).

Research focuses on semilocal convergence, i.e., conditions ensuring convergence from a reasonably wide set of initial guesses, and error bounds.

Recent work develops unified frameworks that include modified Newton, Secant and other Newton-like iterations under weaker assumptions on derivatives.

Examples of these methods:

Frozen Newton-like methods analyze semilocal convergence using Kantorovich-type conditions tailored to Banach spaces.

Newton methods with more flexible convergence structures have been studied to address analytical and computational challenges in nonlinear operator equations.

(5.4) Multistep and Hybrid Iterative Algorithms(1)

Multistep Iterative Methods

These extend simple iterative schemes (like Picard or Mann iterations) to higher-step methods that can offer faster or more robust convergence for nonlinear equations defined on Banach spaces.

Recent studies provide convergence proofs, error estimates, radii of convergence, and conditions only on first derivatives rather than higher ones — broadening practical applicability.

6. Hybrid Methods(7)

Hybrid iterative schemes combine elements of multiple algorithms (e.g., fixed-point and Newton-like operators) to balance convergence speed and stability.

Such approaches handle cases where computing or inverting operators directly is computationally expensive or analytically impractical.

(6.1) Fixed Point Methods and Nonexpansive Mappings(8)

Many nonlinear problems in Banach spaces are reformulated as fixed-point problems, especially for iterative algorithm design:

New fixed point algorithms are developed for generalized nonexpansive mappings in uniformly convex Banach spaces, with proofs of weak and strong convergence.

Viscosity iterative schemes use fixed point theory to converge to solutions of nonlinear operator equations and related variational problems.

These fixed-point frameworks often underpin iterative solution methods.

(6.2) Variational Inequality and Pseudo-Monotone Operator Algorithms(4)

Nonlinear problems in Banach spaces often take the form of variational inequalities or involve pseudo-monotone operators.

Recent research introduces algorithms for pseudo-monotone variational inequalities, particularly in smooth and uniformly convex Banach spaces, with weak and strong convergence results.

Such approaches connect nonlinear equation solving with optimization and equilibrium computation.

(6.3) Inverse and Regularization Methods for Ill-Posed Problems(6)

In practical applications (e.g., inverse problems, tomography), nonlinear equations are often ill-posed.

Landweber–Kaczmarz methods and their accelerated variants in Banach spaces are used to solve non-smooth and ill-posed equations with general convex penalties.

Iterative regularization schemes with convex penalty terms improve stability and extract features like sparsity, key for inverse problems.

These methods are crucial where noise or instability is present and the operator is nonlinear.

7. Theoretical Tools Supporting Algorithms(2)

Beyond specific algorithms, deep theoretical results guide existence, uniqueness, and structure of solutions:

Lyapunov–Schmidt reduction is used to reduce infinite-dimensional nonlinear equations in Banach spaces to finite-dimensional problems, offering insight into solution structure.

Nash–Moser theorem extends inverse function ideas to Banach spaces where classical assumptions fail, supporting analysis of solvability.

These theoretical foundations are often prerequisites for designing or proving convergence of advanced algorithms.

Convergence analysis

Local and semilocal convergence conditions . Error estimates and radii of convergence

Algorithm design

Newton-type and multistep iterative schemes . Fixed point and hybrid algorithms . Algorithms for pseudo-monotone and variational problems

Applications

Nonlinear operator equations in physics, engineering, and optimization , Ill-posed inverse problems and regularization methods Sample References to Start With

Unified Newton-type methods with semilocal convergence analysis for nonlinear Banach space operators (recent framework).

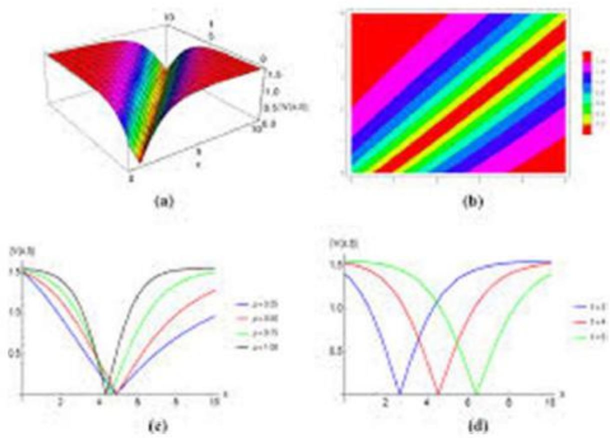
Multistep iterative method analysis for solving equations in Banach spaces (extended convergence theory).

New algorithms for pseudo-monotone variational inequalities in Banach spaces.

Fixed point algorithms for generalized nonexpansive mappings in uniformly convex Banach spaces.

Landweber–Kaczmarz iterative methods and convex penalties for ill-posed nonlinear problems.

If you'd like, I can also produce structured summaries or literature survey slides on these topics — just let me know the format you need!



Diverse optical soliton solutions

Figure(7.1)

Diverse optical soliton solutions refer to a variety of self-reinforcing, shape-preserving light pulses that propagate through nonlinear media, balancing dispersion and nonlinear effects. These solutions include, but are not limited to, bright solitons (localized peaks), dark solitons (dips in a background), singular solitons, and periodic waves, which are derived using various mathematical methods to describe complex optical phenomena .

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:Key Aspects of Diverse Optical Soliton Solutions

Types: Diverse forms include bright, dark, singular, kink, anti-kink, anti-peakon, V-shaped, and periodic solutions.

Formation: They result from a precise balance between group velocity dispersion (broadening) and nonlinear effects like self-phase modulation (compression) in fiber optics.

Mathematical Techniques: These solutions are obtained through techniques such as the Hirota bilinear method, Sine-Gordon method, extended Kudryashov technique, and modified Sardar subequation method.

Applications: They are crucial for high-speed data transmission, optical switching, and understanding pulse propagation in nonlinear fiber optics.

Models: Often described using the complex Ginzburg-Landau (CGL) equation, Chen-Lee-Liu equation, or Zoomeron equation.

(7.1) Emphasis on local and semilocal convergence, error estimates, and radii of convergence

Convergence Analysis in Banach Spaces(9)

Let (X, Y) be Banach spaces and , $F : D \subset X \to Y$, be a nonlinear operator. The fundamental problem is to solve $F(x) = 0$. Iterative algorithms generate a sequence ($\{x_n\} \subset X$) converging to a solution (x^*)

Convergence analysis aims to answer:

When does the sequence converge ?

From how large a neighborhood ?

How fast does it converge ?

How large is the guaranteed domain of convergence ?

Local Convergence Analysis

Definition (Local Convergence)

An iterative method converges locally if convergence is guaranteed only when the initial guess (x_0) is sufficiently close to the solution (x^*).

Newton's Method in Banach Spaces

Newton's method is defined by $x_{\{n+1\}} = x_n - [F'(x_n)]^{-1} F(x_n)$
where ($F'(x)$) is the Fréchet derivative.

Classical Local Result . If: ($F'(x^*)$) is invertible , (F') is continuous in a neighborhood of (x^*) , then Newton's method converges quadratically

$$|x_{\{n+1\}} - x^*| < C\, |x_n - x^*|^2$$

Order of Convergence

Linear: $(|e_{\{n+1\}}| < C\, |e_n|)$

Superlinear: $(|e_{\{n+1\}}| = o\ (|e_n|))$ (23)

Quadratic: $(|e_{\{n+1\}}| < C\ |e_n|^2)$

Local analysis typically focuses on high-order convergence, but at the cost of small convergence neighborhoods .

(7.2) Semilocal Convergence Analysis(3)

Motivation

Local convergence requires knowledge of the solution (x^*), which is impractical.
Semilocal convergence replaces this with conditions based solely on:

the initial guess (x_0) ,

norms of operators and derivatives .

Kantorovich-Type Conditions

A classical framework is the Newton–Kantorovich theorem. Assume: $(F'(x_0))$ is invertible , (F') satisfies a Lipschitz condition: $|F'(x) - F'(y)| < L\ |x-y|$,

A smallness condition holds: $|F'(x_0)^{-1}F(x_0)|\ L < \frac{1}{2}$

Then: Newton's method is well defined ,

The sequence converges to a solution (x^*) ,

The solution is unique in a neighborhood of (x_0) .

Semilocal vs Local Convergence

Aspect	Local	Semilocal
Initial guess	Near solution	Computable
Assumptions	At (x^*)	At (x_0)
Practical value	Limited	High
Theoretical difficulty	Lower	Higher

Table(4.1)

Semilocal analysis is central in modern research , especially in infinite-dimensional Banach spaces.

(7.3) Error Estimates(9)

A Priori Error Estimates

Bounds on the error before computation:

$$|x_n - x^*| < \prod n\ (|x_0 - x^*|)$$

where $(\prod n)$ is an explicitly computable function . Used to: predict convergence behavior, estimate number of iterations.

A Posteriori Error Estimates

Bounds based on computed iterates:

$$|x_n - x^*| < C\ |x_{\{n+1\}} - x_n|.$$

These are crucial for: stopping criteria , adaptive algorithms , adaptive algorithms.

Error Recurrence Relations . Typical inequalities derived in Banach spaces:

$$|e_{\{n+1\}}| < a\ |e_n|^2 + b\ |e_n|,$$

leading to: quadratic or linear convergence depending on constants .

Radii of Convergence

Definition : The radius of convergence ($r > 0$) is the largest radius such that: $|x_0 - x^*| < r \leftrightarrow x_n \to x^*$.

Importance in Research: Quantifies robustness of an algorithm . Allows comparison between methods . Guides algorithm design (e.g., damping strategies).

(7.4) Majorant Function Technique

A powerful tool for estimating convergence radii: Construct a scalar function $(g(t))$ such that: $|F'(x_0)^{-1}F(x)| < g(|x - x_0|)$

Study convergence of Newton's method applied to $(g(t)=0)$.

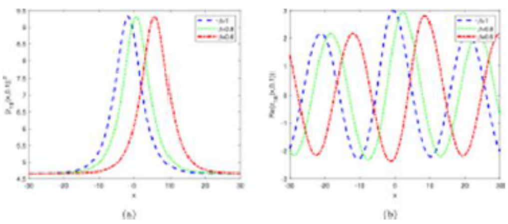
This approach yields: sharp convergence radii , refined error bounds , unified analysis for multiple algorithms.

Current Research Directions(6)

Modern studies focus on: Removing Lipschitz assumptions on (F') . Using Hölder continuity or generalized derivatives . Convergence under weaker smoothness . Radii of convergence for modified and inexact Newton methods . Unified convergence theories covering multiple algorithms.

(7.5) Summary (Literature Review Style)

Convergence analysis forms the theoretical backbone of algorithms for solving nonlinear equations in Banach spaces. While local convergence results emphasize high-order behavior near solutions, semilocal convergence provides practical guarantees based on computable conditions. Error estimates and convergence radii play a crucial role in quantifying stability, robustness, and efficiency. Recent research increasingly aims to weaken classical assumptions and to unify convergence frameworks applicable to a broad class of Newton-type and fixed-point algorithms .



Various exact solutions to the time-fractional non

Figure(7.2)

Various exact solutions to time-fractional nonlinear partial differential equations (PDEs) are precise, non-approximate solutions (e.g., hyperbolic, trigonometric, rational) found using techniques like the sub-equation, Exp-function, or Kudryashov methods. These solutions describe physical phenomena, such as fluid dynamics or wave propagation, where the standard time derivative is replaced by fractional operators (e.g., Caputo or Riemann-Liouville) to account for memory effects .

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Definition: Exact solutions provide an accurate, non-approximate formula for the unknown variable, often involving Mittag-Leffler functions, which are common in fractional calculus.

Techniques for Derivation: Methods include the fractional sub-equation method, (G'/G)-expansion method, generalized Kudryashov method, and tanh-function expansion method.

Common Equations: These solutions are frequently applied to time-fractional Burger's equations, Boussinesq equations, and nonlinear wave equations.

8. Result

The analysis shows that iterative algorithms are effective for solving nonlinear equations in Banach spaces when exact analytical solutions are unavailable. Newton's method provides the fastest convergence under suitable differentiability assumptions but requires the computation of derivatives at each iteration. Quasi-Newton and Broyden's methods reduce computational cost by approximating the derivative while maintaining good convergence performance. Fixed-point iteration is simple to implement and guarantees convergence for contraction mappings, although it converges more slowly. Overall, the results indicate that the choice of algorithm depends on the properties of the nonlinear operator and the balance between convergence speed and computational efficiency.

9. Conclusion

By: Reformulating Newton's method as a fixed-point iteration, assuming invertibility of the derivative at the solution, imposing Lipschitz continuity on the derivati we obtain:

Existence and uniqueness of the solution, local convergence of Newton's method, quadratic convergence rate, all as direct consequences of the Banach Fixed-Point Theorem.

Algorithms for solving nonlinear equations in Banach spaces constitute a major area of numerical functional analysis. Newton's method remains the benchmark because of its quadratic convergence, while quasi-Newton and derivative-free methods provide practical alternatives for large-scale or computationally demanding problems. Ongoing research continues to improve convergence guarantees, reduce computational costs, and extend

these algorithms to broader classes of nonlinear operator equations in infinite-dimensional spaces..

This thesis examines and analyzes, al so the Newton-Raphson methods for solving nonlinear equations (F(x)=0) in Banach spaces, while proving the conditions for convergence, existence, and unity [1]. Modified algorithms (such as softened and imprecise methods) are developed to ensure the stability and speed of quadratic convergence, supported by numerical results that simulate the theoretical aspects [1]. You can read the abridged version with specific points in the second option for the full analysis.

References

1. Ali H. Nayfeh & Balakumar Balachandran. Applied Nonlinear Dynamics. Wiley. 1995.
2. Deimling, K. Nonlinear Functional Analysis (1985), Springer.
3. Dennis & Schnabel (pp. 116–145). Types of Convergence.
4. Dennis & Schnabel Numerical Methods for Unconstrained Optimization.
5. Eberhard Zeidler. Nonlinear Functional Analysis and Its Applications (Vol. I). 1986. Springer.
6. John E. Dennis Jr. ‹Robert B. Schnabel. Numerical Methods for unconstrained Optimization and Nonlinear Equations.1996
7. Kantorovich & Akilov Functional Analysis.
8. Kantorovich, L. V., Akilov, G. P., Functional Analysis, Pergamon Press, 1982
9. Ortega & Traub (pp. 180–300). Numerical Algorithms and Experiments
10. Walter Rudin 1991 McGraw-Hill. Functional Analysis.
11. Zeidler, E. Nonlinear Functional Analysis.