Quest Journals
Journal of Research in Applied Mathematics
Volume 7 ~ Issue 6 (2021) pp: 01-09
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

Quantifying the Attainment of COs and POs for Graduate Programs

Muhammad Shahjalal[1], Md. Zahidul Alam[2]
[1](*Department of Mathematics, Faculty of Science, Bangamata Sheikh Fojilatunnesa Mujib Science and Technology University, Jamalpur-2012, Bangladesh,*
[2](*Faculty of Science, Bangamata Sheikh Fojilatunnesa Mujib Science and Technology University, Jamalpur-2012, Bangladesh ,*
Corresponding Author: Muhammad Shahjalal

ABSTRACT: *One of the important element of Outcome Based Education (OBE) is establishment and attainment of Course Outcomes (COs) and Program Outcome (POs). POs are designed and developed at program level considering knowledge, competencies and skills required for the program. COs are required depth of knowledge and other attributes which are formulated for each course. Every course COs are mapped to target POs and attainment of each PO is calculated by using various tools that take target levels into account. Students should demonstrate the attributes after completion of courses as well as program. The attainment of COs and POs must be quantified in order for the program to be accredited. This paper presents a mathematical framework for quantifying the attainment of program's COs and POs. This approach appears to be more generalized and applicable to the creation of tools for OBEs.*

KEYWORDS: *Attainment of Course outcomes, Program outcomes, Normalized program outcomes and Outcome Based Education*

Received 28 May, 2021; Revised: 08 June, 2021; Accepted 10 June, 2021 *© The author(s) 2021. Published with open access at www.questjournals.org*

I. INTRODUCTION

Globally, there is an increasing demand for decent quality education and an employable workforce. Continuous industry growth, global competition, and contemporary consumer expectations have raised the bar for new engineering graduates' employability and performance in technical careers [1]. Since last decade Outcome Based Education (OBE) has been adopted by the engineering universities and institution in Bangladesh. In the process of accreditation in engineering degree providing institution/university, Board of Accreditation for Engineering & Technical Education (BAETE) strictly observes and assesses the evidence of the practice of outcome base education. Bangladesh Accreditation Council (BAC) generalized 10 standards and 66 criteria for all programs offered by the University in Bangladesh.

By giving priority of national higher educational objectives, needs of stakeholders, and global demand; universities and higher education institutions set institutional vision and mission statements and consequently program offering entities (PoEs) fix-up program education objectives (PEOs), Program Objectives (POs) and design program curriculum to meet the program and institution vision. At the design face of a program curriculum underlying required knowledge, skill, and other behaviour attributes what are defined in National Qualification Framework (NQF) are considered. The activities of the achievement of POs, and PEOs started from teaching-learning process in class room. Course planning, conducting and assessment play a vital role for attaining the POs and PEOs. Every course under a program has a set of course outcomes (COs) and each course outcome aligned to one or more POs with some measurable and attainable skill and attributes. The subjective knowledge of a course is assessed by marks obtained by the students whereas behavioural attributes and skill are measured by qualitative term.

In this paper, a mathematical system is proposed for quantifying the attainment of COs and POs for a defined program, referred to as degrees.

1. 1 Literature review

Students' cognitive domains of learning are usually assessed by formative and summative evaluations process conducted by the institution. The students' collective performance of evaluation has an effect on the attainment of course and program outcomes.

Rahmat, (2011) [2] suggested a comprehensive assessment plan to ensure that all courses and program outcomes are properly assessed for the OBE system. Besides the summative assessment; rubrics are used to measure the affective domain as well as the psychomotor domain. Rubric illustrates a student's level of achievement based on the grades they obtained.

Parimaladevi, (2020) [3] Conducted a study on outcome-based education with the aim of determining and quantifying course and program outcomes. The study gathered input from students about the course and program outcomes. The study concluded that the OBE process facilitates the systematic flow of knowledge necessary for enhancing teaching and learning, and that the accreditation process requires a distinct mapping of outcome-based education. A well-established process for assessing students is essential for enhancing program outcomes and, as a result, achieving program education objectives.

Liu & Zhang, (2020) [4] Assessed the course using indicators such as "attendance rate, activity level, and average score that can represent the course's overall effectiveness." The study suggested a hybrid model of course assessment named "Analytic Hierarchical Process (AHP)," which is used in conjunction with a fuzzy inference method. The study discovered that using the AHP model enables systematic evaluation of a course and ranking against several courses.

Rajak & et al., (2019) [5] addressed the method of obtaining POs and PEOs for AICTE-approved Post Graduate programs. The analysis calculates POs and PEOs by the use of direct and indirect methods. The method was found to be beneficial for evaluating the institute's teaching-learning process.

Yassine et al., (2016) [6] assessed learning outcomes in a smart learning environment and suggested several core features for assessing course learning outcomes, including "Mapping LMS Activities Against Different Learning Outcomes: Course Map Designing, Qualitative Analysis, Benchmarks Identification, and Openness."

Ou Lydia Liu, et al., (2012) [7] investigated the impact of motivation on teaching, learning, and evaluation processes and discovered that motivation assists students in improving their assessment of learning outcomes and leads to data-driven evidence and other institutionally referred criterion.

Vijaya & Arthi, (2019) [8] defined a fuzzy logic-based student assessment method that utilized step-by-step reasoning to overcome the problems associated with fuzzy rule emission. Five distinct characteristics such as academic awareness, communication, behavior, participation, and extracurricular activities are analyzed to determine students' overall performance.

Barlybayev et al., (2016) [9] propose a fuzzy model for evaluating student success on examinations, and the study discovered through experimentation that the approach correlates with other current systems while offering some advantages.

Yıldız and Baba, (2014) [10] suggests a modern approach for evaluating student performance in laboratory courses that is based on fuzzy logic systems. Student performance was evaluated using fuzzy logic and compared to the conventional method. The proposed framework revealed differences in evolution between the classical and fuzzy logic approaches.

Cavus, (2010) [11] defines an evaluation method for learning management systems (LMSs) based on concepts of artificial intelligence and fuzzy logic values. Benefits can accumulate to students and organizations involved in comparing online learning management systems.

Petrudi et al., (2013) [12] develops a method for analyzing fuzzy logic systems that takes into account three characteristics. The research discovered that students' university examination results can be evaluated using a fuzzy logic approach.

McNeil, (2011) [13] proposed a program evaluation model focused on Bloom's Taxonomy to identify outcome indicators in OBE. The model assists program and curriculum planners in identifying, classifying, and specifically expressing specific outcome criteria for program assessment, which is a challenge.

Soragaon and Mahesh , (2016) [1] suggested a simpler approach for assessing or determining course and program outcomes, as well as program-specific outcomes (PSOs). The measured attainment values for POs and PSOs can be compared to the target attainment values, and action plans can be established for those POs and PSOs with an attainment value less than the target value. Additionally, the proposed approach can be used to calculate COs, POs, and PSOs in an independent, non-affiliated organization.

1.2 Attainment and Assessment process

Evaluation plan outlines how learning outcomes' competencies indicators will be thoroughly assessed. Nine major elements of program assessment steps are discussed in [2]. Attainment and assessment process of a course need to define at designing faces of program curriculum that consists of following steps:

1. Defining vision and mission statements of degree offering institution and entity
2. Defining PEOs and POs statements and formulating mapping of PEOs to POs with degree of correlation
3. Define a list of core courses, allied courses with theory, lab/practical, project and other need based courses
4. Define course outcomes (COs) of each types of courses and map the course outcomes with specific one or more program outcomes along with various measurable skills and attributes
5. Formulate mapping with COs to POs with degree of correlation i.e. strong, moderate and weak correlation
6. Evaluate learning outcomes of each course with direct and indirect evaluation system
7. Setting level of attainment target of every courses by the course teacher
8. Assigning Qualitative measure of students gained skill and attributes
9. Plotting learners achievement profiles and compare with the expected attainment defined by the teacher

II. MATHEMATICAL FRAMEWORK

The sections define the relevant definitions, alignment matrices, and various measuring functions.

Definition 1: Let X_{ims} and X_{peos} be two column vectors that reflect the institutional mission statements and the outcomes of educational program respectively.

Definition 2: Let x_i and x^j be a row and column vector respectively. A mapping $f:(x_i \times x^j) \to x_i^j$ where $i = 1,2 \ldots, n$ and $j = 1,2, \cdots, m$; is generated a matrix here we call it relational or align matrix of x_i to x^j .

Definition 3: Let $X_{ims} = x_i$ and $X_{peos} = x^j$ be a row and column vector respectively. Again, let $Q = \{0,1,2,3\}$ be a set and the elements Q determine the level of intensity. A Mapping of PEO to IMS is represented by the function

$$f_{peo2ims} : x_i \times x^j \to \{0,1,2,3\}$$

Table 1: Mapping of PEO to IMS

$x_i \times x^j$	x^1	x^2	x^3	x^4
x_1	3	1	2	2
x_2	2	1	3	2
x_3	2	2	2	1

Alignment matrix of PEO to IMS is depicted in table 1.

Definition 4: Let $X_{PEO} = x_i$ and $X_{PO} = x^j$ be a row and column vector respectively and let $Q = \{0,1,2,3\}$ be a set and the elements Q determine the level of intensity. A Mapping of PEO to PO is represented by the function

$$f_{peo2po}: x_i \times x^j \to \{0,1,2,3\} \text{ or } f_{poe2po}: x_i^j \to \{0,1,2,3\}$$

And one of these mappings is defined by the following table 2:

Table 2: Mapping of PEO to POs

$x_i \times x^j$	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}
x_1	3	1	2	2	3	1	2	2	3	1	2	2
x_2	2	1	3	2	2	1	3	2	2	1	3	2
x_3	2	2	2	1	2	2	2	1	2	2	2	1
x_4	3	1	2	2	3	1	2	2	3	1	2	2

Definition 5: Let $X_{CO} = x_i$ and $X_{PO} = x^j$ be a row and column vector respectively and let $Q = \{0,1,2,3\}$ be a set and the elements Q determine the level of intensity. A Mapping of **CO to PO** is represented by the function

$$f_{co2po}: x_i \times x^j \to \{0,1,2,3\} \text{ or } f_{co2po}: x_i^j \to \{0,1,2,3\}$$

And one of these CO2PO mappings is defined by the following table 3:

Table 3: Mapping COs to POs

$x_i \times x^j$	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}
x_1	3	1	2	2	3	1	2	2	0	1	2	1
x_2	2	1	3	0	2	0	3	0	2	1	3	1

x_3	2	2	2	1	2	2	2	1	2	0	2	1
x_4	3	0	2	2	3	1	0	2	3	1	0	2
x_5	2	1	0	2	2	1	3	2	2	1	3	2

Definition 6: Let x_i^j be a CO2PO matrix. The **Normalized Program Outcome (NPO)** of each PO is calculated by the following formula defined by

$$\text{NPO}(k) = \sum_{i=1}^{m=5} \sum_{j=1}^{n=12} \frac{x_i^j}{3\times m} \; ; \; \forall j \text{ , and } k = 1, \cdots, 12$$

Table 4: Normalized CO2PO Matrix

$x_i \times x^j$	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}
x_1	3	2	1	-	-	-	-	-	-	2	-	
x_2	3	2	1	-	-	-	-	-	-	1	-	
x_3	3	1	2	-	-	-	-	-	-	2	-	1
x_4	3	3	3	-	-	-	-	-	-	2	-	2
x_5	3	1	3	-	-	-	-	-	-	1	-	1
NPO	1	0.53	0.67	0	0	0	0	0	0	0.53	0	0.27

3-High, 2- Medium, 1-Low, 0-No association

Normalized program outcomes of a course are shown in table 4.

Definition7: Let Q={Very strong, strong, moderate, poor and very poor} be a set of qualitative terms and μ be a measurable function expressed by μ: $[0,1] \rightarrow Q$ and defined by

$$\mu(x) = \begin{cases} \text{Very strong} & \text{If } x \in (0.80, 1.00] \\ \text{Strong} & \text{If } x \in (0.60, 0.80] \\ \text{Moderate} & \text{If } x \in (0.30, 0.60] \\ \text{Poor} & \text{If } x \in (0.15, 0.30] \\ \text{Very poor} & \text{If } x \in (0.00, 0.15] \end{cases}$$

The function μ will be used to measure the strength of a course how strongly associated the course outcomes to the program outcomes.

Definition 8: The Uniform grading scale is defined on the basis of percentage of total mark is shown in table 5.

Table 5: Uniform grading scale

Uniform Grading System		
Marks Range in Percentage (%) Scale	**Letter Grade (LG)**	**Credit Point (CP)**
80-100	A+	4.00
75-79	A	3.75
70-74	A-	3.50
65-69	B+	3.25
60-64	B	3.00
55-59	B-	2.75
50-54	C+	2.50
45-49	C	2.25
40-44	D	2.00
00-39	F	0

2.1 Different Measure functions

Various functions for quantifying the achievement of course and program outcomes are defined and described in this section.

2.1.1 Measure function for the attainment of course outcomes

Definition 9: Let X denotes the set of mark range in percentage of the uniform grading scale. Let $x \in X$ be any percentage of marks and Y= { 0,1,2,3 } be a set of different attainment levels. Attainment of a course is measured by the function $M_{CO}: X \to Y$ which is defined by

$$M_{CO}(x) = \begin{cases} 3 & \text{If 80\% of students score more than 60-64\%} \\ 2 & \text{If 70\% of students score more than 60-64\%} \\ 1 & \text{If 60\% of students score more than 60-64\%} \\ 0 & \text{Other wise} \end{cases}$$

2.1.2 Measure for overall COs attainment

Definition 10: Let X denotes the set of all letter grade and credit points of the unified grading system and Y={ 0,1,2,3 } be a set of different attainment levels. The attainment measure of course outcomes (COs) after a semester ended examination from a number of course enrolled by students is calculated by the function $M_{ACOs}: X \to Y$ which is defined by

$$M_{ACOs}(x) = \begin{cases} 3 & \text{If 80\% of students obtained overall grade B or above} \\ 2 & \text{If 70\% of students obtained overall grade B or above} \\ 1 & \text{If 60\% of students obtained overall grade B or above} \\ 0 & \text{Other wise} \end{cases}$$

2.1.3 Measuring the attainments of PO

The achievement of COs leads to the achievement of program outcomes. Attainment of program outcomes is measured by the stated course outcomes of a course.

Definition 11: Let X=[0,1] be an interval of all normalized values of program outcomes of any course; and $Y = \{0,1,2,3\}$ be a set of overall course attainment values in an examination. If Z=[0,3] be any interval then the measure function for the attainment of program outcomes is denoted by the function M_{PO} such that

$$M_{PO}: X \times Y \to Z$$

and M_{PO} is defined by

$$M_{PO}(x, y) = xy = z \quad \text{where } x \in X, y \in Y, z \in Z$$

Example: Suppose that Normalized program outcomes of any PO is x=0.75 and the overall attainment of a course of an examination is y=2 then the measure of the attainment of program outcome is $M_{PO} = 0.75 \times 2 = 1.5$.

According to the $M_{PO}(x, y)$ function the if the overall course outcomes of any course is 2 then the measure of program outcomes of that course is shown in table 6 (in this case NPO is taken from Table 5)

Table 6: Measure of Program Outcomes by a course

$M_{PO}(i,j)$	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	M_{ACOs}
Course i	2	1.06	1.34	0	0	0	0	0	0	1.06	0	0.54	2

2.1.4 Measure of overall PO Attainment

After a semester ended examination to know the measure of the overall PO attainment is helpful for the course teachers and program director or chair of the department.

Definition12: Let the matrices $NPO(i,j)$ and $M_{ACOs}(i)$ denotes the normalized program outcomes and overall course outcomes of a number of i courses respectively. The index i,j are indicated for course and normalized program outcomes. A measure function $M_{APOs} : NPO(i,j) \times M_{ACOs}(i) \to [0,1]$ is defined by

$$M_{APOs}(.,j) = \frac{NPO(i,j) \times M_{ACOs}(i)}{N} \text{ where } N = count(i) \text{ if } NPO(i,j) \neq 0$$

The function M_{APOs} will be used to measure the overall program outcomes of a semester.

After successfully completed all semester of a 4-year program it is important to measure inclusive program outcomes of the program.

Definition 13: Let $APO(i,j)$ be a matrix of overall program outcomes of i number of semesters. Again IPO(.,j) a row matrix denotes the inclusive program outcomes of a 4-year program. Then the IPO is calculated by

$$IPO(.,j) = \sum_{j=1}^{12}\left(\frac{1}{N}\sum_{i=1}^{N} APO(i.,j)\right)$$

where N= count(i) that is total number of semesters in a 4-year program.

Defining attainment level of overall or inclusive program outcomes sometimes make easily understandable for general peoples.

Definition 14: Measure of attainment level for overall or inclusive program outcomes is expressed by the function M_{APOs} where M_{APOs}: [0,1] → {0,1,2,3} and is defined by

$$\mathrm{M_{APOs}(x)} = \begin{cases} 3 & \text{If } x \in [0.75, 1] \\ 2 & \text{If } x \in [0.45, 0.75) \\ 1 & \text{If } x \in [0.20, 0.45) \\ 0 & \text{If } x \in [0, 0.20) \end{cases}$$

The overall or inclusive program outcomes of a degree are measured after producing the final result. According the measure functions M_{APOs} Program outcomes attained with level 1,2 and 3 respectively if the values of POs attainment lies in [0.25,0.45), [0.45,0.75) and [0.75, 1] respectively.

2.1 Student assessment to program outcomes attainment

A well-planned evaluation will evaluate a student's achievement while also motivating him to concentrate on his learning activities in order to achieve the Program Learning Outcome. It has two functions: first, it assesses and verifies the Learning Outcome achievement, and second, it directs a student's attention to it [2].

The most of higher education institutions offered semester-based education in Bangladesh. A four-year program in a bi-semester system comprises of eight semesters, while a four-year program in a trimester system consists of twelve semesters. When students advance through the semesters, the assessment method places a greater focus on more complex issues and a deeper understanding of the cognitive domain. The assessment method emphasizes more complex problems and provides insight further into the cognitive domain as students move through the higher semesters. The table 7 depicted the distribution of marks for cognitive domains from 1st to 4th year exams.

Table 7: Cognitive domain year wise mark distribution

	1st Year	2nd Year	3rd Year	4th Year
Remember	15	10	10	5
Understand	25	25	20	10
Apply	35	35	30	30
Analyse	20	20	25	30
Evaluate	5	10	10	15
Create	-		5	10
Total	100	100	100	100

Note that by using course completion survey we can find out indirect COs and POs attainment and its values.

2.2 Student perspective of PO attainment

Students are taught in a variety of ways, including traditional, online, and blended. Students who successfully complete a course would have gained basic and advanced levels of subjective understanding,

certain competitive abilities and skill sets, as well as social and personal values. Students are evaluated mainly by different forms of formative and summative assessments and are graded according to their received grades under the standardized grading policy.

The achievement of a program's COs and POs is contingent upon the students' assessment results, which is referred to as a direct approach to measuring attainment or achievement. Stakeholder perspectives are important to consider when assessing program outcomes. Integrating stakeholders' opinions through a survey is referred to as an indirect method of assessing achievement. The study employs only a direct evaluation approach. Student results and test scores are used to measure students' achievement of course and program outcomes, as well as student achievement of program outcomes that are related to the student's characteristics profile.

Alignment of Assessment to Program Outcomes is illustrated in figure 1:

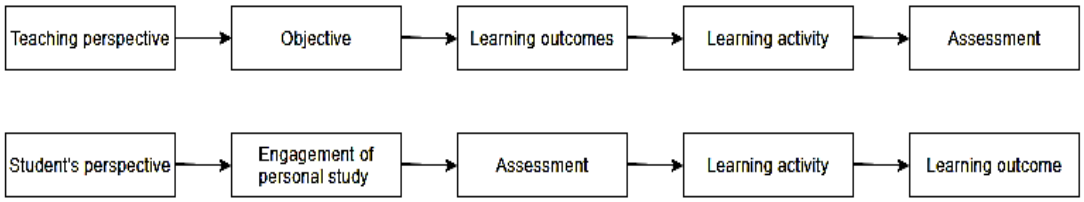

Figure1: Assessment of teachers and students perspective (source: [2])

III. APPLICATION OF THE MATHEMATICAL FRAMEWORK

Suppose a four year program is comprises eight semesters. In any semester there are eight courses of different subjects. The application of the proposed mathematical frame is begin by defining course outcomes to program outcomes of any subject.

3.1 CO2PO Matrix

A course of MAT-1111 defined five course outcomes and the alignment of CO2PO matrix is shown in table 8.

Table 8: CO2PO alignment matrix for the course MAT-1111

MAT-1111	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3	2	1	-	-	-	-	-	-	2	-	-
CO2	3	2	1	-	-	-	-	-	-	1	-	-
CO3	3	1	2	-	-	-	-	-	-	2	-	1
CO4	3	3	3	-	-	-	-	-	-	2	-	2
CO5	3	1	3	-	-	-	-	-	-	1	-	1
NPO(.,j)	1	0.53	0.67	0	0	0	0	0	0	0.53	0	0.27

3.2 Normalized PO Matrix

Every course has the alignment of COs to POs that is the COPO matrix of a course. From the COPO matrix, we can find the normalized values of program outcomes assigned by the course outcomes of each course. Suppose in a program there are eight courses of different kinds. The hypothecated normalized values of courses is calculated and shown in the table 9.

Table 9: COPOs normalized values of all course

NPO **(i,j)**	**PO**	**PO**	**PO**	**PO**	**PO**	**PO**	**PO**	**PO**	**PO**	**PO1**	**PO1**	**PO1**	$\mathbf{M_{ACOs}}$
MAT	1.00	0.53	0.67							0.53		0.27	2
MAT	0.8	0.75	0.5		0.45	0.50							2
MAT				0.8	0.75	0.5		0.45	0.50				3
STA 1141	0.80	0.67					0.75			0.63	0.67	0.35	3
GED 1151	0.35					1.00	0.53	0.67		0.53		0.40	2
CSE 1161	0.87				0.67				0.50	0.45		0.40	3
CSE 1162	0.40		0.87	0.53	1							0.37	3

MAT	0.65	0.75	0.67						0.75	0.85			3

3.3 Overall PO attainment from a semester

The hypothecated normalized values of program outcomes of all courses in a semester is calculated by using the overall course outcomes and shown in the table 10.

Table 10: PO attainment by the courses of a semester

MPO(j)	**PO1**	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PO8**	**PO9**	**PO10**	**PO11**	**PO12**	CO*
MAT 1111	2	1.06	1.34	0	0	0	0	0	0	1.06	0	0.54	2
MAT 1121	1.6	1.5	1	0	0.9	1	0	0	0	0	0	0	2
MAT 1131	0	0	0	2.4	2.25	1.5	0	1.35	1.5	0	0	0	3
STA 1141	2.4	2.01	0	0	0	0	2.25	0	0	1.89	2.01	1.05	3
GED 1151	0.7	0	0	0	0	2	1.06	1.34	0	1.06	0	0.8	2
CSE 1161	2.61	0	0	0	2.01	0	0	0	1.5	1.35	0	1.2	3
CSE 1162	1.2	0	2.61	1.59	3	0	0	0	0	0	0	1.11	3
MAT 1170	1.95	2.25	2.01	0	0	0	0	0	2.25	2.55	0	0	3
Overall	**0.59**	**0.57**	**0.58**	**0.67**	**0.68**	**0.50**	**0.55**	**0.45**	**0.58**	**0.53**	**0.67**	**0.31**	

3.4 Inclusive PO attainment from a four year program

The attainment of program outcomes for each semester can measure accordingly. If the program outcomes are measured and stored in a systematic way then inclusive program outcomes from a 4-year program can be measured and the resultant IPO is shown in table 11.

Table 11: POs attainment from all semesters

Semester	**PO1**	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PO8**	**PO9**	**PO10**	**PO11**	**PO12**
Semester 1	0.59	0.57	0.58	0.67	0.68	0.5	0.55	0.45	0.58	0.53	0.67	0.31
Semester 2	0.59	0.22	0.46	0.97	0.91	0.82	0.59	0.19	0.84	0.57	0.06	0.40
Semester 3	0.35	0.09	0.39	0.77	0.58	0.45	0.58	0.10	0.96	0.15	0.12	0.58
Semester 4	0.63	0.07	0.32	0.03	0.16	0.41	1.00	0.82	0.55	0.09	0.87	0.26
Semester 5	0.82	0.82	0.57	0.94	0.12	0.97	0.14	0.08	0.49	0.30	0.03	0.07
Semester 6	0.23	0.78	0.71	0.35	0.38	0.34	0.64	0.99	0.31	0.68	0.06	0.99
Semester 7	0.97	0.95	0.28	0.43	0.41	0.12	0.24	0.01	0.78	0.12	0.25	0.29
Semester 8	0.83	0.26	0.49	0.77	0.37	0.13	0.66	0.34	0.49	0.43	0.88	0.83
POs Attainment	**0.63**	**0.47**	**0.48**	**0.62**	**0.45**	**0.47**	**0.55**	**0.37**	**0.63**	**0.36**	**0.37**	**0.47**
POs Attainment Levels	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**1**	**2**	**1**	**1**	**2**

The program attainment level is shown the last row of the table 11. The values of inclusive attainment of program outcomes generate the POs attainment levels. In this case $M_{APOs}(x)$ function is applied to find POs attainment levels.

IV. CONCLUSION

This paper has proposed mathematical framework for the components of outcome based education. A program need to be assessed by internal and external bodies of expertize for getting accredited from national and international accreditation authority. This paper used normalized program outcomes to determine the strength of course outcomes for a course. CO2PO alignment matrix is a foundation to make a program strengthen. Students are assessed continuously by formative examination and finally by taking summative examination, mostly known as semester ended examination. Students' exam records are used to determine overall course outcomes of a course. To determine the attainment of assigned program outcomes of a course the normalized program outcomes and the value of overall courses outcome are used. The overall program outcomes are determined using the function $M_{APOs}(.,j)$ after completing a semester-end examination. Finally, determine the inclusive curriculum outcomes and program attainment level after conducting all of the examinations for a 4-year program.

ACKNOWLEDGEMENTS

The authors are thankful to Bangamata Sheikh Fozilatunnesa Mujib Science and Technology University for providing funding and a research grant.

REFERENCES

[1] B. Soragaon and K. Mahesh, "Measuring Attainment of Course Outcomes and Program Outcomes–A Simplified Approach as per Self-Assessment Report-June 2015," *IOSR Journal of Research & Method in Education*, vol. 6, pp. 13-18, 2016.

[2] R. A. A. O. Rahmat, "Achievement of Program Outcomes Using Assessment Plan," *Procedia-Social and Behavioral Sciences*, vol. 18, pp. 87-93, 2011.

[3] D. R. PARIMALADEVI, "AN EMPIRICAL STUDY ON ASSESSMENT AND ATTAINMENT OF COURSE OUTCOME AND PROGRAMME OUTCOME: A STUDENT PERCEPTIVE," *IJRAR-International Journal of Research and Analytical Reviews (IJRAR)*, vol. 7, pp. 26-32-26-32, 2020.

[4] Y. Liu and X. Zhang, "Evaluating the Undergraduate Course based on a Fuzzy AHP-FIS Model," *International Journal of Modern Education & Computer Science*, vol. 12, 2020.

[5] A. Rajak, A. K. Shrivastava, S. Bhardwaj, and A. K. Tripathi, "Assessment and attainment of program educational objectives for post graduate courses," *International Journal of Modern Education & Computer Science*, vol. 2, pp. 26-32, 2019.

[6] S. Yassine, S. Kadry, and M.-A. Sicilia, "Measuring learning outcomes effectively in smart learning environments," in *2016 Smart Solutions for Future Cities*, 2016, pp. 1-5.

[7] O. L. Liu, B. Bridgeman, and R. M. Adler, "Measuring learning outcomes in higher education: Motivation matters," *Educational Researcher*, vol. 41, pp. 352-362, 2012.

[8] M. Vijaya and M. Arthi, "Using Fuzzy Logic Reasoning Approach in Fuzy Decision Tree to Evaluate Students Performance," *International Journal of Applied Engineering Research*, vol. 14, pp. 384-389, 2019.

[9] A. Barlybayev, A. Sharipbay, G. Ulyukova, T. Sabyrov, and B. Kuzenbayev, "Student's performance evaluation by fuzzy logic," *Procedia Computer Science*, vol. 102, pp. 98-105, 2016.

[10] Z. Yıldız and A. F. Baba, "Evaluation of student performance in laboratory applications using fuzzy decision support system model," in *2014 IEEE Global Engineering Education Conference (EDUCON)*, 2014, pp. 1023-1027.

[11] N. Cavus, "The evaluation of Learning Management Systems using an artificial intelligence fuzzy logic algorithm," *Advances in Engineering Software*, vol. 41, pp. 248-254, 2010.

[12] S. H. J. Petrudi, M. Pirouz, and B. Pirouz, "Application of fuzzy logic for performance evaluation of academic students," in *2013 13th Iranian Conference on Fuzzy Systems (IFSC)*, 2013, pp. 1-5.

[13] R. C. McNeil, "A Program Evaluation Model: Using Bloom's Taxonomy to Identify Outcome Indicators in Outcomes-Based Program Evaluations," *Journal of adult education*, vol. 40, pp. 24-29, 2011.