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Review Paper

The Ramsey number of cycles

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Abstract
This investigates problems in a number of different areas of graph theory. These problems are related in the sense that they mostly concern the coloring or structure of the underlying graph.
The first problem we consider is in Ramsey Theory, a branch of graph theory stemming from the eponymous theorem which, in its simplest form, states that any sufficiently. Large graph will contain a clique or anti-clique of a specified size. The problem of finding the minimum size of underlying graph which will guarantee such a clique or anti-clique is an interesting problem in its own right, which has received much interest over the last eighty years but which is notoriously intractable. We consider a generalization of this problem. Rather than edges being present or not present in the underlying graph, each is assigned one of three possible colors and, rather than considering cliques, we consider cycles. Combining regularity and stability methods, we prove an exact result for a triple of long cycles.
Key Words: *Lower Bounds, cycles, Regularity Lemma*

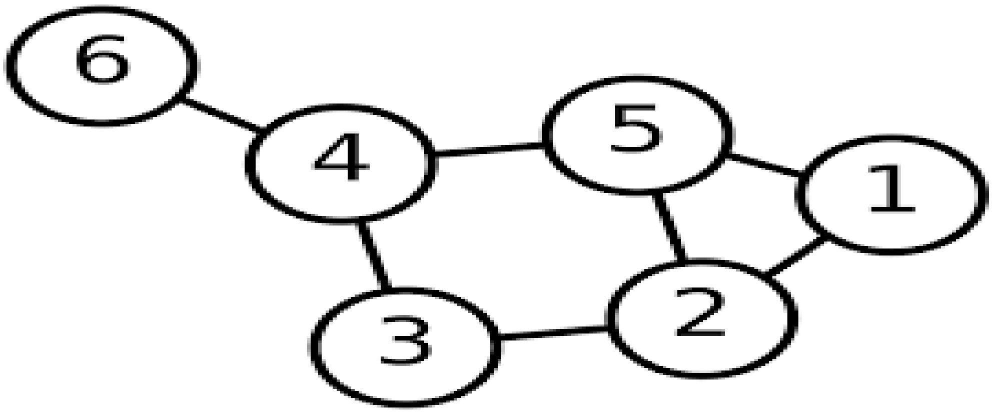
I. Introduction

Recall that, for graphs G1, G2, G3, the Ramsey number R (G1, G2, G3) is the smallest integer N such that every edge-coloring of the complete graph on N vertices with up to three colors, results in the graph having, as a sub graph. a copy of G1 colored with color *i* for some *i* .We will consider the case when G1, G2 and G3 are cycles.

In 1973, Bondy and Erdős [BE73] conjectured that, if n> 3 is odd, then R(Cn. Cn. Cn)= 4***n*** - 3.

Scope and Notation

There is vast amount of literature on Ramsey type problem starting in 1930 with the original paper of Ramsey [Ram]. Graham, Rothschild and Spencer in their “Ramsey Theory" book [GRS], and Stiffer in the 2009 "The Mathematical Coloring Book Mathematic of coloring and the colorful life of its creators" [Soi] present exciting developments in the history, results and people of Ramsey theory. The subject has grown amazingly; in particular with regard to asymptotic bounds for various types of Ramsey numbers (for example see the survey papers [GrRö. Neš, ChGra2]), but the progress on evaluating the basic numbers them- selves has been very unsatisfactory for a long time.



Ramsey Theory studies the conditions of when a combinatorial object necessarily contains some smaller given objects. The role of Ramsey numbers is to quantify some of the general existential theorems in Ramsey Theory. In the case of the so called generalized graph Ramsey numbers one studies partition of the edges of the complete graph, under the Condition that each of the parts avoids some prespecified arbitrary graph, in contrast to classical Ramsey numbers when the avoided graphs are complete.

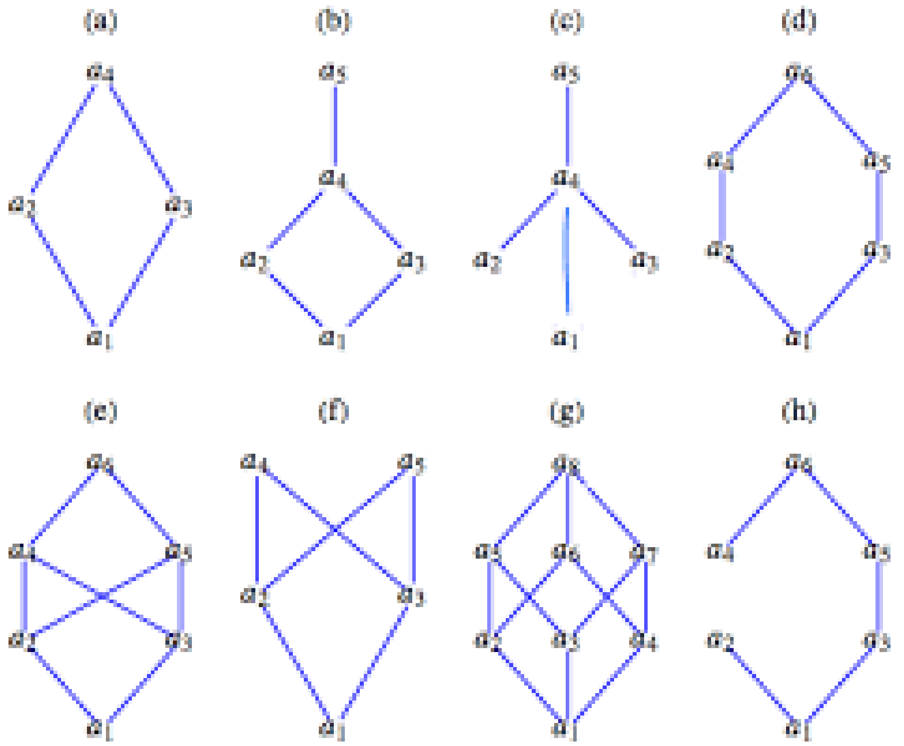
Lower Bounds

Lower and upper bound theory is a mathematical concept that involves finding the smallest and largest possible values for a quantity, given certain constraints or conditions. It is often used in optimization problems, where the goal is to find the maximum or minimum value of a function subject to certain constraints.

A value that is less than or equal to every element of a set of data.

Example: In {3,5,11,20,22} 3 is a lower bound.

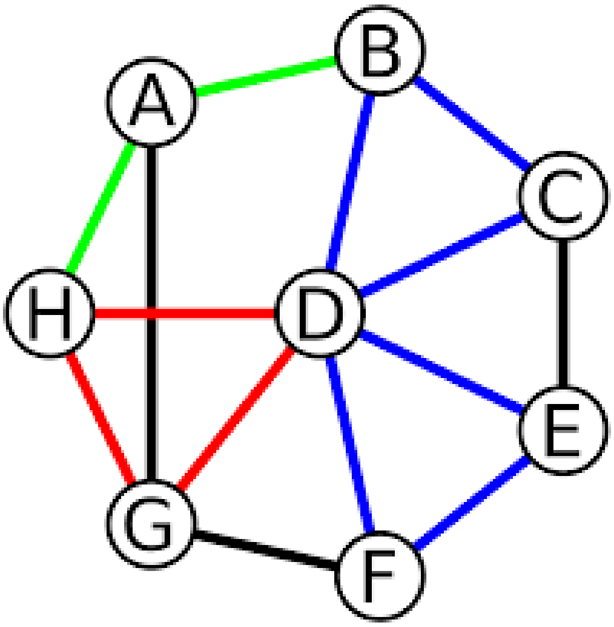
But be careful! 2 is also a lower bound (it is less than any element of that set), in fact any value 3 or less is a lower bound



In mathematics, particularly in order theory, an upper bound or majorant of a subset S of some preordered set (K, ≤) is an element of K that is greater than or equal to every element of S. Dually, a lower bound or minorant of S is defined to be an element of K that is less than or equal to every element of S.

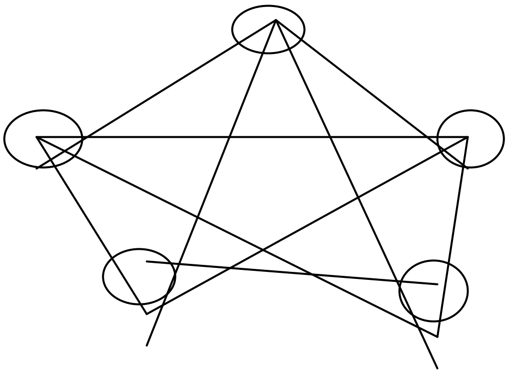
Cycles

Arguably the most widely known classical Ramsey number R (3,3) was mentioned implicitly by Bush [Bush] who was reporting that in the 1953 William Lowell Putnam Mathematical Competition question #2 in Part I asks for the proof of what can be denoted by R(3,3) ≤ 6. The 1955 paper by Greenwood and Gleason [GG] includes the result R(3,3)= 6 with proofs. This is also the first reported case of cycle Ramsey numbers R(C3, C3), since clearly C, K, Chvátal and Harary [CHI] were the first to give the value R(C4, C4)=6. The initial general result for cycles, R(CC)=2n-1 for n 24, was obtained by Chart and Schuster [ChaS] in 1971. The complete solution of the case R(CC) was obtained soon afterwards, independently by Faudree and Schelp [FSI) and Rosta [Ros1]. Both these proofs are somewhat complicated, however a new simpler proof by Károlyi and Rosta was published recently [KáRos], in 2001.



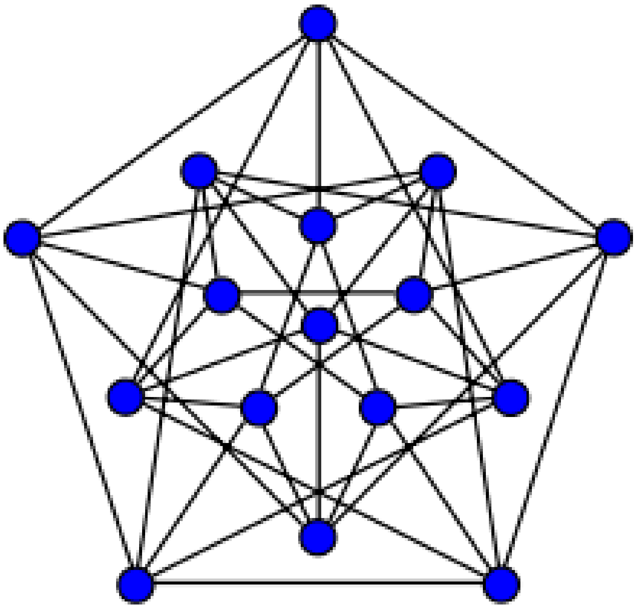
Regularity lemma

It is one of the most powerful tools in extremal graph theory, particularly in the study of large dense graphs. It states that the vertices of every large enough graph can be partitioned into a bounded number of parts so that the edges between different parts behave almost randomly.



The edges between parts behave in a "random- like" fashion.

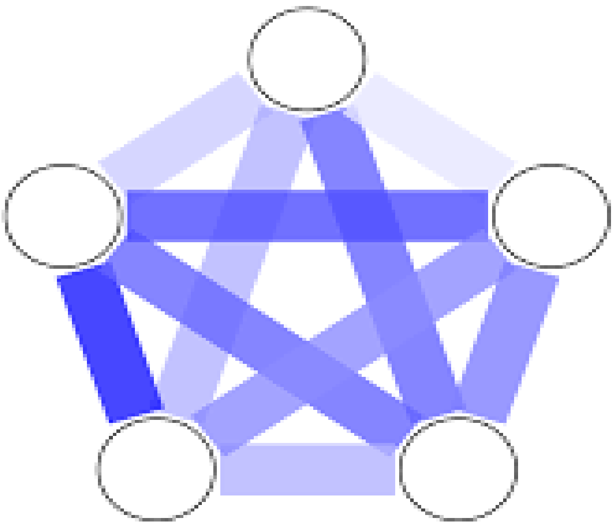
According to the lemma, no matter how large a graph is, we can approximate it with the edge densities between a bounded number of parts. Between any two parts, the distribution of edges will be pseudorandom as per the edge density. These approximations provide essentially correct values for various properties of the graph, such as the number of embedded copies of a given subgraph or the number of edge deletions required to remove all copies of some subgraph.



The regularity lemma roughly states that every graph may be approximated by a union of induced random-like (quasi-random) bipartite sub graphs. Since the quasi-randomness brings important additional information, the regularity lemma proved to be a useful tool.

In the proof of the regularity lemma, we will repeat the process of splitting the graph into smaller and smaller subsets until we obtain the desired partition; |V0| can be thought of as a bin that contains all vertices that are left over from the splitting operations.

In graph theory, a regular graph is a graph where each vertex has the same number of neighbors; i.e. every vertex has the same degree or valency. A regular directed graph must also satisfy the stronger condition that the indegree and outdegree of each internal vertex are equal to each other.



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