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Research Paper

A Mathematical Model of Decision-Making of Airport Taxi Drivers

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ABSTRACT: *In this paper, we analyzes and studies the mechanism of the factors related to the decision-making of airport taxi drivers, comprehensively considers the changing rules of airport passengers and the income of taxi drivers, establishes a taxi driver selection decision model, and gives the driver's choice strategy.*
KEYWORDS: *Airport taxi; Decision-making plan; Analytic hierarchy process*

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I. INTRODUCTION

With the development of economy, flying has become the choice of more and more people, and passengers often do not stay at the airport after getting off the plane, that is, they need to reach the destination in the urban area (or around). Taxi is one of the main means of transportation.

Since the airport's passenger delivery (departure) and pick-up (arrival) channels are separate, taxi drivers who send passengers to the airport will face two choices: A) go to the arrival area to wait in line to return passengers to the city. Taxi must enter the designated “parking pool” to queue passengers in the order of “first come, then arrive”. The length of the queue depends on the number of taxis and passengers waiting in line. The choice requires a certain amount of time cost. B) directly venting back to the city to pull the passenger. At this point the taxi driver will lose the no-load cost and may lose potential passenger revenue.

The number of flights arriving at the airport at a certain time and the number of vehicles already in the storage pool are the exact information that can be observed. The driver makes decisions based on personal experience. If passengers want to take a taxi, they have to wait in line at the designated taxi zone and take the taxi in order. Airport taxi managers are responsible for releasing taxis into the bus area in batches and arranging for a certain number of passengers to board. In real life, there are other factors that affect taxi drivers’ decision-making, the correlation relationship is different, and the influence effect is also quite different.

Combined with the actual situation, the mathematical model is established to solve the following problem: Analyze the relevant factors affecting the decision of the taxi driver, comprehensively consider the change law of the number of passengers in the airport and the income of the taxi driver, establish a taxi driver selection decision model, and give the driver's choice decision.

II. ANALYSIS OF THE PROBLEM

First of all, we need to determine the factors that affect the driver's decision-making. In this paper considers various principles and draws the influencing factors. Secondly, by reasonably analyzing meaning and influence on the decision target of each influence factor, this paper gives method of the value of each influence factor, thus builds a decision matrix that can more accurately reflect the decision goal. Then normalize the elements in the matrix, and use the analytic hierarchy process to assign weights to the indicators. Finally, consider using a linear weighted model to establish a driver decision model, and according to the evaluation value of the plan roll out the driver's final decision.

III. MODELING AND SOLVING OF THE PROBLEM

According to the requirements of the problem, we need to give the relevant factors that affect the taxi driver to make the "stay" or "leave" decision, consider the change of the number of passengers and the driver's income, and then establish the driver's decision model.

(I) Selection principle of influencing factors

Taxi drivers who send passengers to the airport need to make the choice of leaving the airport to return to the city or staying at the airport to wait for pick-up. However, there are many influencing factors for taxi drivers to make decisions. Therefore, the selection of some reasonable influencing factors plays a decisive role in the analysis of taxi driver selection decisions.

The reasonableness and reliability of the influencing factors are related to the following principles:

1) The principle of testability: the selected factors should not only consider its theoretical meaning, but also ensure that the accurate information of the influencing factors can be obtained by combining the reality.

2) The principle of comprehensiveness: the selected factors should be combined with qualitative analysis and quantitative analysis to reflect the characteristics of decision-making objectives as comprehensively as possible.

3) The principle of objectivity: influencing factors cannot be determined with the subjective consciousness of the individual. It must be objective and scientific and can reflect the actual situation.

4) The principle of comparability: all influencing factors need to have strong comparability as a whole, so that the final decision has high credibility.

(II) Determination of influencing factors

Considering the above four principles, this paper selects the following five influencing factors:

Factor 1: Number of flights

This refers to the number of aircraft arriving at the same airport at a certain time. Taxi drivers make subjective judgments based on the number of flights they get. According to real life, the more flights arrive at a certain time (in this paper, 24 hours a day are divided into 12 periods for analysis, the same below), the more likely taxi drivers will stay. Based on the flight data information obtained and through access to the literature, drivers can be provided with a basis for their choice, as shown in the following table:

Table 1. Impact of the number of flights on driver decisions

Number of flights	>40	≤40
Driver tendency to make decisions	Stay	Leave

Table 1 shows that when the number of aircraft arriving at the airport in a certain period is more than 40, the taxi driver is more inclined to stay at the airport waiting for passengers; when the number of aircraft arriving at the airport in a certain period is less than 40, the number of taxi drivers to stay at the airport will be reduced, and more preferred to return to the urban area.

Factor 2: The number of taxis in the pool

This refers to the number of taxis already in the storage pool at a certain time. In theory, the larger the number of taxis in the storage pool, the longer the driver waits, the less willing the driver will stay; on the contrary, the smaller the number of taxis, the greater the willingness of the driver to stay at the airport. But combined with factor one, when there are more taxis in a certain period of time, the number of flights arriving during that period may also be more, so we need to define a variable that can reflect the relationship between the number of taxis and the number of flights, it may be recorded as r_k , which is expressed as:

$$r_k = \frac{T_k}{F_i}.$$

where T_k represents the number of taxis in the storage pool at the k-th moment, F_i represents the number of flights arriving during the i-th time period, r_k is understood that the number of taxi corresponding to an aircraft at a certain time.

According to actual life experience and literature review, we set a critical value for r_k as a reference for taxi selection decisions, as shown in the following table:

Table 2. Impact on driver's decision-making

r_k	>10	≤10
Driver tendency to make decisions	Leave	Stay

Factor 3: Changes in the number of passengers

It means that the number of passengers arriving at the same airport in different periods varies, and the taxi driver can choose to stay at the airport with a large number of passengers and wait at the airport for passengers. Different types of aircraft carry different passenger flows, so only analyzing the number of flights arriving during a certain period of time cannot determine the specific number of passengers arriving at the airport during each period. The change rule for defining the number of passengers is that the number of passengers corresponding to the various aircraft models is multiplied by the sum of the arrival number of various types of aircraft in a certain period, that is:

$$Q_i = \sum_{j=1}^{n} f_{ij} h_j \, (i = 1, 2, \cdots, 12), \tag{1}$$

where Q_i is the number of passengers arriving at the airport during the *i*-th time period, f_{ij} is the number of *j*-type aircraft in the *i*-th time period, h_j is the number of passengers to be carried by the *j*-type aircraft, *n* is the number of aircraft types.

Passengers arriving at the airport choose different modes of transportation to leave the airport at different times. Here, the proportion of passengers arriving at the airport within a certain period of time who choose to leave the airport by taxi is b_i then formula (1) can be converted to:

$$Q_i' = b_i \sum_{j=1}^{n} f_{ij} h_j \, (i = 1, 2, \cdots, 12)$$

Taxi drivers can accord the value of Q_i' making decisions about the number of passengers who choose to leave the airport by taxi at each time.

Factor 4: Length of waiting time

If a taxi driver chooses to stay at the airport and wait for passengers, according to assumption know the driver will have to wait at least two hours before receiving the passengers. When passengers come out, taxi drivers in the storage pool can pick them up at the reception desk through airport managements. The average number of taxis arriving at the airport storage pool at unit time is λ , the average number of taxis arriving in [0, t] time is λt . The time a driver receives a passenger is subject to a negative exponential distribution, so that the average number of drivers receiving a passenger per unit time is μ , then the average time each driver receives a passenger is $1/\mu$. The average waiting time of the taxi in the storage pool is as follows:

$$t_1 = \frac{\lambda}{\mu(\mu - \lambda)}. \tag{2}$$

If a taxi driver chooses to leave the airport, then his cost is the time of from the airport to the city, that is:

$$t_2 = \frac{s}{v},$$

where *s* is the distance from the airport to the city, *v* represents the average speed of the taxi.

Factor 5: Driver income

The income of taxi drivers is closely related to the time consumed and fuel cost, so the average hourly income of taxi drivers is used as the criterion to measure the driver's income. The charges for taxis in Beijing are shown in the following table:

Table 3. Taxi charging standards

Starting price (3 km)	3-10km	More than 10 km
11yuan	2.5 yuan / km	3.75 yuan/ km

When the driver chooses to stay, the cost to the driver is the cost of time. This paper defines the driver's income expression as follows:

$$p_1 = \begin{cases} \dfrac{11 - \alpha d_k}{t_1 + \dfrac{d_k}{v}}, & d_k < 3; \\ \dfrac{11 + 2.5(d_k - 3) - \alpha d_k}{t_1 + \dfrac{d_k}{v}}, & 3 < d_k < 10; \\ \dfrac{11 + 2.5 \times 7 + 3.75(d_k - 10) - \alpha d_k}{t_1 + \dfrac{d_k}{v}}, & d_k > 10. \end{cases} \tag{3}$$

where d_k represents the distance between the destination of the k-th passenger and the airport, α represents the cost of fuel per kilometer of taxi. According to the data found, the average taxi consumes 8.9 liters of fuel per 100 kilometers, and the price of gasoline is 6.49 yuan per liter, it can be concluded that α is 0.58 yuan / km.

When the driver decides to leave, the cost of the driver is his no-load rate, and the cost of fuel. Take Beijing Capital International Airport as an example. The distance from the airport is 30 kilometers from the urban area. From this, the profit of the driver is as follows:

$$p_2 = \begin{cases} \dfrac{11 - \alpha (d_k' + 30)}{t_2 + \dfrac{d_k'}{v}}, & d_k' < 3 \\ \dfrac{11 + 2.5(d_k' - 3) - \alpha (d_k' + 30)}{t_2 + \dfrac{d_k'}{v}}, & 3 < d_k' < 10 \\ \dfrac{11 + 2.5 \times 7 + 3.75(d_k' - 10) - \alpha (d_k' + 30)}{t_2 + \dfrac{d_k'}{v}}, & d_k' > 10 \end{cases} \tag{4}$$

where d_k' represents the distance of the k-th passenger from the boarding point to the destination.

(III) Establishment of multi-criteria decision-making model

The driver needs to make a decision between stay at the airport to wait for passengers and empty taxi return to the city. It is necessary to consider not only the least waiting time for passengers, but also the factors such as the greatest income, so this paper considers to build a Multi-criteria Decision Model, so that the above influencing mechanisms are related to each other and the constraints are well coordinated, so as to obtain the optimal strategy[1].

（1）Determining decision matrix

For the problem, the only choices the driver faces are "leave" and "stop", that is, the driver has only two alternatives. It may be recalled that option 1 is "leaving", that is, the driver is free to return to the urban area to pick up passengers, and option 2 is "stop", that is, the driver goes to the "storage pool" to wait for passengers to be picked up. The impact mechanism described above can be used as the corresponding five evaluation indicators to evaluate the scheme, and the specific corresponding information is as follows:

Table 4. Evaluation indicators

Index serial number	Index
1	Number of flights
2	Number of taxis in storage pool
3	Number of passengers
4	Waiting time
5	Driver's income

The decision matrix consisting of two scenarios and five evaluation indicators is as follows:

$$X = \begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} \end{pmatrix} = \left(x_{ij} \right)_{2\times 5}, \tag{5}$$

where $x_{ij} (i = 1, 2;\ j = 1, 2, \cdots 5)$ denotes the i-th feature to the j-th program index values.

（2）Standardization of influencing factors

Because of the different types of indicators and the different value dimensions, in order to ensure the accuracy of evaluation results, we need to standardize the indicators[2].

（3）Hierarchical analysis to determine the weight of the indicator

Because there are many factors influencing driver's decision-making, and the correlation and restriction degree between different indicators are different, therefore, this paper uses hierarchical analysis to determine the weight of each indicator. The main steps are as follows:

Step 1: Establishing a hierarchy

By dividing the decision-making issues into three levels, the target layer, the guideline layer, and the scenario layer, we can establish a hierarchy diagram as shown in Figure 1:

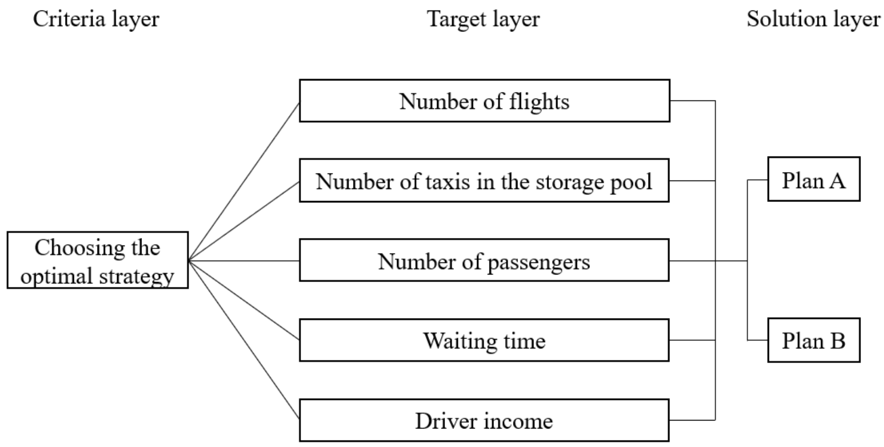

Figure 1. Hierarchy chart

Step 2: Constructing a comparison matrix

Based on the value of the column scale value table, construct the judgment matrix. After analyzing the factors through the references, the following pairwise of comparison matrices are constructed:

$$A = \begin{pmatrix} 1 & 2 & 1/3 & 1/3 & 1/3 \\ 1/2 & 1 & 1/3 & 1/3 & 1/4 \\ 1 & 2 & 1 & 3 & 1/3 \\ 3 & 3 & 1/3 & 1 & 1/2 \\ 3 & 4 & 3 & 2 & 1 \end{pmatrix}.$$

Step 3: Computing the weight vector

Each column vector of matrix A is normalized to obtain a new matrix A*, then A* is summed by rows and the summed vector is standardized to obtain the weight coefficients of each index:

$$W = \frac{\omega_i}{\sum_{i=1}^{5} \omega_i} = \frac{\sum_{j=1}^{n} a_{ij}^*}{\sum_{i=1}^{5} \sum_{j=1}^{n} a_{ij}^*} = (\omega_1, \omega_2, \cdots, \omega_5)'.$$

Step 4: Consistency test

The purpose of the consistency test is to avoid the occurrence of indicators $m > n, n > p, m < p$. To perform the consistency test, first calculate the maximum eigenvalue of the matrix A, then calculate the consistency index CI of the judgment matrix A, and finally calculate the random consistency ratio CI / RI. When the ratio CR<0.1, then, pass the consistency test. RI represents the average random consistency index, which is a constant, and its specific value is queried in the scale according to the matrix order n.

（4）Determining comprehensive evaluation value of scheme

For the problem, the ultimate goal is to calculate the impact of each indicator relative to each scheme, that is, the evaluation value of the comprehensive synthesis of each scheme, by comparing the size of the evaluation values of each scheme to determine the rationality of each scheme. In this paper, the linear weighting method is chosen to calculate the evaluation values of each plan, that is:

$$V = X \cdot W = \begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} \end{pmatrix} \cdot \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}, \quad (6)$$

where $v_i\,(i = 1,2)$ represents the evaluation value of the i-th plan.

（5）Determining decision-making plan

According to the size of the respective evaluation values of plan 1 and 2, the driver's decision-making results can be determined, as *H*. The driver leaves the airport as "0" and the driver stays at the airport means "1", the driver's decision-making result can be expressed as:

$$H = \begin{cases} 0, v_1 > v_2, \\ 1, v_1 < v_2. \end{cases}$$

that is, taking into account the influence of five related factors on driver's decision-making, the comprehensive evaluation value of the two schemes are obtained, and the driver will choose the scheme with large evaluation value as the final decision.

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