Quest Journals
Journal of Research in Applied Mathematics
Volume 7 ~ Issue 8 (2021) pp: 11-16
ISSN(Online) : 2394-0743 ISSN (Print):2394-0735
www.questjournals.org

Research Paper

Problem Solving: A Didactic Approach to 2nd Grade of Primary School

Valakos Ververis[1], Roza Vlachou[1]
[1](*Department of Primary Education,*
Mathematics Education and Multimedia Laboratory,Universityof the Aegean, Greece)
Corresponding Author: *Vlachou Roza*

ABSTRACT:*In this paper, it is presented a didactic approach to the mathematical problems of the 2nd grade of primary school will be attempted, using dynamic mathematical tools of the theory of RhodeScript such as open-ended problem, realistic mathematics, problem posing, counterexample, breach of didactical contract, in order to improving students' performance in problem solving. In more detail, this work has been prepared in order to inform about the application of mathematical tools and their use in teaching practice and specifically in solving mathematical problems. In addition, the individual goals that are attempted to be fulfilled through this elaboration are to make the trainees and the teachers realize that learning the steps of solving a mathematical problem can be achieved through fun, experiential and interdisciplinary activities that will make them students to solve math problems in the most constructive and profitable way.*
KEYWORDS: *Mathematical key tools, problem solving, 2nd grade of primary school*

Received 28 July, 2021; Revised: 10 August, 2021; Accepted 12 August, 2021

I. INTRODUCTION

International assessment programs indicate poor performance in mathematics of students of many countries worldwide with different educational systems, including Greece. Therefore, many researchers worldwide have addressed students' difficulties in mathematics and have proposed methods to diminish them. Specifically, researches findings indicate the critical aspects of the way of teaching mathematics[5], [9], [16], [17], [26], the use of representations of mathematical concepts [6], [21], [25], [27] and teacher' and prospective teachers' attitudes and beliefs towards mathematics [1], [20], [23], [24].

In view of the above, this paper presents a didactic approach to problem solving for the 2nd Grade of primary school, using the theory of RhodeScript, in order to improve students' performance in problem solving, as they have been researched and applied by [2].

II. THEORETICAL BACKGROUND

2.1 The mathematical tools of RhodeScript

The didactic approach to problem solving in the 2nd grade of primary school presented in this paper is based on the theory of RhodeScript, as it has been researched and applied by [2], which uses eleven prototype instructional practices over time implemented by the [2] and its positive effect on mathematical understanding and students' self-confidence in mathematics. In detail, these elements are 1. Realistic Mathematics Education, 2.History of Mathematics, 3.Open-ended and Unusual Problems, 4.Breach of Didactic Contract, 5.Examples, Counterexamples and Non-examples, 6.Spatial Ability and Geometric Transformations, 7.Mental Computations and Estimations, 8.Representations, 9.Interdisciplinary, 10. Problem Posing plus the use of Technology combined with the above practices. The didactical framework, this theory, has been named RhodeScript from the initial letters of the eleven elements.

In this paper using some of these mathematical tools. In particular, the open problem is a mathematical problem that can arise from more than one correct solution [3], [13]. Practicing on this mathematical tool brings about the innovation of the cooperative spirit, the multidimensional thinking and vision, since the mathematical problem cannot be solved by a single correct solution and is more "free" to process, and of pluralism. Another mathematical tool that will be used is the counterexample. A counter-example is an example that shows that a

given hypothesis is not valid and a single counter-example is enough to refute a conjecture [11]. The role of counter-examples has been recognized and discussed in creating cognitive conflict [10] that occurs when a student is confronted with a contradiction or inconsistency in his ideas. Representations are used as a third mathematical tool. There are two types of representations, external and internal [12]. In the present work, we will be concerned only with external representations (symbols, shapes, diagrams, texts), which relate to visualization of mathematical concepts in real time, in order to be understood and more accessible by students.

The breach of didactical contract is an important mathematical tool, as school mathematical problems have many common in terms of presentation and in terms of resolution [4] and students must be equipped with all those necessary knowledge and skills,so that they can solve a mathematical problem that is completely different from the common books. Therefore, subversive and critical thinking on mathematical problems and their multidimensional perspective by students is the key to rupture of the teaching policy. The problem posing is another mathematical tool and lies in the fact that students will be exercised to create their own mathematical problems for the purpose of multilateral and deeper analysis and understanding of these cognitive regions [22].

Another mathematical tool that is being used is interdisciplinarity, as the coupling of one or more sciences is particularly beneficial during the educational process. In addition, realistic mathematics is another mathematical tool of this paper. Realistic mathematics is a philosophical teaching and learning by[7], [8], thinking that mathematics is a human activity that must be associated with reality, to be influential and to society and to be as much as possible more understood to students. The last mathematical tool which used in this paper is creativity. With the sharpening of creativity, it is possible to develop the skill of finding a variety of modes of mathematical resolution and more general problems from the Pupils and Students [19].

2.2The 4 stages of solving mathematical problem according to Polya

In the present paper, beyond the RhodeScriptwe used also the four stages of solving a mathematical problem, according to Polya[15].

Step 1: Understand the problem.Understanding the problem. At this stage, we realize that the child has fully understood the mathematical problem given to him, when he can interpret it in his own words. The maximum form of understanding occurs when the child can tell it to make it a third person.

Step 2: Devise a plan. The student is visualizing and identifying the data of the problem so that all the necessary elements that will lead to his solution are concentrated.

Step 3: Carry out the plan. At this stage, the draft resolution plan created by the student at an earlier phase is carried out, paying attention every time to control their steps and correctness.

Step 4: Retrospective investigation: As last stage, the student has the opportunity to perform the following actions. Initially, he must carry out the control of his answers. That is, the student is called upon to verify his answers in order to ascertain but also to ascertain that they are right. Then he can make a generalization of his answers. This means that you have to go to practice and theory that applied to an extra question or exercise. Then we have the extension. Here, the student is called upon to solve similar or even more complex problems in order to extend its possibilities and knowledge. Finally, the student can proceed with the investigation of other elegant and different solutions but also to the production of new knowledge [2].

III. THE TEACHING PROPOSAL USING DYNAMIC MATHEMATICAL TOOLS

This chapter presents the teaching proposal,duringfour teaching hours, for to problem solving in the 2nd grade of primary school using mathematical tools, as they have been researched and applied by [2] in order to improve students' performance in problem solving. The teaching proposal is structured in the following sevenphases: Check, motivation, discovery, consolidation, extension, evaluation, organization of new knowledge.

3.1 Check (prior knowledge)

Activity 1:The elicitation

Dynamic mathematical tools: Counterexample, representations.

Description: The teacher distributes a piece of paper to each child (Figure 1), who writes on it in the wrong order the 4 steps we follow to solve a mathematical problem (Understand the problem,Devise a plan, , Carry out the plan and Retrospective investigation). Students are asked to put these steps in the correct order they believe. The aim for the teacher is to understand the students' initial perceptions.

Carry out the plan	Understand the problem	Devise a plan	Retrospective investigation

Figure 1: Thepiece of paperwhich teacher gives the students

3.2 Motivation

Activity 2: Socio-cognitive conflict

Dynamic mathematical tools: Representations.

Description: The present activity is divided into 4 phases. More specifically, in the 1st phase the students are divided into groups of four people with one leader from each group. Each one individually represents the 4 correct stages of solving a problem. In the 2nd phase, the group leader gathers all the representations of the steps given by his classmates and himself and hangs them on the board (with the help of the teacher), so that they are visible to the whole class. In the 3rd phase, after all the representations on the board have been hung, the students are asked to have a dialogue with each other and with the coordination and guidance of the teacher to decide which of all these answers is closer to the scientific knowledge. Finally, students, after a process of sociological conflict, are expected to have revised their initial perceptions and to have understood the scientific point of view.

3.3 Discovery

Activity 3: Actor means mathematician

Dynamic mathematical tools: Realistic mathematics, creativity, interdisciplinary.

Description: The students work together and they are asked by teacher to dramatize the following mathematical problem as well as the steps to solve it. "Pavlos wants to play football but he has 3 balls where one is football, the other basketball and the other volleyball. How will he choose the right ball?". One of the students will be the actor of the problem, who will represent the following 4 steps, in order to solve his problem and get the right ball, while the other students will sit at their desks and together with the teacher will give instructions to the actor in order to properly solve the problem. The actor starts to order to finding the 3 balls and he wonders if they all do it for his purpose. The following dramatized problem may alternate with a different protagonist each time. The role of the teacher is guiding and helpful:

Step 1: Understand the problem
- (Teacher): What must Paul do first to solve his problem?
- (Students): Understand the problem he is facing!
- (Paul): He tells his problem to a supposed friend so that he can understand it too.

Step 2: Devise a plan
- (Teacher): After Paul understood his problem, what should he do now?
- (Students): To make a plan to solve his problem.
- (Teacher): To help Paul devise a solution plan, we will make a table of all the data he has.

Step 3: Carry out the plan
- (Teacher): And now what is left for Paul to do?
- (Students): He must execute her solution plan.
- (Pavlos): Pavlos clarifies the data he has with the help of the table and now he will find the right soccer ball by taking it and leaving the rest behind.

Step 4: Retrospective investigation
- (Teacher): And now that Paul is done with his problem, what is left for him to do?
- (Students): He should do a retrospective investigation. Therefore, it should check and verify its result and a generalization.
- (Pavlos): He checks to see if the ball he got was really the right one and after verifying it, he generalizes his problem by saying that the same problem could be faced by a classmate who needed a red tomato while he had three red ones but the two of them were rotten.

3.4Consolidation

Activity 4: A decorated staircase

Dynamic mathematical tools: Interdisciplinary, representations, creativity.

Description: In this activity the children will write in large letters on colorful cardboard the 4 steps which they follow to solve a mathematical problem. They will stick each step on each of the stairs leading to their classroom with the first step of solving a problem sticking to the first step of the ladder. Thus, every time the children go up the stairs to their classroom, they will remember the specific steps and they will follow them in the most realistic way.

Activity 5: Solidarity cycle
Dynamic mathematical tools: Problem posing.
Description: In this activity the students will leave their desks and go out in the school yard together with the teacher. Then they will all sit down together forming a large circle. Starting with a student and then clockwise, this student will narrate his problem, which he has constructed himself, and the rest of his classmates will give him advice on each step to be taken. All students will have a notebook with them to keep the appropriate notes they need.

3.5 Discovery
Activity 6: Save the environment
Dynamic mathematical tools: Open-ended problem, realistic mathematics, breach of didactical contract, interdisciplinary.
Description: In this activity, students clean their school yard. In particular, the teacher will suggest to students to work collectively to clean their school yard by applying the steps of solving a problem. More detail:
1) Students will first tell the problem they will face individually to their classmates for better consolidation of the problem.
2) Before they even leave the classroom, they will make a problem-solving plan where other students will choose to pick up the rubbish with a broom or with gloves or any other way they can think of in order to understand how the problem is solved with different ways. Also, the solution to the problem will not be just one as each student will not pick up the same amount of rubbish. The teacher at this point will emphasize how the use of the data collection table can be omitted.
3) In this step, students will go out into their school yard and start picking up trash in any way they have already built. It is important that the teacher does not mention any expression such as "the one who collects the most garbage wins" because we do not want to provoke competition between students.
4) In this last step, the students will check how much rubbish they each collected individually. Here again teacher will tell the students that they do not need to follow all the steps of solving a problem they had learned before and therefore do not make any generalization of the problem.

3.6Evaluation
Activity 7: Telephone conversations
Dynamic mathematical tools: Counter-example, creativity, problem posing.
Description: The children are divided into groups of two people and they talk to each other referring to the previous activities. In particular, each groupwill talk hypothetically on the phone and ask each other questions about what were the solution steps that followed to carry out this activity. Essentially, it is an indirect evaluation process through dialogue which is performed from student to student with the teacher taking notes on what they say. The general purpose is for each student to recall the steps he followed in the specific problems so that through the narration to his interlocutor, to give him to understand what exactly he did but also to create a new problem of his own with the steps of solving it. Each group of students will be given the following indicative design of questions:
- Which activity did you like the most?
- What were the steps you followed exactly to solve the problem?
- And what exactly did you do at each step?
- Can you tell me a new problem of yours?
- Do you have to suggest me some steps to solve it?

The above short questions will be alternated each time between the two members of the group with the possibility of improvisation.

Activity 8: Student Activity Sheet
Dynamic mathematical tools: Counterexample.
Description: In the following activity, students are asked to evaluate the sentences regarding their correctness, putting R in those that are correct and F in those that are incorrect. Also, the sentences that will characterize them as wrong, will have to justify the reason why the specific sentence is wrong and prove it with a counterexample:

"Mark with R the sentences that are correct and with F the sentences that are wrong. Next, justify those sentences that are marked as incorrect and find their error by proving it with a counterexample.

1. The first step we follow to solve a mathematical problem is "Carry out the plan". R/F
2. The steps we follow to solve a mathematical problem are four. R/F

3. A problem to be solved, it is mandatory to use all its data. R/F
4. The last step we take to solve a mathematical problem is "retrospective investigation". R/F
5. The step of retrospective investigation consists of the control and the conclusion. R/F
6. I can solve a math problem even when I do not understand it. R/F

3.7Organization of new knowledge

The teacher will make a summary table of all the new knowledge on a large board which will be pasted on one of the classroom walls so that it is visible to all students (Figure 2).

The steps to solve a problem	Actions
Step 1: Understand the problem	I say the problem in my own words and I explain it to a friend so that he can understand it too.
Step 2: Devise a plan	I design a problem solving plan by constructing a summary table with all the data I have.
Step 3: Carry out the plan	I implement the solution plan that I created before in order to solve the problem.
Step 4: Retrospective investigation	I check and verify the result I found, as well as a generalization of the problem.

Figure 2: Organization of new knowledge

IV. CONCLUSIONS

International assessment programs (eg PISA) show poor student performance in mathematics in many countries around the world with different education systems, including Greece. Therefore, many researchers around the world have studied the difficulties of students in mathematics and have proposed methods to reduce them. One factor that influences student performance, according to this international research, is the way mathematics is taught.

Based on the above, in the present paper a didactic proposal was presented for the teaching of problem solving for students of the 2nd grade of elementary school using the theory of RhodeScript, as it has been researched and applied by [2], which uses eleven dynamic mathematical tools, such as the open problem, the construction-problem position, the counterexample, the rupture of the didactic contract etc. In addition to the dynamic mathematical tools, utilize all four stages of solving a mathematical problem, according to [15] in order to improve students' performance in problem solving.

Of course, the way a mathematical concept is taught, although very important, is not the only factor responsible for students' difficulty in solving a problem, as textbooks and teachers' knowledge are an important factor in students' difficulties in Mathematics. The harmonious combination of these three elements, books, teaching, teacher, should be the pillar that will support the effort to reduce students' difficulties in solving a problem.

REFERENCES

[1]. Avgerinos, E., Vlachou, R. (2013). The abilities of candidate teachers on concepts of the number line, equal parts of the unit and improper fractions. In proceedings of the 15o Pancyprian Conference on Mathematics Education and Science. Cyprus (pp. 189-201), Cyprus: Cyprian Mathematical Society (in Greek).

[2]. Avgerinos, E., Vlachou, R., & Remoundou, D. (2018). Development and implementation of a didactical framework of 10+1 elements for the reinforcement of students' mathematical ability and attitude towards mathematics: Part I. In Proceedings of International Conference on Educational Research: Confronting Contemporary Educational Challenges through Research, (pp.17-29). University of Patras: Greece.

[3]. Bouvier N. (1992). The way of the world. New York: Marlboro Press.

[4]. Brousseau, K.(1981). Internal locus of control, perceived occupational stress, and cardiovascular health. Journal of occupational behavior, 2 (2), 65-71.

[5]. Chen, X. & Li, Y. (2009). Instructional coherence in Chinese mathematics classroom—a case study of lessons on fraction division. International Journal of Science and Mathematics Education, 8(4), 711-735. doi: 10.1007/s10763-009-9182-y.

[6]. Dreher, A., &Kuntze, S. (2015). Teachers ' professional knowledge and noticing: The case of multiple representations in the mathematics classroom. Educational Studies in Mathematics, 88(1), 89-114. doi:10.1007/s10649-014-9577-8.

[7]. Freudenthal, H. (1973). Mathematics as an Educational Task. Dordrecht: Reidel Publishing Company.

[8]. Freudenthal, H. (1983). Didactical Phenomenology of Mathematical Structures. Dordrecht: Reidel Publishing Company.

[9]. Howe, C., Luthman, S., Ruthven, K., Mercer, N., Hofmann, R., Ilie, S., & Guardia, P. (2015). Rational number and proportional reasoning in early secondary school: towards principled improvement in mathematics. Research in Mathematics Education, 17(1), 38-56. doi:10.1080/14794802.2015.1019914.

[10]. Klymchuk, S. (2001). Counter examples and conflicts as a remedy to eliminate misconceptions and mistakes. A case study. In M. van den Heuvel-Panhuizen (Ed.), Proceedings of the 25th international conference for the psychology in mathematics education, v. 1, (p. 326). Utrecht, The Netherlands.

[11]. Klymchuk, S. (2012). Using Counter-Examples in Teaching & Learning of Calculus: Students' Attitudes and Performance. Mathematics teaching-research journal online, 5(4).

[12]. Lesh, R., Post, T., & Behr, M. (1987). Representations and Translations among Representations in Mathematics Learning and Problem Solving. Problems of Representation in the Teaching and Learning of Mathematics, 21, 33-40.
[13]. Pehkonen E. (1995). Use of open-ended problems in mathematics classroom. University of Helsinki.
[14]. PISA, (2003). Problem Solving for tomorrow's World. Organization for economic cooperation and development.
[15]. Polya, G. (1945). How to solve it, a new aspect of mathematical method.
[16]. Psaras, Ch., Simeonidis, K. & Vlachou, R. (2020). The problem posing as a key tool for the development of students' self-confidence in Mathematics, International Journal of Mathematics Trends and Technology (IJMTT), 66 (6), 1-9.
[17]. Psaras, Ch. (2020). Spatial ability and geometric transformations for the reinforcement of students' mathematical ability and attitude. International Journal of Education Humanities and Social Science, 3(3), 200-214.
[18]. Rønning, F. (2013). Making sense of fractions in different contexts. Research in Mathematics Education, 15(2), 201-202. doi:10.1080/14794802.2013.797741.
[19]. Runco M. (1993). Divergent Thinking, Creativity, and Giftedness. Gifted Child Quarterly, 37 (1).
[20]. Şahin, O., Gökkurt, B., &Soylu, Y. (2016). Examining prospective mathematics teachers' pedagogical content knowledge on fractions in terms of students' mistakes. International Journal of Mathematical Education in Science and Technology, 47(4), 531-551. doi: 10.1080/0020739X.2015.1092178.
[21]. Shahbari, A. J, & Peled, I. (2015). Resolving cognitive conflict in a realistic situation with modeling characteristics: coping with a changing reference in fractions. International Journal of Science and Mathematics Education, 13(4), 891-907.doi: 10.1007/s10763-014-9509-1.
[22]. Silver H. (1994). Social exclusion and social solidarity: Three paradigms. International Labour Review, 1(33), 5-6.
[23]. Thanheiser, E., Olanoff, D., Hillen, A., Feldman, Z., Tobias, M. J., & Welder, M. R. (2016). Reflective analysis as a tool for task redesign: The case of prospective elementary teachers solving and posing fraction comparison problems. Journal of Mathematics Teacher Education, 19(2-3), 123-148.doi: 10.1007/s10857-015-9334-7.
[24]. Whitacre, I., & Nickerson, D. S. (2016). Investigating the improvement of prospective elementary teachers' number sense in reasoning about fraction magnitude. Journal of Mathematics Teacher Education, 19(1), 57-77.doi: 10.1007/s10857-014-9295-2.
[25]. Vlachou, R., & Avgerinos, E. (2016). Visualization and understanding in mathematics education: The case of fractions. The Journal of the ISIS-The Logics of Image, (accepted).
[26]. Vlachou, R., & Avgerinos, E. (2018). Multıple representatıons and development of students' self-confidence on rational number. Experiences of Teaching with Mathematics, Sciences and Technology, 4, 567-586.
[27]. Vlachou, R., & Avgerinos, E. (2019). Current trend and studies on representations in mathematics: The case of fractions. International Journal of Mathematics Trends and Technology (IJMTT), 65(2), 54-72.