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Research Paper

Dynamic Analysis of Orthotropic Micropolar Substrate Under Moving Load

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Abstract: *The present article is aimed to describe the stresses produced in a regular orthotropic micropolar substrate due to moving load. The closed form expressions of normal stress, shear stress and tangential couple stress have been obtained. Effect of moving load on isotropic micropolar substrate has also been deduced as particular case of the problem. The present theory is very effective in inquiring composite ingredients, masonary, boxes, liquid crystals and ingredients which are made up of box-like particle that has the ability to show the microrotation effect and that can sustain body and surface couples.*

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I. INTRODUCTION

Micropolar elasticity theory is the extension of classical theory of elasticity which distinguishes from the latter one in terms of features and concepts. In classical theory, a material particle, within a medium, omits the translation degree of freedom whereas a micropolar elastic material has extra degree of freedom which is irrespective of translation. Microstructure of the material depicts significant effects in the cases of elastic vibrations of higher frequencies and small wavelengths which classical theory failed to explain. Micropolar elastic theory has eliminated the drawback of classical continuum mechanics by asserting that the interaction between two points is transmitted by couple stresses besides the classical force stresses. For this purpose transmission is proposed between two particles of a material through an area element not only by the action of force vector but also by the moment vector which defines the couples stress in an elastic medium.

The classical theory is lacking behind from micropolar concept of elasticity in terms of its usefulness in examining the deformation properties of solids. Whereas, the materials comprising bar-like molecules which demonstrates microrotation effects and also support body as well as surface couples can be examined by micropolar theory.

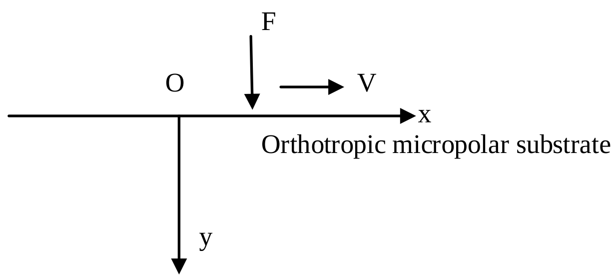
The concept of orthotropic micropolar elasticity attracted many researchers due to its attainable utilization in deformation properties of material which was not possible by the classical theory of elasticity. Wave propagation in orthotropic micropolar elastic soled was studied by Singh (2007). Sachio et at (1984) introduced a finite element method for orthotropic micropolar solids. Steady state response of moving load in micropolar elastic medium with stretch has been investigated by Kumar and Gogna (1972). Response of various sources on orthotropic micropolar elastic medium was analysed by Kumar and Choudhary (2003).

The response of a moving load over a surface is a subject of investigation because of its possible practical application in determining the strength of a structure. Cole and Huth (1958) obtained the steady state solution of the problem of moving load over an elastic half space. Mukherjee (1969) has studied the stresses developed in a transversely isotropic elastic half-space due to normal moving load over a rough surface. The problem of moving load on a plate resting on a layered half space has been solved by Sackman (1961) and Miles (1966). Some notable work concerned with the problem of moving load on an elastic half-space has been done by Achenbach et al. (1967), Olsson (1991), Lee and Ng (1994), Alkeseyeva (2007) etc. The problem of a normal load over a transversely isotropic layer lying on rigid foundation is investigated by Mukhopadhyay (1965) whereas Selim (2007) discussed the static deformation of an irregular initially stressed medium. He used the Eigen value approach to solve the problem. The dynamic response of a normal moving load in the plane of symmetry of a monoclinic half-space was studied by Chattopadhyay et al. (2006). Chattopadhyay et al. (2011) have also studied the stress on a rough irregular isotopic half-space due to normal moving load. Effect of irregularity and heterogeneity on the stresses produced due to a normal moving load on a rough monoclinic half-space has been studied by Singhet al. (2014). The response of moving load on a micropolar half-space with irregularity is investigated by Kaur et al. (2015).Kaur et al. (2016) discussed the dynamic response of normal moving load on an irregular fiber-reinforced half-space.

The present problem is concerned with the effect of moving load in regular orthotropic micropolar substrate. The present study is useful to the seismologists and geophysicists for understanding and predicting the behavior of earth medium at different margins. The closed form expressions for the normal stress, shear stress and tangential couple stress produced at any point of orthotropic micropolar substrate due to moving load are obtained. The effect of friction of the rough surface has been studied by introducing frictional coefficient (R). The response of moving load on an isotropic micropolar substrate has been discussed as particular case of the study.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

We consider a model which consists of a regular orthotropic micropolar substrate under the influence of normal moving load which is moving with constant velocity V in the direction of x-axis and independent of y-axis.



For two-dimensional problem, we assumed a plain strain parallel to xy-plane, with displacement vector $v = (v_1, v_2, 0)$ and microrotation vector $(0,0,\varphi_3)$.

The basic governing equations for homogeneous and orthotropic micropolar substrate in the absence of body forces and couples are given by (Eringen (1966) and Insan (1974)) :

$$A_{11}v_{1,11} + (A_{12} + A_{78})v_{2,12} + A_{88}v_{1,22} - K_1\varphi_{3,2} = \rho\, \ddot{v}_1 \quad , \qquad \text{.... (1)}$$

$$(A_{12} + A_{78})v_{1,12} + A_{77}v_{2,11} + A_{22}v_{2,22} - K_2\varphi_{3,1} = \rho\, \ddot{v}_2 \quad , \qquad \text{.... (2)}$$

$$B_{66}\varphi_{3,11} + B_{44}\varphi_{3,22} + \chi\varphi_3 + K_1 v_{1,2} + K_2 v_{2,1} = \rho\, j\, \ddot{\varphi}_3 \quad , \qquad \text{.... (3)}$$

where A_{11}, A_{12}, A_{22}, A_{77}, A_{88}, B_{44} and B_{66} are material constants. ρ is the density of the material and j is the micro inertia.

Here

$$\chi = K_2 - K_1, \qquad K_1 = A_{78} - A_{88}, \qquad K_2 = A_{77} - A_{78}$$

The normal stress, shear stress and tangential couple stress are given by

$$\left.\begin{aligned} \sigma_{yy} &= A_{12}\frac{\partial v_1}{\partial x} + A_{22}\frac{\partial v_1}{\partial y} \\ \sigma_{xy} &= A_{78}\frac{\partial v_2}{\partial x} + A_{88}\frac{\partial v_1}{\partial y} + (A_{88} - A_{78})\varphi_3 \\ m_{xz} &= B_{66}\frac{\partial \varphi_3}{\partial x} \end{aligned}\right\} \qquad \text{.... (4)}$$

The boundary conditions at $y = 0$ may be written as:

$$\sigma_{yy} = -F\delta(x - Vt)\,, \qquad \sigma_{xy} = -FR\delta(x - Vt) \quad \text{and} \quad m_{xz} = 0 \qquad \text{.... (5)}$$

where $$\delta(x - Vt) = \frac{1}{\pi}\int_0^\infty \cos k(x - Vt)dk,$$

and k =wave number, t= time, R=frictional coefficeient

III. SOLUTION OF THE PROBLEM

The solution of the equations (1), (2) and (3) may be assumed as:

$$v_1 = \int_0^\infty [A\, e^{-kqy}\ cosk(x - Vt) + B\, e^{-kqy} \sin k(x - Vt)]\, dk\,, \qquad \text{.... (6)}$$

$$v_2 = \int_0^\infty [C\, e^{-kqy}\ cosk(x - Vt) + D\, e^{-kqy} \sin k(x - Vt)]\, dk\,, \qquad \text{.... (7)}$$

$$\varphi_2 = \int_0^\infty k[E\, e^{-kqy}\ cosk(x - Vt) + F\, e^{-kqy} \sin k(x - Vt)]\, dk \quad , \qquad \text{.... (8)}$$

where k is the wave number, q is the unknown positive real number independent of k and A, B, C, D, E and F are arbitrary constants.
Using solutions given in equations (6), (7) and (8) in the equation of motion (1), (2) and (3), we have

$$\beta_1 A - \beta_2 D + K_1 E q = 0 \quad , \qquad \ldots (9)$$
$$\beta_1 B - \beta_2 C + K_1 F q = 0 \ , \qquad \ldots (10)$$
$$\beta_2 A + \beta_3 D + K_2 E = 0 \ , \qquad \ldots (11)$$
$$-\beta_2 B + \beta_2 C - K_1 F = 0, \qquad \ldots (12)$$
$$\beta_4 E = 0 \quad , \qquad \ldots (13)$$
$$\beta_4 F = 0 \quad , \qquad \ldots (14)$$
$$-\chi E - K_1 A q + K_2 D = 0 \ , \qquad \ldots (15)$$
$$-\chi F - K_1 B q - K_2 C = 0 \ , \qquad \ldots (16)$$

where
$$\beta_1 = (-A_{11} + A_{88}\, q^2 + \rho\, V^2) \ ,$$
$$\beta_2 = (A_{12} + A_{78})\, q \ ,$$
$$\beta_3 = (-A_{77} + A_{22}\, q^2 + \rho\, V^2) \ ,$$
$$\beta_4 = (-B_{66} + B_{44}\, q^2 + \rho\, j\, V^2) \ ,$$
$$\beta_5 = \frac{1}{\rho j} \ ,$$
$$V^2 = (B_{66} - B_{44}\, q^2)\, \beta_5 \ ,$$

The non-trivial solution of above equations gives the following equation
$$A' q^4 + B' q^2 + C' = 0 \qquad \ldots (17)$$
where
$$A' = \{A_{22}A_{88} - \rho\beta_4\, B_{44}A_{22} - \rho B_{44}\beta_5 A_{88} + \rho^2\beta_5^2 B_{44}^2\}\,\chi^2 - K_1^2\chi A_{22} + K_1^2\chi\rho\beta_5 B_{44} \ ,$$
$$B' = \{A_{77}A_{88} + \rho\beta_5 B_{44}A_{77} - A_{11}A_{22} + \rho\beta_5 A_{22}B_{66} + A_{11}\rho\beta_5 B_{44} + A_{88}\rho\beta_5 B_{66} - 2\rho^2\beta_5^2 B_{44}B_{66}\}\chi^2 + K_1^2\chi A_{77} - K_1^2\chi\rho\beta_5 B_{66} + \chi\, K_2^2\, A_{88} - \chi\,\rho\,\beta_5 K_2^2 B_{44} + (A_{12} + A_{78})^2\,\chi^2 - 2\chi K_1 K_2\,(A_{12} + A_{78})] \ ,$$
$$C' = [\{A_{11}A_{77} - \rho\,\beta_5\, A_{77}\, B_{66} - \rho\,\beta_5\, A_{11}B_{66}\}\,\chi^2 - A_{11}\chi\, K_2^2 + \chi K_2^2\rho\beta_5 B_{66}] \ ,$$

Let q_1^2 and q_2^2 be the two roots of equations (17), we get
$$q_i^2 = \frac{-B' \pm \sqrt{B'^2 - 4A'C'}}{2A'}, \qquad i = 1,2 \ldots. \qquad \ldots (18)$$

In view of (18), equations (6), (7) and (8) can be written as:

$$v_1 = \int_0^\infty [\{A_1\, e^{-kq_1 y} + A_2\, e^{-kq_2 y}\}\, cos k(x - Vt) + \{B_1\, e^{-kq_1 y} + B_2\, e^{-kq_2 y}\}\, sin\, k(x - Vt)]dk \qquad \ldots (19)$$

$$v_2 = \int_0^\infty [\{-B_1\alpha_1 e^{-kq_1 y} - B_2\alpha_2 e^{-kq_2 y}\}\, cos k(x - Vt) + \{A_1\alpha_1 e^{-kq_1 y} + A_2\alpha_2 e^{-kq_2 y}\}\, sin k(x - Vt)]dk \qquad \ldots (20)$$

$$\varphi_3 = \int_0^\infty k[\{\frac{A_1(K_2\alpha_1 - K_1 q_1)}{\lambda} e^{-kq_1 y} + \frac{A_2(K_2\alpha_2 - K_1 q_2)}{\lambda} e^{-kq_2 y}\} cos k(x - Vt) + \left\{\frac{B_1(K_2\alpha_1 - K_1 q_1)}{\lambda} e^{-kq_1 y} + \frac{A_2(K_2\alpha_2 - K_1 q_2)}{\lambda} e^{-kq_2 y}\right\} sin k(x - Vt)]dk \qquad \ldots (21)$$

where
$$\alpha_1 = \frac{\beta_1\chi - K_1^2 q_1^2}{\beta_1\chi - K_1 K_2 q_1}, \qquad \alpha_2 = \frac{-(\beta_2\chi - K_1 K_2 q_2)}{\beta_3\chi + k^2},$$

Applying boundary conditions (5) on (19), (20) and (21), we get
$$\xi_1 A_1 + \xi_2 A_2 = 0 \quad , \qquad \ldots (22)$$
$$\xi_1 B + \xi_2 B_2 = \frac{F}{\pi k} \quad , \qquad \ldots (23)$$
$$\xi_3 B_1 + \xi_4 B_2 = 0 \ , \qquad \ldots (24)$$
$$\xi_3 A_1 + \xi_4 A_2 = \frac{FR}{\pi k} \quad , \qquad \ldots (25)$$
$$T_1 A_1 + T_2 A_2 = 0 \ , \qquad \ldots (26)$$
$$T_1 B_1 + T_2 B_2 = 0 \ , \qquad \ldots (27)$$

where
$$\xi_1 = A_{12} + A_{22}\alpha_1 q_1 \ ,$$

$$\xi_2 = A_{12} + A_{22}\alpha_2 q_2 ,$$
$$\xi_3 = A_{78}\alpha_1 - A_{88}q_1 + (A_{88} - A_{78})T_1 ,$$
$$\xi_4 = A_{78}\alpha_2 - A_{88}q_2 + (A_{88} - A_{78})T_2 ,$$

$$T_1 = \frac{K_2\alpha_1 - K_2 q_1}{\chi}, \qquad T_2 = \frac{K_2\alpha_2 - K_1 q_2}{\chi},$$

Solving above equations, we get

$$A_1 = \frac{FR\,\xi_2}{k\pi D}, \quad A_2 = \frac{-FR\,\xi_1}{k\pi D}, \quad B_1 = \frac{-F\,\xi_4}{k\pi D}, \quad B_2 = \frac{F\,\xi_3}{k\pi D}, \qquad \text{…. (28)}$$

where $\quad D = \xi_1\xi_4 - \xi_2\xi_3$

With the help of obtained values of arbitrary constants and displacement components, we get non-vanishing natural stress, shear stress and tangential couple stress as follows;

$$\frac{\sigma_{yy}}{F} = \left[\frac{R(x-Vt)}{\pi D}\xi_1\xi_2\left(\frac{1}{\psi_2} - \frac{1}{\psi_1}\right) + \frac{1}{\pi D}\left(\frac{\xi_2\xi_3 q_2 y}{\psi_2} - \frac{\xi_1\xi_4 q_1 y}{\psi_1}\right)\right], \qquad \text{…. (29)}$$

$$\frac{\sigma_{xy}}{F} = \left[\frac{-\xi_3\xi_4(x-Vt)}{\pi D\psi_1} + \frac{\xi_3\xi_4(x-Vt)}{\pi D\psi_2} + \frac{R\xi_2\xi_3 q_1 y}{\pi D\psi_1} - \frac{R\xi_1\xi_4 q_2 y}{\pi D\psi_2}\right], \qquad \text{…. (30)}$$

and

$$\frac{m_{xz}}{F} = B_{66}\left[\frac{-2Ry(x-Vt)\xi_2 T_1 q_1}{\pi D\psi_1^2} + \frac{2Ry(x-Vt)\xi_1 T_2 q_2}{\pi D\psi_2^2} - \frac{\xi_4 T_1\psi_3}{\pi D\psi_1^2} + \frac{\xi_3 T_2\psi_4}{\pi D\psi_2^2}\right], \qquad \text{…. (31)}$$

where
$$\psi_1 = [(x-Vt)^2 + q_1^2 y^2],$$
$$\psi_2 = [(x-Vt)^2 + q_2^2 y^2],$$
$$\psi_3 = [q_1^2 y^2 - (x-Vt)^2],$$
$$\psi_4 = [q_2^2 y^2 - (x-Vt)^2]$$

IV. PARTICULAR CASE

In the absence of orthotropic effect, the expressions (29), (30) and (31) reduces to

$$\frac{\sigma_{yy}}{F} = \frac{R(x-Vt)\xi_1'\xi_2'}{\pi D}\left(\frac{1}{\psi_2} - \frac{1}{\psi_1}\right) + \frac{1}{\pi D'}\left(\frac{\xi_2'\xi_3' q_2 y}{\psi_2} - \frac{\xi_1'\xi_4' q_1 y}{\psi_1}\right), \qquad \text{…. (32)}$$

$$\frac{\sigma_{xy}}{F} = \left[\frac{-\xi_3'\xi_4'(x-Vt)}{\pi D'\psi_1} + \frac{\xi_3'\xi_4'(x-Vt)}{\pi D'\psi_2} + \frac{R\xi_2'\xi_3' q_1 y}{\pi D'\psi_1} - \frac{R\xi_1'\xi_4' q_2 y}{\pi D'\psi_2}\right], \qquad \text{…. (33)}$$

$$\frac{m_{xz}}{F} = \frac{\gamma}{\pi}\left[\left(\frac{-2\xi_2' T_1 q_1}{\psi_1^2} + \frac{2\xi_1' T_2 q_2}{\psi_2^2}\right)\frac{Ry(x-Vt)}{D'} - \frac{\xi_4' T_1\psi_3}{D'\psi_1^2} + \frac{\xi_3' T_2\psi_4}{D'\psi_2^2}\right], \qquad \text{…. (34)}$$

where
$$\xi_1' = \{\lambda + (\lambda + 2\mu + K)\alpha_1 q_1\}$$
$$\xi_2' = \{\lambda + (\lambda + 2\mu + K)\alpha_2 q_2\}$$
$$\xi_3' = \{\mu\alpha_1 - (\mu + K)q_1 + KT_1\}$$
$$\xi_4' = \{\mu\alpha_2 - (\mu + K)q_2 + KT_2\}$$
$$D' = \xi_1'\xi_4' - \xi_2'\xi_3'$$

Expressions (32), (33) and (34) give normal stress, shear stress and tangential couple stress produced in regular isotropic micropolar substrate.

V. CONCLUSION

The present study investigates the response of moving load on an orthotropic micropolar substrate. The expressions of normal stress, shear stress and tangential couple stress are derived in closed form. As a particular case of the problem, effect of moving load on isotropic micropolar substrate has also been discussed. The analysis of moving load plays a vital role in the field of transportation. For example, the structural elements which support the moving masses like bridges, overhead cranes, railway, runways, pipelines etc. are influenced by moving loads.

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