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Research Paper

Process Monitoring Schemes for Correlated Data

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ABSTRACT: *When the basic assumption of independence and identical distribution of the observations is violated the performance of the traditional control charts are seriously affected. Many alternative monitoring schemes are suggested in the literature for accommodating serial correlation. Traditional control charts with modified control limits and time series based monitoring schemes are the most popular monitoring schemes. This article discuss about various monitoring schemes that exist in the literature for monitoring correlated data.*
KEY WORDS*: Control charts, serial correlation, AR(1) process, Run length*

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I. INTRODUCTION

The essential tool in SPC is the Control chart. A basic assumption in traditional control charts is that the observations from the manufacturing process are independent and identically distributed. However, in practice this assumption is frequently violated [1]. With improvements in measurements and data collection technology, processes can be sampled at higher rates and the high frequency of sampling leads to data correlation. Also, in continuous flow processes like chemical processes, the data are correlated [2]. Many authors discussed about the properties of standard control charts when applied to correlated observations. The assumptions made about the time series model, occurrence of a special cause, knowledge of process parameters etc. varies from study to study. The exact nature of the effect of autocorrelation also varies in these studies depending on the assumptions made. A basic conclusion that can be drawn from their studies is that correlation has a significant effect on the properties of the control charts that were investigated. When correlation is present in the data there are serious problems of not detecting the special causes that truly exist and giving false signals when there is no special cause.

II. TRADITIONAL CHARTS IN PRESENCE OF CORRELATION

Alwan [3] studies the performance of standard Shewhart control chart for individual observations with fixed control limits. He considers an ARMA(p,q) model and derives the probability of false positive (not detecting an assignable cause that truly exist) and false negative (detecting an assignable cause that does not exist) signals. Their results indicate that even milder levels of autocorrelation, which are often difficult to notice without formal time-series machinery can deteriorate the ability of the standard control charts to identify the special causes.

Alwan, Champ and Maragah [4] studies the effect of autocorrelation on individuals chart with supplementary runs rule. They assume an AR(1) model and their study shows that for an individuals chart with no supplementary runs rule the in-control ARL's tend to increase as the autoregressive parameter φ_1 moves away from 0. That is, for both positive and negative correlation, the in-control ARL tend to increase. But with the addition of supplementary runs rule the ARLs decreases monotonically as φ_1 approaches 1. i.e. For the case of a positively auto correlated AR(1) process, a chart supplemented with runs rule has a significantly lower in-control ARL than when the process is iid and the in-control ARL's of negatively correlated process are significantly larger than the iid case.

VanBrackle and Reynolds [5] consider EWMA and CUSUM control chart when the observation are from an AR(1) process with an additional random error. They consider only positive values of φ_1 and it is shown that the presence of positive correlation significantly reduces the in-control ARL of the EWMA and CUSUM charts. The reduction in the in-control ARL is more severe for large values of φ_1. As φ_1 increases the correlation between successive values of the EWMA, and CUSUM statistic also increases and the in-control ARL of the charts decreases rapidly. In the out-of-control situation, the correlation shortens the time required to detect small to moderate shifts and lengthens the detection time of large shifts. The shortened detection time for a small to moderate shifts is desirable but there is a very high probability of false alarm. Suggestions for choosing the design parameters of EWMA and CUSUM charts when the observations are correlated are also given in their paper. Schmid and Schone [6] studies the properties of EWMA control chart in the presence of correlation and prove theoretically that the tail probabilities of the run length in the in-control state for the correlated process are larger than in the case of independent observations if all the autocovariances are greater than or equal to zero.

In view of the lowered performance characteristics of the various control charts in the presence of correlation several remedial process monitoring schemes have been proposed in the literature. Traditional control charts with modified control limits and time series based monitoring schemes are the most popular methods.

III. CHARTS WITH MODIFIED CONTROL LIMITS

The first approach and the simplest method among the procedures developed for monitoring autocorrelated data is adjusting the control limits of the conventional charts to accommodate the systematic non random behaviour of the autocorrelated process.

Vasilopoulos and Stamboulis [7] modifies the control limits of a shewhart $\overline{X}$ chart when the observations follow an AR(2) process. For an $\overline{X}$ chart, when the process is an iid sequence the 3σ limits are given by $\overline{X} \pm 3\sigma_x/\sqrt{n}$. When the process observations are not independent and normally distributed, the standard deviation of $\overline{X}$ is not $\sigma_x/\sqrt{n}$ but it is to be obtained in terms of the process parameters and the process variance. The true variance of $\overline{X}$ is used to compute the control limits of the chart.

Using such an approach English, Krishnamurthi and Sastri [2] modifies the control limits of an individuals chart for the AR(p) process:

$$Y_t=\varphi_1 Y_{t-1}+\varphi_2 Y_{t-2}+\ldots+\varphi_p Y_{t-p}+\varepsilon_t$$

where φ_i s are the autoregressive parameters and ε_t follows $N(0,\sigma_\varepsilon^2)$. The control limits suggested are $\mu\pm3\sqrt{var(Y_t)}$ where μ is the process mean . For an AR(1) process

$$Var(Y_t) = \sigma\varepsilon^2/(1-\varphi_1)^2$$

and for an AR(2) process

$$Var(Y_t) = \frac{1-\varphi_2}{1+\varphi_2}\sigma^2{}_\varepsilon \;/((1-\varphi_2)^2-\varphi^2{}_1)$$

Wardell, Moskowitz and Plante [8] studies an EWMA chart with modified limit and their study reveals that an EWMA chart with properly adjusted limits performs at least as good as and often better than the special cause chart. Lu and Reynolds (1999) also studies the EWMA chart applied to observation from an AR(1) process with an additional random error and they conclude that if the control limits of the charts are adjusted to account for the correlation, it is preferable to plot EWMA of the observations rather than the forecast residuals. For low to moderate levels of autocorrelation, EWMA chart performs equally good as residuals chart and only for high levels of autocorrelation and large shifts the EWMA for residuals is marginally quicker in detecting lack of control.

Yaschin [9] propose a method to assess the run length distribution of a CUSUM chart when the data show moderate autocorrelation. He shows that the actual observations can be replaced by an iid sequence for which the run length distribution is approximately the same. When the serial correlation is not too large the approximation is good. He also proposes a method to modify the control limits of a CUSUM chart by using the autocorrelation structure of the process.

IV. RESIDUAL- BASED CONTROL SCHEMES

The most commonly used methods of SPC for autocorrelated data are the time-series modelling approach proposed by Alwan and Roberts[1]. The idea is to fit an appropriate time-series model for the process observations and obtain the one-step-ahead forecast. The typical time series model used for modelling autocorrelated data is the Auto Regressive Integrated Moving Average (ARIMA) model of Box and Jenkins [10]. If the fitted model is exact, the forecast residuals are uncorrelated and follow $N(0, \sigma_\varepsilon^2)$ distribution so that any conventional chart can be employed to monitor the sequence of the residuals.. If a shift in the process occurs, the identified model will no longer be correct and this model misspecification will be reflected as signals on a control chart applied to the residuals.

It is proposed to use two charts simultaneously : a common cause chart and a special cause chart. The common cause chart is a run chart of the forecasted values based on the fitted ARIMA model. This chart is without any control limit and it helps to view the level of the process and evolution of that level through time. Thus, it provides a better understanding of the process and helps in achieving real-time process control. The control decisions of this chart can be made on some economic calculations to balance the expected loss of bad product over some specified period of the time against the cost of re-centering. The special cause chart is a standard control chart for the forecast residuals. Since the residuals are assumed to be independent and normally distributed any traditional chart can be applied to the sequence of residuals. The latter ensures that the original process is under quality surveillance.

For example, if the process is identified to follow an AR(1) model, then the actual observation Y_t is given by, $Y_t=\varphi_1 Y_{t-1}+\varepsilon_t$ where $\varepsilon_t \sim N(0,\sigma_\varepsilon^2)$ and the forecast residual at time t is,

$$e_t = Y_t - \hat{\varphi}_1 Y_{t-1}$$

where $\hat{\varphi}_1$ is the estimate of φ_1. It is assumed that $\hat{\varphi}_1=\varphi_1$ and therefore e_t follows normal distribution with mean 0 and variance $\sigma\varepsilon^2$. Any standard control chart can be applied to the sequence e_t.

V. FORECASTING USING EWMA

In situations where time-series modeling approach becomes impractical, Montgomery and Mastrangelo [11] suggest using the EWMA to get a suitable one-step ahead forecast of the process. The EWMA statistic which is equivalent to an ARIMA (0,1,1) model in its correlation structure provides an excellent one-step ahead predictor for the model. The optional value of the EWMA parameter λ in terms of the mean square error of prediction is shown to be $(1-\theta_1)$. Cox [12] has derived the expression of the optimal value of λ for predicting the ARIMA(1,0,0) model. (AR(1) model). The value of λ which minimizes the m.s.e is derived as

$$\lambda = \begin{cases} (3\varphi-1)/(2\varphi) & 1/3\le\varphi<1 \\ 1 & \varphi<1/3 \end{cases}$$

In general if the observations from the process are positively autocorrelated and the process mean does not drift too quickly, the EWMA with an appropriate value for λ is suggested as a suitable one-step ahead predictor.

It is suggested to maintain two charts simultaneously. The forecast errors from EWMA can be plotted on any traditional chart. This chart is accompanied by a run chart of the original observations on which the EWMA forecast is superimposed. This allows visualization of the process dynamics. They also propose a chart which combines the information about the state of statistical control and process dynamics. The control limits for this chart is obtained as follows.

The prediction errors from the EWMA, $e_t = Y_t - \hat{Y}_t$ are assumed to be iid $N(0, \sigma_\varepsilon^2)$. The control limits can be obtained using the probability statement

$$P\{-U_{\alpha/2}\sigma_\varepsilon \le e_t \le U_{\alpha/2}\sigma_\varepsilon\}=1-\alpha$$

where $u_{\alpha/2}$ is the upper $\alpha/2$ percentage point of the unit normal distribution.

$$P\{\hat{Y}_t - U_{\alpha/2}\sigma_\varepsilon \le Y_t \le \hat{Y}_t + U_{\alpha/2}\sigma_\varepsilon\}=1-\alpha$$

Thus the actual observations Y_t can be plotted on a chart with the upper and lower control limits $\hat{Y}_t \pm U_{\alpha/2}\sigma_\varepsilon$ and central line $\hat{Y}_t$.

Hunter [13] also discuss about the use of EWMA as a forecasting tool.

VI. FORECASTING USING KALMAN FILTER

Monitoring forecast residuals from Kalman Filter is a similar approach [2]. The Kalman Filter is commonly used by the control engineers and other physical scientists in diverse areas such as the processing of signals in aerospace tracking and underwater sonar and statistical process control [14].

Let $Y_t, Y_{t-1}, ...Y_1$ denote the observed values of a variable of interest at times t, t-1, ...1. Y_t is assumed to be dependent on an unobservable quantity μ_t known as the state of nature. The Kalman Filter is a recursive procedure for inference about the state of nature μ_t given data upto time t. A general K. F model can be described by the matrix equations:

$$Y_t = F_t\mu_t+\varepsilon_t \quad (1)$$

$$\mu_t = G_t\,\mu_{t-1}+\upsilon_t \quad (2)$$

where F_t and G_t are known quantities. The first equation known as the observation equation describes the relationship between Y_t and the state of nature μ_t. The observation error ε_t is assumed to follow $N(0, W_t)$. The state of nature μ_t is assumed to change over time according to the second equation known as the system equation. The system equation error υ_t is assumed to follow $N(0, V_t)$. Both the variances V_t and W_t are assumed to be known and ε_t and υ_t are assumed to be independent. The recursive equations of K. F can be derived in many ways. The K.F recursive equations are derived using a Bayesian approach. The posterior distribution of μ_t given the data $Y^t = (Y_1, ..., Y_t)$ has been derived as $L(\mu_t / Y^t) \sim N(\hat{\mu}_t, Q_t)$ where

$$\hat{\mu}_t = G_t \hat{\mu}_{t-1} + R_t F_t'(W_t + F_t R_t F_t')^{-1} e_t$$
$$Q_t = R_t - R_t F_t'(W_t + F_t R_t F_t')^{-1} F_t R_t \qquad (3)$$
$$e_t = Y_t - F_t G_t \hat{\mu}_{t-1}$$

$R_t = G_t Q_{t-1} G_t' + V_t$, is the variance of μ_t prior to observing Y_t. Equation (3) is known as the recursive equations of the Kalman Filter. It is begun at time zero by choosing μ_0 and Q_0 to be best guesses about the mean and variance of μ_0 respectively.

A special case of the KF model is proposed [2] for recursively estimating the autoregressive parameters of an AR(p) model and to monitor the forecast residuals from the KF model. Let G_t =I and υ_t=0 equations (1) and (2) reduces to

$$Y_t = F_t \mu_t + \varepsilon_t \qquad (4)$$
$$\mu_t = \mu_{t-1}$$

where $F_t = (Y_{t-1}\ Y_{t-2}\ ...\ Y_{t-p})$, $\mu_t = (\varphi_1, \varphi_2, ..., \varphi_p)_t'$ and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. The recursive equations in this case reduces to

$$K_t = Q_{t-1} F_t'(\sigma_\varepsilon^2 + F_t Q_{t-1} F_t F_t')^{-1} \qquad (5)$$
$$e_t = Y_t - F_t \hat{\mu}_{t-1} \qquad (6)$$
$$\hat{\mu}_t = \hat{\mu}_{t-1} + K_t e_t \qquad (7)$$
$$Q_t = Q_{t-1} - K_t F_t Q_{t-1} \qquad (8)$$

where K_t is called the Kalman gain. The recursive procedure is

- step(1) Initialize the Kalman Algorithm with initial estimates of $(\varphi_1, \varphi_2, ..., \varphi_p)$, σ_ε^2 and Q_t
- step(2) Increment the time index one unit and compute the Kalman gain vector given by equation(5)
- step(3) After observing Y_t compute the prediction error using equation (6) and revise the autoregressive vector using equation (7)
- step(4) Revise the associated parameter error covariance matrix using equation (8)
- step(5) Repeat steps 1 to 4 as long as the process is in control

Kalman Filter recursive algorithm is recommended for the accurate estimation of the process mean and to apply traditional control charts to the residuals which are expected to be Gaussian white noise [2].

VII. OTHER MONITORING SCHEMES FOR CORRELATED DATA

Runger and Willemain [15] propose unweighted batch mean (UBM) chart as an alternative to time-series modelling. Since it requires no time series modeling, he calls it as a model free approach. The UBM chart plots arithmetic average of successive observations and exploits the large number of observations available in a data rich environment. The averaging of the observations dilutes the autocorrelation.

The j th unweighted batch mean is given by

$$V_j = \frac{\sum_{i=1}^{b} Y_{(j-1)b+i}}{b} \qquad j=1,2,.....$$

Procedures for determining the appropriate batch size b is given in Law and Carson [16]. Runger et. al. [17] provide a detailed analysis of batch size for an AR(1) model.

Weighted batch means (WBM) chart is also proposed [15] when the process is modelled as a ARMA model. This chart plots weighted averages of consecutive data values. Given an ARMA model, one can compute weights that make the batch means uncorrelated . The jth batch mean is calculated as

$$X_j = \sum_{i=1}^{b} w_i Y_{(j-1)b+i} \qquad j=1,2,.....$$

The batch size b is selected to tune performance against a specified shift s. If , $\sum_{i=1}^{b} w_i = 1$, then Xj is an unbiased estimate of the process mean. For an AR(1) model, the optimal weights are

$$w_1 = \frac{-\varphi_1}{(b-1)(1-\varphi_1)}$$

$$w_i = \frac{1}{(b-1)} \qquad \text{for} \quad i=1,2,.....,b\text{-}1$$

$$w_b = \frac{1}{(\text{b}-1)(1-\varphi_1)}$$

The EWMAST chart is proposed by Zhang [18]. It is an EWMA chart for stationary process data. The control limits of the chart are calculated considering the correlation structure of the data. The variance of the EWMA statistic Z_t is calculated as

$$\sigma_z^{\ 2} = \frac{\lambda}{(2-\lambda)}\sigma_y^{\ 2}\left\{1+2\sum_{k=1}^{m}\rho(k)(1-\lambda)^k(1-(1-\lambda)^{2m-k})\right\}$$

Where $\rho(k)$ is the autocorrelation function of Y_t at lag k. 'm' is a large integer (m is recommended as 25) and λ is the EWMA smoothing constant. The control limits for the chart are $\mu \pm L\sigma_z$. It is shown that this chart performs better than the residuals chart when the process autocorrelation is not very positively strong and mean shifts are small to medium.

The Auto Regressive Moving Average (ARMA) chart proposed by Jiang , Tsui and Woodall [19] plots ARMA statistic of the original observations. It is an extension of the EWMAST chart proposed by Zhang. The ARMA chart performs better than EWMAST chart when the process has strong positive correlation.

The Autoregressive T^2 chart is proposed by Apley and Tsung [20]. Here Hotelling's T^2 statistic is used for monitoring univariate autocorrelated process. It is shown that T^2 statistic can be decomposed into sum of the squares of the residual errors for various order autoregressive time series models fitted to the process. The performance of the chart is compared with CUSUM and Shewhart charts applied to residuals. It is shown that in certain ranges of the paramater values of the ARMA model, the T^2 chart performs better than either CUSUM or Shewhart chart applied to residuals. An additional advantage of the chart is that it provides some robustness with respect to model uncertainty.

Certain non parametric approaches are also in practice. Peihua Qiu et. Al [21] and Li & Qiu [22] discuss about a non parametric approach for monitoring serially correlated data.

VIII. CONCLUSION

All the charts discussed in this article provide methods for accommodating serial correlation in the data. Every procedure has its own advantages and limitations. Among the schemes the most commonly used one is the residual based control scheme. But studies shows that the run length properties of the traditional charts applied to residuals are different from the run length properties of those charts applied to iid observations. Therefore new monitoring schemes have to be designed for monitoring serially correlated data.

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