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Research Paper

Some Weaker Forms of Continuous Functions in Topological Spaces

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Abstract - In this paper we introduce a new class of continuous functions called $sg\omega\alpha$- continuous functions and $sg\omega\alpha$- irresolute maps in topological spaces and studied some of their properties. Further, we also study the concept of quasi-$sg\omega\alpha$-open and quasi $sg\omega\alpha$-closed function and their properties. Finally, a new class of closed graphs called $sg\omega\alpha$-closed graphs and related properties are studied in topological spaces.

Keywords - *$sg\omega\alpha$-closed sets, $sg\omega\alpha$-continuous functions, $sg\omega\alpha$-irresolute maps, $sg\omega\alpha$-closed maps.*

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I. INTRODUCTION

Continuous functions stands among the most fundamental point in the whole of the Mathematical Science. Different forms of stronger and weaker forms of functions have been introduced over the years. Balachandran et al [1] introduced the concept of generalized continuous maps and generalized irresolute maps in topological spaces and Benchalli et al [2], [3], [5] introduced and studied the concept of $\omega\alpha$-closed sets, $\omega\alpha$-continuous maps and $g\omega\alpha$-continuous maps in topological spaces.
Recently Rajeshwari K. and T.D.Rayanagoudar [16] introduced the concept of semi generalized $\omega\alpha$-closed (briefly $sg\omega\alpha$-closed) sets in topological spaces.

In this paper, authors introduce the concept of $sg\omega\alpha$-continuous functions and $sg\omega\alpha$-irresolute maps in topological spaces. Further, we also introduce quasi $sg\omega\alpha$-open, quasi $sg\omega\alpha$-closed functions and $sg\omega\alpha$-closed graphs in topological spaces.

II. PRELIMINARY

Throughout this paper spaces (X, τ) and (Y, σ) (or simply X and Y) always denote topological spaces on which no separation axioms are assumed unless explicitly stated.

Definition 2.1. *A subset A of a topological space X is called a*
(i) semi-open [8] if $A\subseteq cl(int(A))$ and semi-closed if $int(cl(A))\subseteq A$.
(iii) α-open [13] if $A\subseteq int(cl(int(A)))$ and α-closed if $cl(int(cl(A)))\subseteq A$.

Definition 2.2. *[16] A subset A of a topological space X is called a $T_{sg\omega\alpha}$-space if every $sg\omega\alpha$-closed set is closed.*

Definition 2.3. *Let X be a topological space. A subset A of X is said to be*
(i) g-closed [9] (respectively αg-closed [6]) if $cl(A) \subseteq U$ (respectively $\alpha cl(A) \subseteq U$) whenever $A \subseteq U$ and U is open in X.
(iii) ω-closed [17] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.
(iv) $\omega\alpha$-closed [2] ($g\omega\alpha$-closed [4]) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ω-open

($\omega\alpha$-open) in X.
(v) $g^*\omega\alpha$-closed [14] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\omega\alpha$-open in X.
(vi) $sg\omega\alpha$-closed [16] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\omega\alpha$-open in X.

Definition 2.4. *A function $f : X \to Y$ is called*
(i) g-continuous [1] (respectively α-continuous [12], ω-continuous [17], αg-continuous [6]) if $f^{-1}(G)$ is g-closed (respectively α-closed, ω-closed, αg-closed) set in X for every closed set G of Y.
(ii) g-closed [11] (respectively ω-closed [17], αg-closed [7]) if f(G) is g-closed (respectively ω-closed, αg-closed) in Y for every closed set G in X.
(iii) $\omega\alpha$-closed [3] ($g\omega\alpha$-closed [5]) if f(G) is $\omega\alpha$-closed ($g\omega\alpha$-closed) for every closed set G in X.
(iv) $g\omega\alpha$-continuous [5] (resp. $g^*\omega\alpha$-continuous [15]) if $f^{-1}(G)$ is $g^*\omega\alpha$-closed set in X for every closed set G of Y.
(v) ω-irresolute [17] ($\omega\alpha$-irresolute [3]) if $f^{-1}(G)$ is ω-closed ($\omega\alpha$-closed) in X for each ω-closed ($\omega\alpha$-closed) set G of Y.

Definition 2.5. *[16] If A is $sg\omega\alpha$-closed, then $sg\omega\alpha$-cl(A) = A.*
If A is $sg\omega\alpha$-open then $sg\omega\alpha$-int(A) = A.

Definition 2.6. *[16] Let $x \in X$ and $V \subset X$, then V is called $sg\omega\alpha$-neighborhood of x in X if there exists $sg\omega\alpha$-open set U of X such that $x \in U \subseteq V$.*

Theorem 2.7. *[16] Let A be a subset of X. Then $x \in sg\omega\alpha cl(A)$ if and only if for any $sg\omega\alpha$-nbd N_x of x in X such that $N_x \cap A \neq \phi$.*

Definition 2.8. *[10] Let $f : X \to Y$ be a function. Then*
(i) the subset { (x, f(x)) : $x \in X$ } of the product space $X \times Y$ is called the graph of f and is denoted by G(f).
(ii) a closed graph, if its graph G(f) is closed set in the product space $X \times Y$.

Lemma 2.9. *[10] A function $f : X \to Y$ has a closed graph if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exists $U \in O(X, x)$ and $V \in O(Y, y)$ such that $f(U) \cap V = \phi$.*

3 $sg\omega\alpha$ - Continuous Functions in Topological Spaces

In this section, we introduce the concept of semi generalized $\omega\alpha$-continuous (briefly $sg\omega\alpha$-continuous) functions in topological spaces and study their properties.

Definition 3.1. *A function $f : X \to Y$ is called $sg\omega\alpha$-continuous if the inverse image of every closed set in Y is $sg\omega\alpha$-closed in X.*

Example 3.2. *Let $X = Y = \{1, 2, 3\}$, $\tau = \{X, \tau, \{1\}, \{2, 3\}\}$ and $\sigma = \{Y, \phi, \{2\}, \{1, 3\}\}$. Define a function $f : X \to Y$ by f(1) = 3, f(2) = 1 and f(3) = 2. Then f is $sg\omega\alpha$-continuous.*

Theorem 3.3. *Every continuous function is $sg\omega\alpha$-continuous function.*
However the converse of the above theorem need not be true as seen from the following example.

Example 3.4. *Let $X = Y = \{1, 2, 3\}$, $\tau = \{X, \phi, \{1\}, \{2\}, \{1, 2\}\}$ and $\sigma = \{Y, \phi, \{2\}, \{1, 3\}\}$. Let $f : X \to Y$ be the identity function. Then f is $sg\omega\alpha$-continuous but not a continuous, since for the open set $\{2\}$ in Y, $f^{-1}(\{2\}) = \{2\}$ is not open in X but it is $sg\omega\alpha$-open in X.*

Remark 3.6. *If the function $f : X \to Y$ is α-continuous, ω-continuous, $g\omega\alpha$-continuous, $g^*\omega\alpha$-continuous and αg-continuous then f is $sg\omega\alpha$-continuous, but the converse is not true in general as seen from the following examples.*

Example 3.7. *In the Example 3.4, the function f is $sg\omega\alpha$-continuous but not α-continuous, $\omega\alpha$-continuous, $g\omega\alpha$-continuous, $g^*\omega\alpha$-continuous and αg-continuous for a closed set $\{2\}$ in Y, $f^{-1}(\{2\}) = \{2\}$ is $sg\omega\alpha$-closed but not α-closed, ω-closed , $g\omega\alpha$-closed, $g^*\omega\alpha$-closed and αg-closed in X.*

Theorem 3.8. *A function $f : X \to Y$ is $sg\omega\alpha$-continuous if and only if the inverse image of every open set in Y is $sg\omega\alpha$-open in X.*
Proof: *The proof is obvious.*

Theorem 3.9. *The composition of $sg\omega\alpha$-continuous functions need not be $sg\omega\alpha$-continuous as seen from the following example.*

Example 3.10. *$X = Y = Z = \{ 1, 2, 3 \}$, $\tau = \{ X, \phi, \{ 1 \}, \{ 1, 2 \}\}$, $\sigma = \{ Y, \phi, \{ 1, 2 \}\}$ and $\eta = \{ Z, \phi, \{ 1 \}\}$. Let $f : X \to Y$ be the identity function and define $g : Y \to Z$ by $g(1)=2$, $g(2)=1$ and $g(3)=3$. Then f and g are $sg\omega\alpha$-continuous functions but $gof : X \to Z$ is not $sg\omega\alpha$-continuous, since for the closed set $\{ 2, 3 \}$ in Z, $(gof)^{-1}(\{ 2, 3 \}) = f^{-1}(g^{-1}(\{ 2, 3 \})) = f^{-1}(\{ 1, 3 \}) = \{ 1, 3 \}$ is not $sg\omega\alpha$-closed set in X.*

Theorem 3.11. *Following statements are equivalent for the function $f : X \to Y$:*
(i) f is $sg\omega\alpha$-continuous.
(ii) the inverse image of each open set in Y is $sg\omega\alpha$-open in X.
(iii) the inverse image of each closed set in Y is $sg\omega\alpha$-closed in X.
(iv) for each x in X, the inverse image of every neighborhood of $f(x)$ is a $sg\omega\alpha$-neighborhood of x.
(v) for each x in X and each neighborhood N of $f(x)$ there is a $sg\omega\alpha$-neighborhood W of x such that $f(W) \subseteq N$.
(vi) for each $A \subset X$, $f(sg\omega\alpha cl(A)) \subseteq cl(f(A))$.
(vii) for each $B \subset Y$, $sg\omega\alpha cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$.

Proof: *(i) $\to$ (ii) Follows from the theorem 3.8.*
(ii) $\to$ (iii) Follows from the result $f^{-1}(A^c) = (f^{-1}(A))^c$.
(ii) $\to$ (iv) Let $x \in X$ and let N be a neighborhood of $f(x)$. Then there exists $V \in O(Y)$ such that $f(x) \in V \subseteq N$. Consequently $f^{-1}(V) \in sg\omega\alpha$-$O(X)$ and $x \in f^{-1}(V) \subseteq f^{-1}(N)$. Thus $f^{-1}(N)$ is $sg\omega\alpha$ neighborhood of $f(x)$.
(iv) $\to$ (v) Let $x \in X$ and let N be a neighborhood of $f(x)$. Then by assumption $W = f^{-1}(N)$ is a $sg\omega\alpha$ neighborhood of x and $f(W) = f(f^{-1}(N)) \subseteq N$.
(v) $\to$ (vi) Let $y \in f(sg\omega\alpha$-$cl(A))$ and let N be any neighborhood of y. Then $\exists\ x \in X$ and a $sg\omega\alpha$ neighborhood W of x such that $f(x) = y$, $x \in W$. Hence $x \in sg\omega\alpha$-$cl(A)$ and $f(W) \subseteq N$. By theorem 2.7, $W \cap A \neq \phi$ and hence $f(A) \cap N \neq \phi$. Thus $y \in f(x) \in cl(f(A))$. Therefore $f(sg\omega\alpha$-$cl(A)) \subseteq cl(f(A))$.
(vi) $\to$ (vii) Let $B \subset Y$. Then replacing A by $f^{-1}(B)$ in (vi), we obtain $f(sg\omega\alpha$-$cl(f^{-1}(B)) \subseteq cl(f(f^{-1}(B))) \subseteq cl(B)$. That is $sg\omega\alpha$-$cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$.
(vii) $\to$ (i) Let $G \in O(Y)$, then $Y \setminus G \in C(Y)$. Therefore, $f^{-1}(Y - \setminus G) = f^{-1}(cl(Y \setminus G)) \subseteq sg\omega\alpha$-$cl(f^{-1}(Y \setminus G)) = X \setminus (sg\omega\alpha$-$int(f^{-1}(G))$. This implies that $sg\omega\alpha$-$int(f^{-1}(G)) \subseteq X \setminus f^{-1}(Y \setminus G)) = f^{-1}(G)$. Thus, $sg\omega\alpha$-$int(f^{-1}(G)) \subseteq f^{-1}(G)$. But $f^{-1}(G) \subseteq sg\omega\alpha$-$int(f^{-1}(G))$ is always true. Therefore $f^{-1}(G) = sg\omega\alpha$-$int(f^{-1}(G))$. This implies, $f^{-1}(G) \in sg\omega\alpha$-$O(X)$. Therefore f is $sg\omega\alpha$-continuous.

4 $sg\omega\alpha$-Irresolute Maps in Topological Spaces

This section gives the concept of sg$\omega\alpha$-irresolute maps and their properties in topological spaces.

Definition 4.1. *A map $f : X \to Y$ is called $sg\omega\alpha$-irresolute if $f^{-1}(V)$ is $sg\omega\alpha$-closed set in X for every $sg\omega\alpha$-closed set V in Y.*

Theorem 4.2. *A map $f : X \to Y$ is $sg\omega\alpha$-irresolute if and only if for every $sg\omega\alpha$-open set A in Y, $f^{-1}(A)$ is $sg\omega\alpha$-open in X.*
Proof: *The proof is obvious.*

Theorem 4.3. *If $f : X \to Y$ is $sg\omega\alpha$-irresolute then for every subset A of X, $f(sg\omega\alpha\text{-}cl(A)) \subseteq cl(f(A))$.*
Proof: *If $A \subseteq X$, then $cl(f(A)) \in sg\omega\alpha\text{-}C(Y)$. As f is $sg\omega\alpha$-irresolute, $f^{-1}(cl(f(A))) \in sg\omega\alpha\text{-}C(X)$. Further $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(cl(f(A)))$. Therefore by $sg\omega\alpha$-closure operator, $sg\omega\alpha\text{-}cl(A) \subseteq f^{-1}(cl(f(A)))$. Consequently, $f(sg\omega\alpha\text{-}cl(A)) \subseteq f(f^{-1}(cl(f(A)))) \subseteq cl(f(A))$.*

Theorem 4.4. *Every $sg\omega\alpha$-irresolute map is $sg\omega\alpha$-continuous.*
Proof: *Let f be $sg\omega\alpha$-irresolute map and $V \in C(Y)$. Then $V \in sg\omega\alpha\text{-}C(Y)$. Since f is $sg\omega\alpha$-irresolute map, $f^{-1}(V) \in sg\omega\alpha\text{-}C(Y)$. Therefore f is $sg\omega\alpha$-continuous.*
The converse of the above theorem need not be true as seen from the following example.

Example 4.5. *$X = Y = \{ 1, 2, 3 \}$, $\tau = \{ X, \phi, \{ 1 \}, \{ 1, 3 \}\}$ and $\sigma = \{ Y, \phi, \{ 1, 2 \}\}$. A function $f : X \to Y$ is defined as by $f(1)=1$, $f(2)=3$ and $f(3)=2$. Then f is $sg\omega\alpha$-continuous but not $sg\omega\alpha$-irresolute, since for the set $A = \{ 1, 3 \}$ in Y, $f^{-1}(\{ 1, 3 \}) = \{ 1, 2 \}$ is not $sg\omega\alpha$-closed in X.*

Theorem 4.6. *Let $f : X \to Y$ be surjective, $sg\omega\alpha$-irresolute and a closed map. If X is $T_{sg\omega\alpha}$-space then Y is also $T_{sg\omega\alpha}$-space.*
Proof: *Let $A \in sg\omega\alpha\text{-}C(Y)$. Since f is $sg\omega\alpha$-irresolute, then $f^{-1}(A) \in sg\omega\alpha\text{-}C(X)$. As X is $T_{sg\omega\alpha}$-space, then $f^{-1}(A) \in C(X)$. Then $A = f(f^{-1}(A)) \in C(Y)$ as f is closed and injective. Hence Y is also $T_{sg\omega\alpha}$-space.*

Theorem 4.7. *Let $f : X \to Y$ is bijective, closed and $\omega\alpha$-irresolute map. Then $f^{-1} : Y \to X$ is $sg\omega\alpha$-irresolute.*
Proof: *Let $G \in sg\omega\alpha\text{-}C(X)$. Let $(f^{-1})^{-1}(G) = f(G) \subseteq U$ where $U \in \omega\alpha\text{-}O(Y)$. Then $G \subseteq f^{-1}(U)$. Since $f^{-1}(U) \in \omega\alpha\text{-}O(X)$ and $G \in sg\omega\alpha\text{-}C(X)$, then $cl(G) \subseteq f^{-1}(U)$ and hence $f(cl(G)) \subseteq U$. As f is closed and $cl(G) \in C(X)$, so $f(cl(G)) \in C(Y)$. Thus $f(cl(G)) \in sg\omega\alpha\text{-}C(Y)$. Therefore $cl(f(cl(G)))) \subseteq U$ and $cl(f(G)) \subseteq U$. Thus $f(G) \in sg\omega\alpha\text{-}C(Y)$. Hence $f^{-1} : Y \to X$ is $sg\omega\alpha$-irresolute.*

5 Quasi $sg\omega\alpha$-Open functions

In this section authors introduced the concept of quasi sg$\omega\alpha$-open functions in topological spaces and some of their characterizations.

Definition 5.1. *A function $f : X \to Y$ is said to be quasi $sg\omega\alpha$-open if the image of every $sg\omega\alpha$-open set in X is open in Y.*

Theorem 5.2. *A function $f : X \to Y$ is quasi sgωα-open if and only if $f(sg\omega\alpha\text{-}int(U)) \subset int(f(U))$, $\forall U \subset X$.*
Proof: *Let f be quasi sgωα-open. We have, $int(U) \subset U$ and $sg\omega\alpha\text{-}int(U) \in sg\omega\alpha\text{-}O(X)$. Thus, $f(sg\omega\alpha\text{-}int(U)) \subset f(U)$. As $f(sg\omega\alpha\text{-}int(U))$ is open, then $f(sg\omega\alpha\text{-}int(U)) \subset int(f(U))$.*
Conversely, assume that $U \in sg\omega\alpha\text{-}O(X)$. Then $f(U) = f(sg\omega\alpha\text{-}int(U)) \subset int(f(U))$. But always $int(f(U)) \subset f(U)$ holds. Thus, $f(U) = int(f(U))$ and hence f is quasi sgωα-open.

Lemma 5.3. *If $f : X \to Y$ is quasi sgωα-open, then $sg\omega\alpha\text{-}int(f^{-1}(U)) \subset f^{-1}(int(U))$ where $U \subset Y$.*
Proof: *Let $U \subset Y$. Then $sg\omega\alpha\text{-}int(f^{-1}(U)) \in sg\omega\alpha O(X)$. Since f is quasi sgωα-open, then $f(sg\omega\alpha\text{-}int(f^{-1}(U))) \subset int(f(f^{-1}(U))) \subset int(U)$. Hence $sg\omega\alpha\text{-}int(f^{-1}(U)) \subset f^{-1}(int(U))$.*

Theorem 5.4. *The following statements are equivalent for a function $f : X \to Y$:*
(i) f is quasi sgωα-open
(ii) $f(sg\omega\alpha\text{-}int(U)) \subset int(f(U))$, $U \subset X$
(iii) for each $x \in X$ and each sgωα-neighbourhood U of x in X, $\exists$ a neighbourhood V of $f(x)$ such that $V \subset f(U)$.
Proof: *(i) $\to$ (ii) Proof follows from Theorem 5.2.*
(ii) $\to$ (iii) Let $x \in X$ and U be an arbitrary neighbourhood of x. Then there exists a $V \in sg\omega\alpha O(X)$ with $x \in V \subseteq U$. But from (ii), $f(V) = f(sg\omega\alpha\text{-}int(V)) \subset int(f(U))$. Hence $f(V) = int(f(V))$. Thus, $f(V)$ is open in Y with $f(x) \in f(V) \subset f(U)$.
(iii) $\to$ (i) Let $U \in sg\omega\alpha O(X)$. Then for each $y \in f(U)$, there exists a neighbourhood V_y in Y with $V_y \subset f(U)$. But, as V_y is a neighbourhood of y, $\exists$ an open set $W_y \in Y$ with $y \in W_y \subset V_y$. Thus, $f(U) = \cup\{W_y : y \in f(U)\} \in O(Y)$. Hence f is quasi sgωα-open function.

Theorem 5.5. *A function $f : X \to Y$ is quasi sgωα-open if and only if for any subset $B \in Y$ and any $F \in C(X, f^{-1}(B))$, $\exists$ $G \in C(Y, B)$ such that $f^{-1}(G) \subset F$.*
Proof: *Assume that f is quasi sgωα-open. Let $B \subset Y$ where $F \in C(X, f^{-1}(B))$. Put $G = Y \setminus f(X \setminus F))$. Then $f^{-1}(B) \subset F$, that is $B \subset G$. By quasi sgωα-open function, we get $G \in C(Y)$. Then $f^{-1}(G) \subset F$.*
Conversely, let $U \in sg\omega\alpha O(X)$. Put $B = Y \setminus f(U)$. Then $X \setminus U \in sg\omega\alpha O(X, f^{-1}(B))$. From hypothesis, $\exists$ $F \in C(Y)$ with $B \subset F$ and $f^{-1}(F) \subset B \setminus U$. Thus, $f(U) \subset Y \setminus F$. On the other hand, we have $B \subset F$ and $Y \setminus F \subset Y \setminus B = f(U)$. Thus, $f(U) = Y \setminus F \in O(Y)$. Thus f is quasi sgωα-open.

Theorem 5.6. *A function $f : X \to Y$ is quasi sgωα-open if and only if $f(cl(B)) \subset sg\omega\alpha\text{-}cl(f(B))$, $\forall B \in Y$.*
Proof: *Assume that f is quasi sgωα-open. Then $\forall B \in Y$, $f-1(B) \subset sg\omega\alpha\text{-}cl(f^{-1}(B))$. From theorem 5.5, $\exists$ $F \in C(Y)$ such that $B \subset F$ and $f^{-1}(F) \subset sg\omega\alpha\text{-}cl(f^{-1}(B))$. Thus, $f^{-1}(cl(B)) \subset f^{-1}(F) \subset sg\omega\alpha\text{-}cl(f^{-1}(B))$.*
Conversely, let $B \subset Y$ where $F \in sg\omega\alpha C(X, f^{-1}(B))$. Let $W = cl_y(B)$, then $B \subset W$ where $W \in C(X)$. Hence $f^{-1}(W) \subset sg\omega\alpha\text{-}cl(f^{-1}(B)) \subset F$. From theorem 5.5, f is quasi sgωα-open.

6 Quasi $sg\omega\alpha$-Closed functions

In this section, we introduce the concept of quasi $sg\omega\alpha$-closed functions in topological spaces and studied their properties.

Definition 6.1. *A function $f : X \to Y$ is said to be quasi $sg\omega\alpha$-closed if the image of each $sg\omega\alpha$-closed set in X is closed in Y.*

Remark 6.2. *Every $sg\omega\alpha$-closed function need not be closed follows from the following example.*

Example 6.3. *Let $X = Y = \{1, 2, 3\}$, $\tau = \{X, \phi, \{1, 2, \}\}$ and $\sigma = \{Y, \phi, \{1\}, \{2, 3\}\}$. Define a function $f : X \to Y$ as $f(1) = 2$, $f(2) = 1$ and $f(3) = 3$. Then f is $sg\omega\alpha$-closed but not quasi $sg\omega\alpha$-closed. Since the $sg\omega\alpha$-closed set $\{3\}$ in X, $f(\{3\}) = \{3\}$ is not closed in Y.*

Lemma 6.4. *A function $f : X \to Y$ is quasi $sg\omega\alpha$-closed if and only if $f^{-1}(int(B)) \subset sg\omega\alpha\text{-}intf^{-1}(B))$, where $B \subset Y$.*

Theorem 6.5. *A function $f : X \to Y$ is quasi $sg\omega\alpha$-closed if and only if for any $B \subset Y$ and $G \in sg\omega\alpha O(Y, f^{-1}(B))$, $\exists\, U \in O(Y, B)$ such that $f^{-1}(U) \subset G$.*
Proof: *Similar to theorem 5.5.*

Theorem 6.6. *Let $f : X \to Y$ and $g : Y \to Z$ be any two functions with $g \circ f : X \to Z$ is quasi $sg\omega\alpha$-closed. Then g is closed if f is $sg\omega\alpha$-irresolute with surjective function.*
Proof: *Let $F \in sg\omega\alpha C(Y)$. As f is $sg\omega\alpha$-irresolute, $f^{-1}(F) \in sg\omega\alpha C(X)$. As $g \circ f$ is quasi $sg\omega\alpha$-closed and f is surjective, $(g \circ f)(f^{-1}(F))) = g(F)$ is closed in Z. Thus g is closed function.*

Definition 6.7. *A function $f : X \to Y$ is called $sg\omega\alpha^*$-closed if the image of every $sg\omega\alpha$-closed subset of X is $sg\omega\alpha$-closed in Y.*

Theorem 6.8. *Let $f : X \to Y$ and $g : Y \to Z$ be any two functions. Then*
(i) $g \circ f$ is $sg\omega\alpha^*$-closed if f is quasi $sg\omega\alpha$-closed and g is $sg\omega\alpha$-closed.
(ii) $g \circ f$ is quasi $sg\omega\alpha$-closed if f is $sg\omega\alpha^*$-closed and g is quasi $sg\omega\alpha$-closed.
Proof: *Proofs are obvious.*

Theorem 6.9. *Let X and Y be topological spaces. Then $f : X \to Y$ is quasi $sg\omega\alpha$-closed if and only if $g(X)$ is closed in Y and $g(V) \setminus g(X \setminus V)$ is open in $g(X)$ whenever $V \in sg\omega\alpha\text{-}O(X)$.*
Proof: *Suppose $g : X \to Y$ is quasi $sg\omega\alpha$-closed. As X is $sg\omega\alpha$-closed, $g(X)$ is closed in Y and $g(V) \setminus g(X \setminus V) = g(X) \setminus g(X \setminus V)$ is open in $g(X)$, where $V \in sg\omega\alpha\text{-}O(X)$. Suppose, $g(X)$ is closed in Y, then $g(V) \setminus g(X \setminus V)$ is open in $g(X)$, where $V \in sg\omega\alpha\text{-}O(X)$. Let $C \in C(X)$. Then $g(C) = g(X) \setminus (g(X \setminus C) \setminus g(C))$ is closed in $g(X)$. Hence $g(C)$ is closed in Y.*

Corollary 6.10. *Let X and Y be any two topological spaces. A surjective function $g : X \to Y$ is quasi $sg\omega\alpha$-closed if and only if $g(V) \setminus g(X \setminus V) \in O(Y)$ where $U \in sg\omega\alpha\text{-}O(X)$.*

Definition 6.11. *A topological space (X, τ) is said to be $sg\omega\alpha$-normal if for any $A, B \in sg\omega\alpha\text{-}C(X)$ with $A \cap B = \phi$, $\exists\, U, V \in O(X)$ with $U \cap V = \phi$ such that $A \subset U$ and $B \subset V$.*

Theorem 6.12. *Let X and Y be any two topological spaces with X is sgωα-normal and $g : X \to Y$ be sgωα-continuous, quasi sgωα-closed surjective function. Then Y is normal.*
Proof: *Let $A, B \in C(Y)$ with $A \cap B = \phi$. Then $g^{-1}(A), g^{-1}(B) \in sg\omega\alpha\text{-}C(X)$. As X is normal, then $\exists\ U, V \in O(X)$ such that $g^{-1}(A) \subset U$ and $g^{-1}(B) \subset V$. Then $A \subset g(U) \setminus g(X \setminus U)$ and $B \subset g(V) \setminus g(X \setminus V)$. From corollary 6.10, $(g(U) \setminus g(X \setminus U)), (g(V) \setminus g(X \setminus V)) \in O(Y)$. Then $(g(U) \setminus g(X \setminus U)) \cap (g(V) \setminus g(X \setminus V)) = \phi$. Thus Y is normal.*

7 $sg\omega\alpha$-Closed Graphs in Topological Spaces

Definition 7.1. *A topological space X is said to be*
(i) $sg\omega\alpha$-T_1 space if for each pair of distinct points x and y of X, there exist disjoint $sg\omega\alpha$-open sets U containing x but not y and V containing y but not x.
(ii) $sg\omega\alpha$-T_2 space if for each pair of distinct points x and y of X, there exist disjoint $sg\omega\alpha$-open sets U and V such that $x \in U$ and $y \in V$.

Definition 7.2. *A function $f : X \to Y$ has sgωα-closed graph if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in sg\omega\alpha O(X, x)$ and $V \in O(Y, y)$ such that $(U \times cl(V)) \cap G(f) = \phi$.*

Theorem 7.3. *Let $f : X \to Y$ be a function. Then the following properties are equivalent.*
(i) f is sgωα-closed graph.
(ii) for each $(x, y) \in (X \times Y) \setminus G(f)$, there $\exists U \in sg\omega\alpha O(X, x)$ and $V \in O(Y, y)$ with $f(U) \cap cl(V) = \phi$.
(iii) for each $(x, y) \in (X \times Y) \setminus G(f)$, there $\exists U \in sg\omega\alpha O(X, x)$ and $V \in sg\omega\alpha O(Y, y)$ with $(U \times sg\omega\alpha\text{-}cl(V)) \cap G(f) = \phi$.
(iv) for each $(x, y) \in (X \times Y) \setminus G(f)$, there $\exists U \in sg\omega\alpha O(X, x)$ and $V \in sg\omega\alpha O(Y, y)$ with $f(U) \cap sg\omega\alpha\text{-}cl(V) = \phi$.
Proof: *(i) $\Rightarrow$ (ii): Suppose (i) holds. Let f be a sgωα-closed graph. Then $(x, y) \in (X \times Y) \setminus G(f)$, so $\exists U \in sg\omega\alpha O(X, x)$ and $V \in O(Y, y)$ with $(U \times cl(V)) \cap G(f) = \phi$. Thus, $\forall\ x \in X, \in sg\omega\alpha\text{-}O(X, x)$, that is $f(x) \neq y$. Hence $f(U) \cap cl(V) = \phi$. Thus (b) holds.*
(ii) $\to$ (i): Suppose (ii) holds. Then $(x, y) \in (X \times Y) \setminus G(f)$, $\exists U \in sg\omega\alpha O(X, x)$ and $V \in O(Y, y)$ with $f(U) \cap cl(V) = \phi$, that is $U \in sg\omega\alpha\text{-}O(X, x)$ and $f(x) \neq y$. Thus $(U \times cl(V)) \setminus G(f) = \phi$.
(i) $\to$ (iii) Suppose (i) holds. Then $(x, y) \in (X \times Y) \setminus G(f)$, $\exists U \in sg\omega\alpha O(X, x)$ and $V \in O(Y, y)$ with $(U \times cl(V)) \cap G(f) = \phi$. Since every open set is sgωα-open, $sg\omega\alpha\text{-}cl(V) \subseteq cl(V)$. Therefore $(U \times sg\omega\alpha\text{-}cl(V)) \cap G(f) = \phi$. Thus, (iii) holds.
(ii) $\to$ (iv): Suppose (ii) holds, that is $(x, y) \in (X \times Y) \setminus G(f)$. Then $\exists\ U \in sg\omega\alpha O(X, x)$ and $V \in O(Y, y)$ with $f(U) \cap cl(V) = \phi$. Since every open set is sgωα-open, $sg\omega\alpha\text{-}cl(V) \subseteq cl(V)$. So $f(U) \cap sg\omega\alpha\text{-}cl(V) = \phi$. Thus (iv) holds.
(i) $\to$ (iv): It follows from (ii).

Theorem 7.4. *Let $f : X \to Y$ is surjective sgωα-closed graph. Then Y is a T_1.*
Proof: *Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Let $x_0 \in X$. Then $f(x_0) = y_2$ by surjectiveness of f. Therefore $(x_0, y_1) \in (X \times Y) \setminus G(f)$. Then $\exists\ U_1 \in sg\omega\alpha O(X, x_0)$ and $V_1 \in O(Y, y_1)$ with $f(U_1) \cap cl(V_1) = \phi$ by sgωα-closed graphs. Since $x_0 \in U_1$ and $f(x_0) = y_1 \in$*

$f(U_1)$ and $f(U_1) \cap cl(V_1) = \phi$. Thus $y_2 \notin V_1$.
Let $x_1 \in X$. Since f is surjective $f(x_1) = y_1$. Thus $(x_1, y_2) \in (X \times Y) \setminus G(f)$. Then $\exists$ $U_2 \in sg\omega\alpha O(X, x_1)$ and $V_2 \in O(Y, y_2)$ such that $f(U_2) \cap cl(V_2) = \phi$. Since $x_1 \in U_2$ and $f(x_1) = y_2 \in f(U_2)$ and $f(U_2) \cap cl(V_2) = \phi$. Thus $y_1 \notin V_2$. Therefore for each y_1, $y_2 \in Y$, $\exists$ $V_1, V_2 \in O(Y)$ with $y_1 \in V_1$, $y_2 \notin V_1$ and $y_1 \notin V_2$, $y_2 \in V_2$. Hence Y is T_1 space.

Corollary 7.5. *Let $f : X \to Y$ is surjective $sg\omega\alpha$-closed graph. Then Y is $sg\omega\alpha$-T_1 space.*

Theorem 7.6. *Let $f : X \to Y$ is injective $sg\omega\alpha$-closed graph. Then X is $sg\omega\alpha$-T_1 space.*

Proof: Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Since f is injective, $f(x_1) \neq f(x_2)$, implies $(x_1, f(x_2)) \in (X \times Y) \setminus G(f)$. Then exists $U_1 \in sg\omega\alpha O(X, x_1)$ and $V_1 \in O(Y, f(x_2))$ with $f(U_1) \cap cl(V_1) = \phi$ by $sg\omega\alpha$-closed graphs. As $x_1 \in U_1$, then $f(x_1) \in f(U_1)$. So $f(x_2) \notin f(U_1)$ and $x_2 \notin U_1$.
Let $(x_2, f(x_1)) \in (X \times Y) \setminus G(f)$. Then $\exists$ $U_2 \in sg\omega\alpha O(X, x_2)$ and $V_2 \in O(Y, f(x_1))$ with $f(U_2) \cap cl(V_2) = \phi$. As $x_2 \in U_2$, then $f(x_2) \in f(U_2)$, so $f(x_1) \notin f(U_2)$ and $x_1 \notin U_2$. Therefore $\forall$ $x_1, x_2 \in X$, $\exists$ $U_1, U_2 \in sg\omega\alpha$-$O(X)$ such that $x_1 \in U_1$, $x_2 \notin U_1$ and $x_1 \notin U_2$, $x_2 \in U_2$. Hence X is $sg\omega\alpha$-T_1 space.

Corollary 7.7. *Let $f : X \to Y$ be bijective with $sg\omega\alpha$-closed graph. Then both X and Y are $sg\omega\alpha$-T_1 space.*

Theorem 7.8. *Let $f : X \to Y$ be surjective $sg\omega\alpha$-closed graph. Then Y is T_2-space.*
Proof: Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since f is surjective, $\forall$ $x_1 \in X$, $f(x_1) = y_1$. Now $(x_1, y_2) \in (X \times Y) \setminus G(f)$. Since f is $sg\omega\alpha$-closed graph, $\exists$ $U \in sg\omega\alpha O(X, x_1)$, $V \in O(Y, y_2)$ with $f(U) \cap cl(V) = \phi$. Now $x_1 \in U$, then $f(x_1) = y_1 \in f(U)$. So $y_1 \neq cl(V)$ as $f(U) \cap cl(V) = \phi$. Therefore, $\exists$ $W \in O(Y, y_1)$ with $W \cap V = \phi$. Hence, Y is T_2 space.
Corollary 7.9. *Let $f : X \to Y$ be surjective $sg\omega\alpha$-closed graph. Then Y is $sg\omega\alpha$-T_2 space.*

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