Quest Journals
Journal of Research in Applied Mathematics
Volume 7 ~ Issue 11 (2021) pp: 33-39
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

The de-Sitter Model with Dark Energy

Prashant R. Dhongle
Seth Kesarimal Porwal College, Kamptee, Nagpur, India

ABSTRACT: *The de-Sitter model with dark energy has been found in stationary space-time and it is realized that the model is completely filled with dark matter and it represents dark energy star, for the value of M greater than zero. In particular, for zero value of M, the model goes over to de-Sitter empty space-time and then it describes open flat universe of special relativity. The model is the generalization of empty de-Sitter universe. Further geometrical and physical aspects of the model are studied.*
KEYWORDS*: General Relativity, Einstein Field Equations, Dark Energy, Dark Matter.*

Received 08 November, 2021; Revised: 22 November, 2021; Accepted 24 November, 2021

I. INTRODUCTION

General relativity has developed into an essential tool in Cosmology and modern Astrophysics and it is the basis of current cosmological models of the expanding universe. The spherical symmetry has its own importance in general theory of relativity due to its comparative simplicity. Many important space-times like Schwarzschild solutions (exterior and interior), the Robertson – Walker model etc. are spherically symmetric in nature. Most of the spherically symmetric space-times are also stationary space-times. Stationary space-times or time – independent gravitational fields are very significant in general relativity and in it the metric tensor components g_{ij} , are all independent of the time coordinate, *i.e.*, there exists a coordinate system in which we can express the metric tensor independent of the time coordinate. Borkar et al. [1] have studied the spherically symmetric space-time with charged perfect fluid distribution in stationary space-time. Borkar et al. [2] and Pandya et al. [3] have studied the spherically symmetric space-time in bimetric theory of gravitation.

In the universe, 96 % of energy content is in exotic form, out of which 70 % of energy is repulsive in nature, called dark energy and 23 % of energy is attractive in nature, called dark matter [4] and [5]. Although dark matter is the most popular theory to explain the various astronomical observations of galaxies and galaxy clusters, there has been no direct observational evidence of dark matter. It has been noted that the names "dark matter" and "dark energy" serve mainly as expressions of human ignorance. Dark matter is matter that is inferred to exist from gravitational effects on visible matter and background radiation, but is undetectable by emitted or scattered electromagnetic radiation [6] whereas dark energy is a hypothetical form of energy that permeates all of space and tends to increase the rate of expansion of universe [7]. Generally, dark energy has a strong negative pressure (*i.e.*, effects, acting repulsively) in order to explain the observed acceleration in the expansion rate of the universe. The models of dark energy have created a lot of interest in the research area and many researchers ([8] to [16]) have developed the dark energy cosmological models of the universe. Dark energy is one of the popular ways to explain recent observations and experiments that the universe appears to be expanding at an accelerating rate. A star is self luminous, massive celestial body that generates the nuclear energy in its core. Dark energy star is the hypothetical compact star, which is believed to be the alternative explanation for observations of astronomical black hole candidate. The theory states that, the in falling matter from the event horizon is converted into vacuum energy or dark energy. The dark energy star theory proposed by Chapline [17] is the generalization of picture of gravastar (gravitational vacuum star) and dark energy star has interior solution with matter and is governed by equation of state $p = \omega\rho, \quad \omega \le -1/3$ which is matched with exterior vacuum Schwarzschild solution.

Many researchers are studying the nature and properties of the different cosmological models of the universe in stationary space-times. Beig et al. [18] have thrown light on the far – field behavior of stationary space-times. Garcia et al. [19] have discussed the conformally flat stationary axisymmetric space-times. Bartolo et al. [20] have studied the existence and multiplicity results for orthogonal trajectories on stationary space-times under intrinsic assumptions, with some examples and applications. Many other researchers ([21] to [28]) have studied in detail various cosmological models in stationary space-times with different physical parameters and have discussed the behavior of the models physically and geometrically.

The de-Sitter space-time has an important role in describing the geometrical and physical features of the universe. It is deduced on the condition $(\rho + p) = 0$ of Zel'dovich matter; it is completely empty and describes the geometry of empty space-time [29]. In this paper, we study the de-Sitter universe in stationary space-time in order to verify its geometrical and physical behavior. It is found that our model is analogous to de-Sitter universe and it is completely filled with dark matter and it represents dark energy star, since $p = (-1/3)\rho$ in our model. In particular for $M = 0$, our model goes over to empty de-Sitter universe of Tolman [29] and then it describes open flat universe of special relativity. So our model is the generalization of empty de-Sitter universe. Other geometrical and physical aspects of the model have been studied.

II. THE METRIC AND ITS SOLUTION

We consider the spherically symmetric metric

$$ds^2 = -e^{\lambda}dr^2 - r^2 d\theta^2 - r^2 Sin^2\theta d\phi^2 + e^{\nu}dt^2 , \qquad (1)$$

where λ and ν are functions of *r* and *t*. We assume that the space-time is filled with matter consisting of perfect fluid. The Einstein's field equations are

$$R_{ij} - \frac{1}{2} R\, g_{ij} = 8\pi T_{ij} . \qquad (2)$$

The energy-momentum tensor T_{ij} for a perfect fluid is given as

$$T_{ij} = (\rho + p)\, u_i u_j - p g_{ij} . \qquad (3)$$

Here ρ is the density and p is the pressure of the perfect fluid. We choose the components of four-velocity u^i as $u^1 = 0 = u^2 = u^3,\ u^4 \neq 0$ and we have

$$g_{ij}\, u^i u^j = 1 . \qquad (4)$$

The Einstein's field equations (2) in stationary space-time take the form

$$P_{\alpha\beta} + \frac{h}{2} f_{\alpha}^{\ \gamma} f_{\beta\gamma} - \frac{1}{2}\gamma_{\alpha\beta} P = 8\pi T_{\alpha\beta} , \qquad (5)$$

$$\frac{3}{8} h\, f_{\alpha\beta} f^{\alpha\beta} + \frac{1}{2} P = \frac{8\pi}{h} T_{44}, \qquad (6)$$

$$\frac{\sqrt{h}}{2} f_{\alpha\ |\beta}^{\ \beta} + \frac{3}{2} f_{\alpha\beta} (\sqrt{h})^{|\beta} = \frac{8\pi}{\sqrt{h}} T_{4\alpha}, \qquad (7)$$

in which

$$\gamma_{\alpha\beta} = - g_{\alpha\beta} + \frac{g_{4\alpha}\, g_{4\beta}}{h} \qquad (\alpha, \beta = 1, 2, 3), \qquad (8)$$

is the three – dimensional metric tensor determining the geometry of space, $f_{\alpha\beta}$ is the three – dimensional anti-symmetric tensor given by

$$f_{\alpha\beta} = g_{\beta\,|\,\alpha} - g_{\alpha\,|\,\beta} = \frac{\partial g_{\beta}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha}}{\partial x^{\beta}} , \qquad (9)$$

$$h = g_{44} , \qquad g_{\alpha} = \frac{-g_{4\alpha}}{h} \qquad (\alpha = 1, 2, 3). \qquad (10)$$

(Here the stroke '|' stands for covariant differentiation). *P* is the three - dimensional scalar curvature given by

$$P = \gamma^{\alpha\beta} P_{\alpha\beta} , \qquad (11)$$

where $P_{\alpha\beta}$ is the three - dimensional Ricci tensor constructed from the three – dimensional metric tensor $\gamma_{\alpha\beta}$ in the same way as R_{ik} is constructed from the g_{ik} (for details, refer [30]).

In stationary space-times, the gravitational potentials g_{ij} are independent of time *t*. So that, for the metric (1) in stationary space-time, λ and ν are functions of *r* alone. Thus the Einstein's field equations (5 – 7) in stationary space-times becomes

$$\frac{1}{r^2}e^{\lambda} - \frac{1}{r^2} = -8\pi\, p e^{\lambda} , \qquad (12)$$

$$\frac{1}{2}\frac{\lambda'}{r} = -8\pi p e^{\lambda}, \qquad (13)$$

$$\frac{1}{r^2}e^{\lambda} - \frac{1}{r^2} + \frac{\lambda'}{r} = 8\pi \rho e^{\lambda}. \quad (14)$$

The system of three differential equations (12 – 14) in three unknowns λ, p and ρ has a unique solution given by

$$e^{\lambda} = 1/(1 - M r^2) \qquad (15)$$

and

$$8\pi\rho = 3M, 8\pi p = -M, \qquad (16)$$

where M is the constant of integration. Assuming $\lambda = -\nu$, we write

$$e^{\lambda} = e^{-\nu} = 1/(1 - M r^2). \qquad (17)$$

So that our metric (1) becomes

$$ds^2 = -\frac{1}{(1 - M r^2)} dr^2 - r^2 d\theta^2 - r^2 Sin^2\theta d\phi^2 + (1 - M r^2) dt^2 . \qquad (18)$$

This is the spherically symmetric metric in stationary space-time having the matter of perfect fluid with density and pressure satisfying the equation (16).

III. GEOMETRICAL AND PHYSICAL ASPECTS OF MODEL

Choosing $M = 1/R^2$, then our model (18) goes over to the de-Sitter universe

$$ds^2 = -\left(1 - \frac{r^2}{R^2}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 Sin^2\theta d\phi^2 + \left(1 - \frac{r^2}{R^2}\right) dt^2 . \qquad (19)$$

The de-Sitter universe is deduced on the assumption of Zel'dovich matter (stiff matter) given by the condition $(\rho + p) = 0$ and it is completely empty describing the geometry of empty space-time. For our model, it is realized from equation (16) that the density ρ and pressure p obeys the equation

$$p = (-1/3)\,\rho. \qquad (20)$$

This suggests that the de-Sitter universe is not completely empty but it contains dark energy and it represents dark energy star. This is one of the implications which we can add in the geometry of de-Sitter universe that it not only describes the geometry of empty space-time but it also describes the geometry of the universe containing dark energy and matter and it represents dark energy star. In particular for $M = 0$, our dark energy de-Sitter universe (18) goes over to empty de-Sitter universe of Tolman [29] and then it represents open flat universe of special relativity.

3.1 The nature of our de-Sitter universe in relates to coordinates transformations

I. In the coordinate transformation $r = \left(\sin\beta/\sqrt{M}\right)$, our model (18) has the geometry

$$ds^2 = -\frac{1}{M}\left\{d\beta^2 + \sin^2\beta\,(d\theta^2 + Sin^2\theta d\phi^2)\right\} + \cos^2\beta\, dt^2 . \qquad (21)$$

II. In the coordinate transformation, $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$ with $a + b = \frac{1}{\sqrt{M}} e^{\sqrt{M}\,t}\left(1 - M r^2\right)^{\frac{1}{2}}$ and $a - b = \frac{1}{\sqrt{M}} e^{-\sqrt{M}\,t}\left(1 - M r^2\right)^{\frac{1}{2}}$, the model (18) goes over to

$$ds^2 = -\left\{dx^2 + dy^2 + dz^2 + da^2\right\} + db^2 , \qquad (22)$$

and in view of the transformation $U_1 = ix, U_2 = iy, U_3 = iz, U_4 = ia, U_5 = ib$,we write

$$ds^2 = dU_1^2 + dU_2^2 + dU_3^2 + dU_4^2 + dU_5^2 , \qquad (23)$$

with

$$U_1^2 + U_2^2 + U_3^2 + U_4^2 + U_5^2 = \left(\frac{i}{\sqrt{M}}\right)^2 . \qquad (24)$$

The equation (23) suggests that the geometry of our model is one which holds on the surface of sphere embedded in a Euclidean space of five dimensions.

III. On applying the Lemaitre Robertson transformation

$$\bar{r} = \frac{r e^{-\sqrt{M}\, t}}{\sqrt{1 - M r^2}} \quad \text{and} \quad \bar{t} = t + \frac{1}{\sqrt{M}} \log\sqrt{1 - M r^2}\,,$$

the model (18) takes the form as

$$ds^2 = -\, e^{2\sqrt{M}\,\bar{t}} \left\{ d\bar{r}^2 + \bar{r}^2 (d\theta^2 + Sin^2\theta d\phi^2) \right\} + d\bar{t}^2 \,. \qquad (25)$$

Removing bars, we get

$$ds^2 = -\, e^{2\sqrt{M}\, t} \left\{ dr^2 + r^2 (d\theta^2 + Sin^2\theta d\phi^2) \right\} + dt^2 \,. \qquad (26)$$

Its Cartesian equivalent is

$$ds^2 = -\, e^{2\sqrt{M}\, t} \left\{ dx^2 + dy^2 + dz^2 \right\} + dt^2 \,. \qquad (27)$$

Thus, with the help of this transformation, we see that our static line element is converted into a non-static one. It is realized that the geometry of all these models of our dark energy de-Sitter universe is depending upon *M* and have the similar features as that of the geometry of empty de-Sitter space-time.

3.2 Motion of test particle

The motion of the test particles is described by the geodesic equations given by

$$\frac{d^2 x^i}{ds^2} + \Gamma^i_{jk} \frac{dx^j}{ds} \frac{dx^k}{ds} = 0 \,. \qquad (28)$$

In our model (18) (for $\theta = \pi/2$ initially), the geodesic equation (28) yields

$$\frac{d^2\theta}{ds^2} = 0\,, \qquad (29)$$

$$\frac{d^2\phi}{ds^2} + \frac{2}{r}\frac{dr}{ds}\frac{d\phi}{ds} = 0\,, \qquad (30)$$

$$\frac{d^2 t}{ds^2} - \frac{2 M r}{(1 - M r^2)} \frac{dr}{ds}\frac{dt}{ds} = 0\,. \qquad (31)$$

Equation (29) shows that a particle which starts moving in the plane $\theta = \pi/2$ continues to move in the same plane. Equations (30) and (31) after simplification gives

$$r^2 \frac{d\phi}{ds} = A \qquad (32)$$

and

$$(1 - M r^2) \frac{dt}{ds} = B\,, \qquad (33)$$

where *A* and *B* are constants of integration.
Using equations (32) and (33), from the line element (18), we obtain

$$\frac{dr}{ds} = \left\{ M\left(A^2 + r^2\right) + \left(B^2 - 1 - \frac{A^2}{r^2}\right) \right\}^{\frac{1}{2}} . \quad (34)$$

We write

$$\frac{dr}{dt} = \frac{dr}{ds}\frac{ds}{dt} = \frac{1}{B}(1 - M r^2) \left\{ M\left(A^2 + r^2\right) + \left(B^2 - 1 - \frac{A^2}{r^2}\right) \right\}^{\frac{1}{2}} , \qquad (35)$$

which on differentiating with respect to *t* gives

$$\frac{d^2 r}{dt^2} = -\frac{2 M r}{(1 - M r^2)} \left(\frac{dr}{dt}\right)^2 + \frac{1}{B^2}(1 - M r^2)^2 \left(M r + \frac{A^2}{r^3} \right). \qquad (36)$$

Also we have

$$\frac{d\phi}{dt} = \frac{d\phi}{ds}\frac{ds}{dt} = \frac{A}{B}\left(\frac{1}{r^2} - M\right). \qquad (37)$$

From equation (35), $\left(dr/dt\right) = 0$ gives the value of *M* as

$$M = \frac{A^2 - (B^2 - 1)r^2}{r^2(A^2 + r^2)}, \tag{38}$$

for which velocity is zero and

$$\frac{d^2r}{dt^2} = \frac{1}{B^2}(1 - Mr^2)^2\left(Mr + \frac{A^2}{r^3}\right) > 0, \text{ for all } M \geq 0. \tag{39}$$

This shows that $\left(d^2r/dt^2\right) > 0$ (always) for $M \geq 0$ and therefore a particle once reaches perihelion starts to move away from the perihelion and would never return for value of M given by equation (38).

In particular for $r = 1/\sqrt{M}$, we have $dr/dt = 0 = d\phi/dt$ which suggests that all motion will vanish at a radius $1/\sqrt{M}$. This radius is the apparent horizon of the model.

3.3 Shift in spectral lines

A ray of light is emitted from a distant star travelling among the radial direction towards the origin. Then we have $ds = 0 = d\theta = d\phi$. So that, the line element (18) becomes

$$\frac{dr}{dt} = \pm(1 - Mr^2), \tag{40}$$

which gives the path *dt* as

$$dt = -\frac{dr}{(1 - Mr^2)}. \tag{41}$$

If *t* is the time for a ray of light to travel from $r = 1/\sqrt{M}$ to $r = 0$ then we have

$$t = \int_0^{1/\sqrt{M}} \frac{dr}{(1 - Mr^2)}, \tag{42}$$

and then (using the transformation $r = \sin\psi/\sqrt{M}$), the equation (42) yield $t = \infty$. This infers that a light ray would take an infinite time as measured by an observer at the origin to travel between the origin and the horizon. So, the observer will never have any information about the events occurring at the horizon.

Let δt_1 be the interval of two successive wave crests emitted from a distant star and δt_2 the corresponding interval of their reception by an observer at rest at the origin so that the proper time interval and the coordinate time interval are same at the origin. Thus

$$\delta t_2^0 = \delta t_2. \tag{43}$$

A light ray leaving a particle located at the radius *r* at time t_1 and reaching at the origin at the later time t_2 is given (from equation (41)) as

$$t_2 - t_1 = \int_0^r \frac{dr}{(1 - Mr^2)}, \tag{44}$$

which on differentiating with respect to t_1 gives

$$\frac{\delta t_2}{\delta t_1} = 1 + \frac{dr/dt}{(1 - Mr^2)}, \tag{45}$$

where dr/dt denotes the radial velocity at $t = t_1$, *i.e.*, $dr/dt = \left(dr/dt\right)_{t = t_1}$. From equation (33), we have

$$(1 - Mr^2)\delta t_1 = B\delta t_1^0 \text{ OR } \delta t_1^0 = \frac{1}{B}(1 - Mr^2)\delta t_1. \tag{46}$$

From equations (43), (45) and (46), we write

$$\frac{\delta t_2^0}{\delta t_1^0} = \frac{B}{(1 - Mr^2)} + \frac{B\left(dr/dt\right)}{(1 - Mr^2)^2}. \tag{47}$$

We have $B > 0$, $(1 - Mr^2) > 0$ and therefore, the sign of $\delta t_2^0/\delta t_1^0$ depends upon the sign of radial velocity dr/dt at time $t = t_1$. From equation (35), it is observed that $\left(Mr^2(A^2 + r^2)\right)/\left(A^2 - (B^2 - 1)r^2\right) > 1 \Rightarrow (dr/dt) > 0 \Rightarrow \delta t_2^0/\delta t_1^0 > 0$ which means that there exists red shift. If $\left(Mr^2(A^2 + r^2)\right)/\left(A^2 - (B^2 - 1)r^2\right) < 1$, then $(dr/dt) < 0$ yield $\delta t_2^0/\delta t_1^0 > 0$ or

$\delta t_2^0 / \delta t_1^0 < 0$, which shows that there may exist red shift or violet shift. Thus there is a possibility of both red and violet shift. But the possibility of red shift is greater.

Let λ_0 and $\lambda_0 + \delta\lambda_0$ be the wavelengths of waves corresponding to the time δt_1^0 and δt_2^0 respectively. Then

$$\frac{\delta t_2^0}{\delta t_1^0} = \frac{c\,\delta t_2^0}{c\,\delta t_1^0} = \frac{\lambda_0 + \delta\lambda_0}{\lambda_0} = \left(1 + \frac{\delta\lambda_0}{\lambda_0}\right). \qquad (48)$$

For the line element (26), we find that $\frac{\delta t_2^0}{\delta t_1^0} = e^{(t_2 - t_1)\sqrt{M}} = 1 + \sqrt{M}\,(t_2 - t_1)$ upto first approximation. If *r* is the distance travelled in time $t_2 - t_1$ then $t_2 - t_1 = r$ as $c = 1$. So that, we have

$$\left(1 + \frac{\delta\lambda_0}{\lambda_0}\right) = \frac{\delta t_2^0}{\delta t_1^0} = 1 + \sqrt{M}\, r \Rightarrow \quad \frac{\delta\lambda_0}{\lambda_0} = \sqrt{M}\, r\,. \qquad (49)$$

This shows that red shift is proportional to the distance measured from the origin, for which the proportionality constant *M* is found as $M = 2.45 \times 10^{-14}$ cm^{-1}, from Hubble's law.

IV. OUR FINDINGS

1. The de-Sitter universe has been investigated in stationary space-time and it is realized that our model is completely filled with dark matter and it represents dark energy star, since $p = (-1/3)\rho$ in our model.
2. For $M = 0$, our model goes over to empty de-Sitter universe. Thus our model is the generalization of empty de-Sitter universe.
3. In relates to the coordinate transformations, it is realized that the geometry of our dark energy de-Sitter universe is depending upon *M* and have the similar features as that of the geometry of empty de-Sitter universe.
4. The motion of the test particle is studied in our model and it is seen that the particle once reaches perihelion starts to move away from the perihelion and would never return for $M = \left(A^2 - (B^2 - 1)r^2\right) / \left(r^2 (A^2 + r^2)\right)$. Further at a radius $r = 1/\sqrt{M}$, the motion vanishes and this radius $r = 1/\sqrt{M}$ is the apparent horizon of the model.
5. In our model, the red shift of the spectrum will appear for $\left(M r^2 (A^2 + r^2)\right) / \left(A^2 - (B^2 - 1)r^2\right) > 1$ and there is a possibility of red shift and violet shift for $\left(M r^2 (A^2 + r^2)\right) / \left(A^2 - (B^2 - 1)r^2\right) < 1$.
6. The red shift is proportional to the distance measured from the origin for which the proportionality constant *M* is found as $M = 2.45 \times 10^{-14}$ cm^{-1}.

V. CONCLUSION

The de-Sitter model has been investigated in stationary space-time and it is observed that our model is completely filled with dark matter and it represents dark energy star. In particular for zero value of *M*, our model goes over to empty de-Sitter universe. Thus our model is the generalization of empty de-Sitter universe. Other geometrical and physical aspects of the model are studied in related to coordinate transformations, motion of test particles and red shift of spectral lines.

REFERENCES

[1]. Borkar, M. S. and Dhongle, P. R., Spherically Symmetric Charged Perfect Fluid Distribution in Stationary Space-time, Tensor N. S., 2011. Vol. 73(2), pp. 127 – 136.

[2]. Borkar, M. S. and Gayakwad, P. V., Evaluation of Schwarz Child's Exterior and Interior Solutions in Bimetric Theory of Gravitation, Int. J. Applications & Applied Mathematics, 2014. Vol. 9, No. 1, pp. 260 – 271.

[3]. Pandya, D. N. and Hasmani, A. H., Spherically Symmetric Charged Anisotropic Solution in Higher Dimensional Bimetric General Relativity, Int. J. Applications & Applied Mathematics, 2020. Vol. 15, No. 1, pp. 58 – 68.

[4]. Perlmutter, S. et al., Discovery of a Supernova Explosion at Half the Age of the Universe, Nature, 1998. Vol. 391, pp. 51 – 54.

[5]. Riess, A. G. et al., Type Ia Supernova Discoveries at $z > 1$ from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution, Astrophys. J., 2004. Vol. 607, pp. 665 – 687.

[6]. Trimble, V., Existence and Nature of Dark Matter in the Universe, Annual Review of Astronomy and Astrophysics, 1987. Vol. 25, pp. 425 – 472.

[7]. Peebles, P. J. E. and Ratra, B., The Cosmological Constant and Dark Energy, Reviews of Modern Physics, 2003. Vol.75 (2), pp. 559 – 606.

[8]. Lobo, F. S. N., Stable Dark Energy Stars, Class. Quantum Gravit., 2007. Vol. 23, pp. 1525 – 1541.

[9]. Padmanabhan, T., Dark Energy and Gravity, Gen. Relativ. Gravit., 2008. 40, pp. 529 – 564.

[10]. Chan, R., da Silva, M. F. A. and da Rocha, J. F. V., Star Models with Dark Energy, Gen. Relativ. Gravit., 2009. Vol. 41, pp. 1835 – 1851.
[11]. Ghezzi, C. R., Anisotropic Dark Energy Stars, Astrophys. Space Sci., 2011. Vol. 333, pp. 437 – 447.
[12]. Rahaman, F. et al., Singularity-Free Dark Energy Star, Gen. Relativ. Gravit., 2012. Vol. 44, pp. 107 – 124.
[13]. Chirde, V. R. and Shekh, S. H., Plane Symmetric Dark Energy Models in the Form of Wet Dark Fluid in f(R, T) Gravity, J. Astrophys. Astr., 2016. Vol.37, pp. 15.
[14]. Khurshudyan, M. and Khurshudyan, A. Z., Some Interacting Dark Energy Models, Symmetry, 2018. Vol. 10, pp. 577.
[15]. Tiwari, R. K., Beesham, A. and Shukla, B. K., Scenario of Two-Fluid Dark Energy Models in Bianchi Type-III Universe, Int. J. Geometric Methods in Modern Physics, 2018. Vol. 15, pp. 1850189.
[16]. Naidu, R. L., Aditya, Y. and Reddy, D. R. K., Bianchi Type-V Dark Energy Cosmological Model in General Relativity in the Presence of Massive Scalar Field, J. Heliyon, 2019. e01645.
[17]. Chapline, G., Dark Energy Stars, Proceedings of Texas Conference of Relativistic Astrophysics, Stanford, 2004.
[18]. Beig, R. and Schmidt, B., Time-Independent Gravitational Fields, Lect. Notes Phys., 2000. Vol. 540, pp. 325 – 372.
[19]. Garcia, A. A. and Campuzano, C., Conformally Flat Stationary Axisymmetric Space-times, Physical Review D, 2002. 66, pp. 124018.
[20]. Bartolo, R., Germinario, A. and Sanchez, M., Orthogonal Trajectories on Stationary Space-times under Intrinsic Assumptions, Journal of the Juliusz Schauder Center, 2004. Vol. 24, pp. 239 – 268.
[21]. Katz, J., Lynden-Bell, D. and Bicak, J., Gravitational Energy in Stationary Space-times, J. Class. Quant. Grav., 2006. Vol. 23, pp. 7111 – 7128.
[22]. Herberthson, M., Bounds for, and Calculation of, The Multipole Moments of Stationary Space-times, J. Phys. Conference Series, 2007. 66, pp. 012064.
[23]. Chrusciel, P. T. and Delay, E., Non – Singular, Vacuum, Stationary Space-times with a Negative Cosmological Constant, Annales Henri Poincare, 2007. 8, pp. 219 – 239, arXiv: gr-qc/0512110v2.
[24]. Javaloyes, M. A. and Sanchez, M., Existence of Standard Splittings for Conformally Stationary Space-times, J. Class. Quant. Grav., 2008. Vol. 25, pp. 168001.
[25]. Nayak, K. R., Einstein Equations and Inertial Forces in Axially Symmetric Stationary Space-times, Gen. Relativ. Gravit., 2009. Vol. 41, pp. 2737 – 2756.
[26]. Borkar, M. S. and Dhongle, P. R., Bianchi Type I String Dust Model with Magnetic Field in Stationary Space-Time, Int. J. Appl. Math., 2010. Vol. 23, No. 6, pp. 977 – 988.
[27]. Borkar, M. S. and Dhongle, P. R., Cylindrically Symmetric Inhomogeneous String Cosmological Model with Magnetic Field in Stationary Space-Time, American Journal of Pure Applied Mathematics, 2013. Vol. 2, No. 1, pp. 37 – 42.
[28]. Borkar, M. S. and Dhongle, P. R., Pre – Hawking Radiating Gravitational Collapse in Stationary Space-Time, International Journal of Theoretical & Applied Sciences, 2013. Vol.5(2), pp. 27 – 31.
[29]. Tolman, R. C., Relativity Thermodynamics and Cosmology, Clarendon Press, Oxford, 1934.
[30]. Landau, L. D. and Lifshitz, E. M., The Classical Theory of Fields, Fourth Revised English Edition, Pergamon Press, Oxford, 1975.